Shih, JL. et al. (Eds.) (2023). Proceedings of the 31st International Conference on Computers in Education. Asia-Pacific Society for Computers in Education

A Mathematical App for the Conceptual Understanding of Area and Perimeter

Jumela F. SARMIENTO[a,*], Debbie Marie B. VERZOSA[b], Maria Alva Q. ABERIN[a], Ma. Louise Antonette N. DE LAS PEÑAS[a], Agnes D. GARCIANO[a], Juan Carlo F. MALLARI[a], & Mark Anthony C. TOLENTINO[a]

[a]*Department of Mathematics, Ateneo de Manila University, Philippines*

[b]*Department of Mathematics and Statistics, University of Southern Mindanao, Philippines*

*jsarmiento@ateneo.edu

Abstract: This paper discusses an app that was developed to build a strong understanding of the concepts of area and perimeter in students. An important feature of the app is the three-component feature which highlights progressive learning: *Explore,* designed for the learning of the conceptual understanding of area and perimeter; *Apply*, where area and perimeter concepts are applied; and *Create* intended for constructing representations to develop higher order thinking skills. The pedagogical basis for the creation of the app, the game design elements employed in the app as well as the integration of the app in the classroom will be presented.

Keywords: Mathematical app, area and perimeter, area, perimeter

1. Introduction

There is a longstanding and general consensus worldwide (Hiebert & Carpenter, 1992; Skemp, 2006) and in the Philippines (Department of Education, 2016) that mathematics must be learned relationally; that is, as an integrated network of connections between and among mathematical concepts. Teaching area and perimeter meaningfully entails going beyond the teaching of formulas or procedures so that students could justify these formulas and re-construct them in case they forget (Lithner, 2017). Toward this end, designing instructional materials to facilitate students' learning of area and perimeter is essential.

A problem arises in emerging economies whereby many schools routinely cater to ill-prepared students, which forces teachers to resort to rote-based strategies in teaching area and perimeter. In (Bansilal, 2011), classroom discussion facilitated by a teacher in South Africa focuses largely on helping her students remember the formula rather than on allowing her students to reason about the authentic performance-based assessment mandated by their national board of education. The reason for this teachers' actions was due to her learners' non-readiness for the task, preventing the students from engaging in more meaningful mathematical activity. In Taiwan, which has historically performed well in international assessments, there are teachers who present formulas directly instead of engaging their learners to explore the geometric basis for these formulas (Huang, 2017).

The potential of technological tools for teaching area and perimeter has been explored (Hwang et al., 2020). This paper discusses the *Area and Perimeter* app aimed to provide students the opportunity to understand the conceptual underpinnings of area and perimeter formulas. This app was designed in consideration of scholarly literature and was one of the outputs of a government-funded project in the Philippines (De Las Peñas et al., 2023). It was designed in a game-like environment to address particular learning competencies in mathematics. It is compatible with and runs efficiently in a wide range of Android devices (e.g., smartphones, tablets) and Windows PC (laptops or desktops) without the internet.

2. Pedagogical Basis

The *Area and Perimeter* app was designed based on the concrete-pictorial-abstract approach developed by Jerome Bruner (1996). The idea is to carefully sequence representations that would enable learners to transition from concrete to abstract (Ding & Li, 2014). As such, area formulas (which are abstract concepts) are strongly tied to geometric shapes (which are pictorial representations). The app also consists of three levels: *Explore*, *Apply*, and *Create*. By *exploring* geometric objects, students can think about area and perimeter using particular cases. By playing the app repeatedly, they can generalize, understand the conceptual basis for the area and perimeter formulas, and *apply* them to compute the area and perimeter of a given geometric figure. Finally, students are provided an opportunity to develop higher-order thinking by *creating* geometric objects given the area or perimeter. This sequence of *Explore*, *Apply*, and *Create* allows the student to develop relational understanding instead of relying solely on procedural formulas of area and perimeter.

3. The *Area and Perimeter* App

3.1 App Description

In the *Area and Perimeter* app, students have the option to choose the following topics: *Area*, *Perimeter,* or *Area and Perimeter*. The *Area* component includes the following subtopics: *Introduction to Area*, *Rectangle/Square*, *Triangle*, *Parallelogram* or *Trapezoid*. The app allows the student to investigate area properties of rectangles, triangle, parallelogram and trapezoid. The *Perimeter* component includes *Introduction to Perimeter*, *Rectangle* or *Rectilinear Figures.* Herein, a student can learn perimeter properties of rectangle or rectilinear figures. In the *Area and Perimeter* component, students find the area and perimeter of rectilinear shapes and construct rectilinear shapes of given area and perimeter.

There are three levels in the app, to reflect the pedagogical intention to enable students to develop deep understanding of area and perimeter, beyond knowing formulas by rote.

Explore. In this level, students learn the conceptual underpinnings of both area and perimeter. Under the topic *Area* and subtopics *Introduction to Area* and *Rectangle/Square*, students are shown a rectilinear shape then they are supposed to click on each unit square within the shape (Figure 1(a)) and are asked how many squares are enclosed in the figure. This way, they connect the idea of area to the measure of the surface inside the given enclosed region.

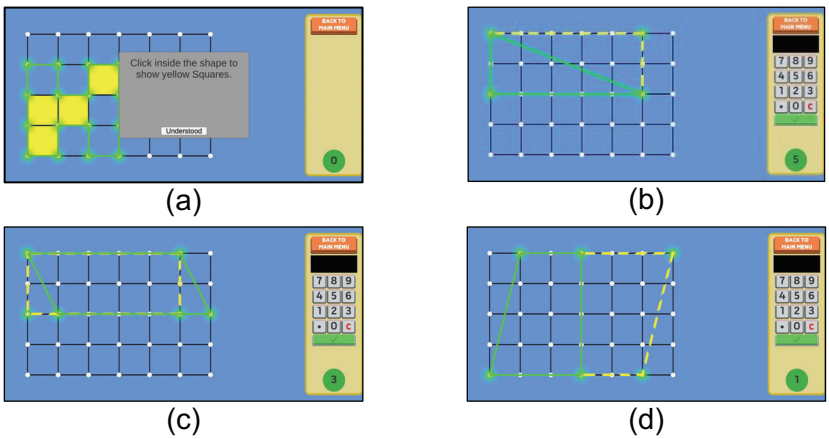


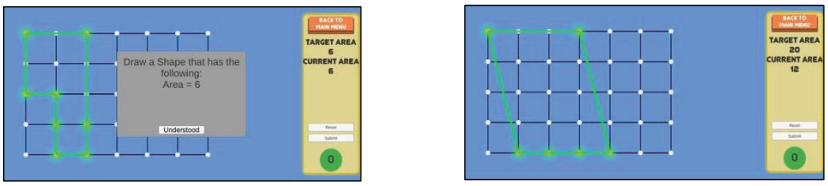
Figure 1. Screenshots of the app; Topic: *Area* and Level: *Explore.*

For the sub-topics of triangles, parallelograms, and trapezoids the aim of the *Explore* component is to allow students to learn the reasoning behind the area formulas. For example, for *Triangle*, a rectangle is first shown and then a diagonal is drawn (Figure 1(b)) to show that half of the rectangle is a triangle. This reinforces the idea that the area of a triangle is half the

area of a corresponding rectangle. For *Parallelogram*, an auxiliary line segment is shown (Figure 1(c)) to illustrate that translating the triangle at one end of the parallelogram to the other end would result in a rectangle. This reinforces the idea that the area of a parallelogram is the same as that of a rectangle with the same height and width. Finally, for *Trapezoid* (Figure 1(d)), a trapezoid (in green border) is shown and is duplicated (whole figure) and placed adjacent to the former so that the combined figure is a parallelogram. This reinforces the idea that the area of a trapezoid is half the area of a parallelogram with the same height and base. A score on the lower right corner of the screen allows the student to monitor his or her own progress and decide if he or she can choose another feature of the app.

Apply. Unlike the *Explore* level, in the *Apply* level, there are no extra steps, students are directly asked, "How many square units are in the figure?" At the *Apply* level, students use the ideas in the *Explore* component to determine the area of given geometric figures.

Create. In this level, students construct representations themselves, enabling them to develop higher-order thinking skills. Specifically, students are asked to draw a shape by clicking and connecting vertices to produce the required shape with a given area (Figure 2(a)). On the right side of the screen, the target area and current area are given. Thus, if a student constructs a shape with an incorrect area (Figure 2(b)), there is an instant feedback that provides an opportunity for the students to construct a new shape until they get one with the correct area.



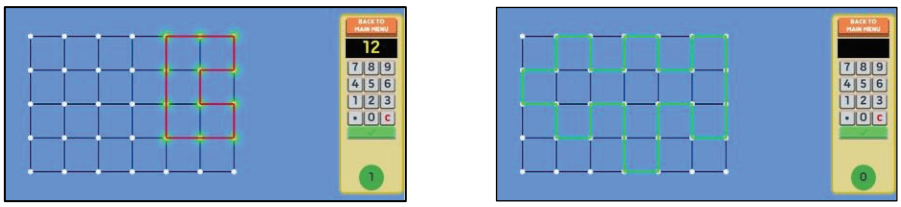
(a) (b)

Figure 2. Screenshots of the app, Topic: *Area* and Level: *Create*

Under the *Perimeter* option, the same levels (*Explore, Apply* or *Create*) are also available.

Explore. In this level, the students are asked to click on the edges of the shape and count the number of red edges such as in an example under *Rectilinear Figure* that is shown in Figure 3(a). When the correct answer is given, the app affirms that the shape has the given number as its perimeter. Through this, students connect the idea of perimeter with the number of edges around the shape.

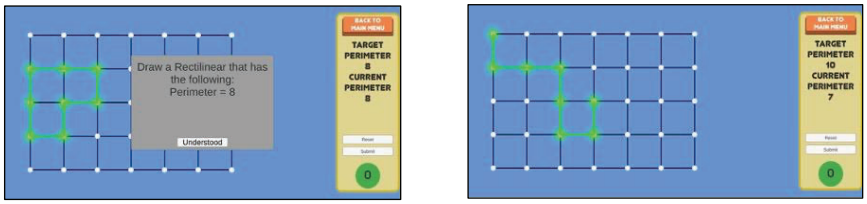
Apply. In this level, students must directly input the perimeter of the given shape (Figure 3(b)) using the ideas from the *Explore* component to be able to find the perimeter.



(a) (b)

Figure 3. Screenshots of the app, Topic: *Perimeter* and Levels: *Explore* (a), *Apply* (b)

Create. Here, students are tasked to construct a rectilinear shape with a given perimeter (Figure 4(a)). On the right side of the screen, both the target perimeter and the current perimeter are given. As the students add edges to form a shape, they will see that the current perimeter is updated, and this will guide them in constructing the correct shape having the desired target perimeter. For example, as shown in Figure 4(b), the target perimeter is 10 while the current perimeter is 7 and it is not possible to continue adding edges to construct a rectilinear shape having the target perimeter. Thus, students who meet such situations will construct new shapes until they successfully meet the target perimeter.

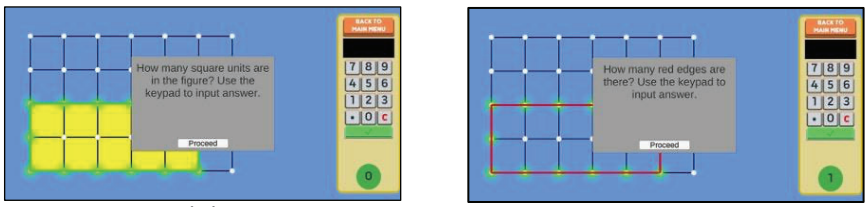


(a) (b)

Figure 4. Screenshots of the app, Topic: *Perimeter* and Level: *Create*.

When the topic *Area and Perimeter* is chosen from the Main Menu, students may perform tasks that involve both concepts, to help them attain the reasoning skills to distinguish between these two concepts (Huang, 2017).

Explore. Under the *Explore* level, students first click on each unit square within the figure to show yellow squares and then find the number of unit squares (Figure 5(a)). When the answer is correct, they now click on the edges of the shape to show red edges and then count them (Figure 5(b)). When the answer is correct, the app confirms that the area of the shape is the number of unit squares given and the perimeter is the number of red edges.

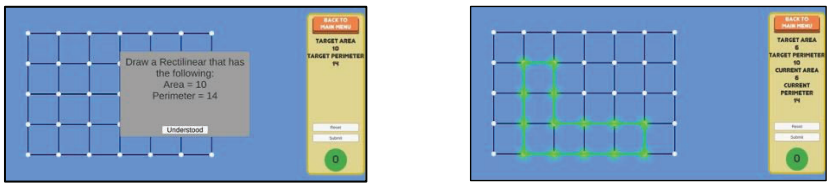


(a) (b)

Figure 5. Screenshots of the app, Topic: *Area and Perimeter* and Level: *Explore*

Apply. In this level, students directly input the area and perimeter of the given shape.

Create. In this level, students draw a rectilinear shape with the given area and perimeter (Figure 6(a)). There may be instances when students construct a rectilinear shape with the correct perimeter but incorrect area or correct area but incorrect perimeter (Figure 6(b)). The current area and current perimeter given on the right side of the screen provide the immediate feedback that will enable the students to construct new shapes until they successfully meet the target area and perimeter.



(a) (b)

Figure 6. Screenshots of the App, Option: *Area and Perimeter* and Difficulty Level: *Create.*

3.2 Game Design Factors

Creating an innovative instructional app in a game-like environment such as *Area and Perimeter* requires a sound framework on which its development can be based. Focusing on how the game is aimed at enabling players to make the leap from the concrete to the abstract, we used Shi and Shih's game-based learning (GBL) design factors (Shi and Shih, 2015) as a guide, apart from "sociality," which *Area and Perimeter* does not inherently feature as it is a single-player game.

The game goal is to develop an understanding of area and perimeter formulas. This is achieved via a combination of the abovementioned *Explore*, *Apply*, and *Create* difficulty levels, which constitute the game mechanism. The player interacts with the game by tapping squares to highlight areas, dragging between points to construct edges, and tapping a simple number

pad to provide answers to area questions. As mentioned above, players are free to select what concept (area or perimeter) and what shape to focus on; the *Explore*, *Apply*, and *Create* difficulties are also available for all these concepts from the beginning. Moreover, in the *Create* difficulty, players are tasked to construct any shape that has particular area and perimeter values, giving them autonomy and testing their creativity. Each concept, shape, and difficulty combine to form an inherently challenging level that requires the player to accomplish a task (e.g., highlight areas, tap edges).

While the *Area and Perimeter* app does not have a narrative by the strictest definition, it does feature a sense of progression through the various topics and difficulties. The game fantasy centered around an interactable grid, which is also the main medium with which the game provides sensation. The game's mystery arises from the variety of the topics and levels, as well as the randomness in the questions generated. These factors taken together give rise to the game value of *Area and Perimeter* as a learning tool for mastering abstract concepts such as area and perimeter formulas using concrete examples.

4. Integration and Use of the *Area and Perimeter* App

This section explores the use of the *Area and Perimeter* app as a pedagogical tool based on the RAT (Replacement, Amplification, Transformation) framework (Hughes et al., 2006). The app can replace the traditional approach of teaching area and perimeter without modifying the content or learning outcomes. For instance, teachers frequently ask students to calculate the areas and perimeters of hand-drawn figures of triangles, squares, rectangles and other quadrilaterals. These figures are now presented in the lattice configuration of the app. Finding the area or perimeter still must be done, but instead of doing it in a static way, geometric figures are now shown in a colorful and dynamic way. Teachers can use the app during drill and practice time.

Secondly, learning may be enhanced by the integration of the app since it can explain perimeter and area ideas with ease and efficiency. The lattice design accurately represents unit squares, which can help learners comprehend the conceptual underpinnings of area and perimeter calculations. This understanding is also possible since students engage in a variety of activities that the app can generate.

Finally, the app may be used to change instructional methods and the learning process. Teachers may conduct classroom activities that are more student-centered by using the app as a tool for group inquiry and projects that promote cooperation among students and provide a supportive learning environment. The *Create* option has the potential to improve student engagement and alter the way that students learn. Thus, teachers are encouraged to utilize this feature during class discussions and activity periods to expose students to questions about area and perimeter that aren't commonly asked.

Teachers can screencast to a smart TV or projector if students don't have their own devices. The proposed exercises can be completed by the entire class at the same time.

The app's three learning levels—Explore, Apply, and Create—can accommodate various learning demands of the students. While the levels on Explore and Apply can be utilized for remedial work, the level on Create is suitable for student enrichment activities.

5. Conclusion and Future Direction

This paper describes the features of the *Area and Perimeter* app designed to help students gain a better understanding of the notions of area and perimeter and help them make sense of the usual formulas associated with these concepts. The three levels of difficulty of the app leads students from basic exploration of the area or perimeter of polygons to application of these ideas in calculating these parameters, and finally to the creation of shapes with a given area or perimeter.

Efforts to improve numeracy and literacy among Filipino learners are even more relevant at this time considering the recognition by the Department of Education (DepEd) of

the Philippines that there have been setbacks in learning targets caused by school closures during the Covid-19 pandemic. The DepEd issued the *Adoption of the National Learning Recovery Program* (DepEd, 2023) to help address these setbacks. The *Area and Perimeter* app may be considered as a tool that can contribute to this recovery plan.

The *Area and Perimeter* app may be expanded to include the exploration of areas and perimeters of non-polygonal figures such as circles and other irregular shapes. This will further help students understand the concepts and not resort to sheer memorization of formulas. An avenue for future work would be the translation of the app into Filipino to make it more accessible to students in the Philippines. Finally, the authors plan to conduct further research to evaluate the effectiveness of the app in the learning of students.

Acknowledgements

This paper is one of the outputs of the Ateneo Mathplus Resources Laboratory housed at the Department of Mathematics, School of Science and Engineering, Ateneo de Manila University. The authors thank the Department of Science and Technology-Philippine Council for Industry, Energy, and Emerging Technology Research and Development (DOST-PCIEERD) and the University Research Council (URC), Ateneo de Manila University for the support of the development of the mathematical resources for Grades 1 to 11 Mathematics. Many thanks go to our app developers Victor Antonio M. Ortega, Mr. Nigel Benedict Cargo, Mr. Jose Teodoro Lacson, and our artist Mr. Amiel Damian F. Justiniani.

References

Bansilal, S. (2011). Assessment reform in South Africa: opening up or closing spaces for teachers? *Educational Studies in Mathematics, 78,* 91-107.

Bruner, J. (1966). *Toward a theory of instruction*. Cambridge: Belknap Press.

De Las Peñas, M. L. A. N., Aberin, M. A. Q., Garciano, A. D., Sarmiento, J. F., Tolentino, M. A. C., Verzosa, D. M. B., Mallari, J. C. (2023). *Terminal report (Department of Science and Technology-Philippine Council for Industry, Energy and Emerging Technology Research and Development): Mathematical resources for distance learning utilizing community LTE networks and television frequencies*.

Department of Education (2016). *K to 12 curriculum guide: mathematics*. www.deped.gov.ph/wp-content/uploads/2019/01/Math-CG_with-tagged-math-equipment.pdf

Department of Education. (2023). *Adoption of the National Learning Recovery Program in the Department of Education.* www.deped.gov.ph/wp-content/uploads/DO_s2023_013.pdf

Ding, M., & Li, X. (2014). Transition from concrete to abstract representations: the distributive property in a Chinese textbook series. *Educational Studies in Mathematics, 87,* 103-121.

Hiebert, J., & Carpenter, T. P. (1992). Learning and teaching with understanding. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 65-98). New York: Macmillan.

Huang, H. E. (2017). Curriculum interventions for area measurement instruction to enhance children's conceptual understanding. *International Journal of Science and Mathematics Education, 15,* 1323–1341.

Hughes, J. E., Thomas, R., & Scharber, C. (2006). Assessing technology integration: The RAT – Replacement, Amplification, and Transformation – Framework. In C. M. Crawford, R. Carlsen, K. McFerrin, J. Price, R. Weber, & D. A. Willis (Eds.), *Proceedings of SITE 2006--Society for Information Technology & Teacher Education international conference* (pp. 1616-1620). Orlando, Florida, USA: Association for the Advancement of Computing in Education (AACE).

Hwang, W., Hoang, A., & Tu, Y. (2020). Exploring authentic contexts with ubiquitous geometry to facilitate elementary school students' geometry learning. *Asia-Pacific Education Researcher, 29,* 269–283.

Lithner, J. (2017). Principles for designing mathematical tasks that enhance imitative and creative reasoning. *ZDM*, *49*, 937-949.

Shi, Y.-R., & Shih, J.-L. (2015). Game factors and game-based learning design model. *International Journal of Computer Games Technology, 2015*, 1-11.

Skemp, R. R. (2006). Relational understanding and instrumental understanding. *Mathematics Teaching in the Middle School*, *12*(2), 88–95. Originally published in *Mathematics Teaching* (1976).

Shih, JL. et al. (Eds.) (2023). Proceedings of the 31st International Conference on Computers in Education. Asia-Pacific Society for Computers in Education

A Visualization App on Proving Geometric Concepts

Ma. Louise Antonette N. DE LAS PEÑAS[a*], Debbie Marie B. VERZOSA[b], Maria Alva Q. ABERIN[a], Agnes D. GARCIANO[a], Jumela F. SARMIENTO[a], Mark Anthony C. TOLENTINO[a] & Juan Carlo F. MALLARI[a]
[a]*Department of Mathematics, Ateneo de Manila University, Philippines*
[b]*Department of Mathematics and Statistics, University of Southern Mindanao, Philippines*
*mdelaspenas@ateneo.edu

Abstract: This paper discusses the description, design and pedagogical basis of a mathematical app called *Two Column Proof*, which provides students a framework for writing proofs of geometric statements. The app focuses on proving concepts on: properties of parallelograms, conditions that determine when a quadrilateral is a parallelogram, properties on trapezoids and kites for high school mathematics. It employs visual representations for students to understand the statement or reason of each given line in the proof, strengthening their logical and mathematical knowledge needed in the proof construction.

Keywords: Mathematical app, two column proof, STEM, mathematical proofs

1. Introduction

The abilities to carry out analytical and critical thinking are necessary for students to succeed in STEM education. In the STEM fields, analytical and critical thinking skills are used to find patterns in data, determine the meaning of these patterns, and then use the data to support a claim (Adams, 2017). Mathematics educators have been recommending increasing the role of proofs in the mathematics education of students at all levels (Stylianou et al, 2009). Moreover, it has been suggested that the role of proofs be opportunities for students to observe patterns, make conjectures and provide justification, not just merely exercises in logical thinking (De Villers, 2003). Geometry is one of the subject areas where mathematical argumentation and proving is taught, developing students' deductive reasoning and critical thinking (Kunimune, Fujita, & Jones, 2010). The various geometric facts and theorems may not have much use to students in all aspects of STEM education, but the critical thinking capability developed through the rigor of mathematical proofs would certainly make them more successful problem solvers. Moreover the concept of proof is used frequently in STEM education, where students need to write reasons and justifications based on evidence in a logical manner and organized manner.

This paper discusses a mathematical application (app) called *Two Column Proof* (TCP), designed and created to support the proving of geometry concepts. Zbiek (2003) emphasizes that the design and appropriate use of technology is essential in developing mathematical intuition, understanding math concepts and proving assumptions. TCP is one of the mathematical resources developed under government-funded projects (De Las Peñas et al., 2022, 2023) with one of the primary objectives to strengthen mathematical learning competencies in students. In engagement in schools under these projects, one of the greatest challenges of teachers is the teaching of geometric proofs. Students have difficulties completing proofs. There is not much student engagement with the topic, and the students have difficulty learning as well as enjoying the learning process.

This paper presents the design and pedagogical basis for the TCP app, where the visualization of geometric representations is integrated into the app with the purpose of helping students understand the proof construction process; allowing them to have a better

understanding of the arguments to use to support statements and claims. Employing visualization technological tools stimulates interest in learning and developing cognitive interest (Andreev et al., 2021). Visualization representations and the role of visualization play an important role in geometry learning (Žakelj & Klančar, 2022). The use of visualization in proofs was found to be effective in discovery learning models (Kristiyajati & Wijaya, 2019). Visualization is a powerful tool to explore mathematical problems, give meaning to mathematical concepts and find relationships between them (Rösken & Rolka, 2006).

The TCP app was designed to run primarily on mobile devices (Android) and desktops (Windows). The Android version can run on devices with Android 5.1 until 13, 2 GB RAM, and approximately 120 MB available storage. The Windows version can run on 32- or 64-bit systems with Windows 7 or later, 1 GHz processor or higher, 4 GB RAM, and approximately 300 MB available storage. In the past two years of the pandemic, the app has been useful in support of the blended learning modality in schools called for by the Philippine Department of Education (DepEd). The app can be used with minimal supervision, and is apt for remote learning. An interested reader is invited to visit the Mathplus website at https://mathplusresources.wordpress.com/ for access to the app and other mathematical resources developed by the authors.

2. Scientific Framework and Pedagogical Basis

The design of the *Two Column Proof* app was grounded on the use of visualization and imagery in mathematics, which has drawn the attention of experts in the field of psychology of mathematics education (Presmeg, 2006). Drawing on the extensive body of work on mathematical visualization, Rivera (2011) presented a strong rationale for a visually-oriented mathematics curriculum. An important aspect of this curriculum is the use of diagrams. Pantzaria et al. (2009) found that diagrams were not universally helpful. In fact, the presence of diagrams constrained the range of strategies employed by students when solving problems. However, presenting a problem together with a diagram significantly enabled students with initial difficulties to solve problems successfully. Rivera argued that the main issue is not whether to present diagrams or not, but more importantly, how diagrams are used by the students. Drawing on the idea of diagrams as effective visual scaffolds provided that students use them meaningfully, the app presents a diagram integrated into a feedback mechanism representing a target theorem. More specific details on how the app facilitates effective use of diagrams towards the proof of a theorem are discussed in the next section.

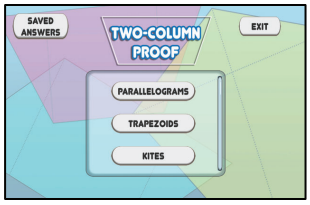
3. The *Two Column Proof* (TCP) App

3.1 App Description

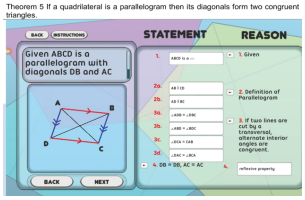
A *two column proof* is a structured way of coming up with a proof of a mathematical result, where in one column, a statement is written, and in the second column a corresponding justification and reason is given for the statement. The two-column proof remains the most common proof format in many secondary school textbooks. The first encounter of students in proving geometry concepts is usually through the two-column proof.

The *Two Column Proof* (TCP) app was designed primarily to help students in writing a geometric proof. The application is divided into three main topics: parallelograms, trapezoids and kites (Figure 1(a)). The app addresses the following learning competencies in the Philippine high school curriculum: i) “determine the properties of a parallelogram”; ii) “determine the conditions that make a quadrilateral a parallelogram”; and “prove theorems on trapezoids and kites” (Department of Education [DepEd], 2020). The first part of the parallelogram sub-menu gives statements pertaining to properties of parallelograms; the second part pertains to the converse of these statements, where a property of a quadrilateral is given and one has to prove the quadrilateral is a parallelogram. For the trapezoid and the kite sub-menus, statements on properties of trapezoids and kites are given respectively. While

writing a proof in the app, the student clicks the “Next” button to proceed to the next Statement/Reason pair. A line of the proof appears, and the student types the answer in the blank provided. To visualize a statement/phrase, the right arrow button is clicked (Figure 1(b)).



(a)



(b)

Figure 1. (a) Main menu; (b) “Statement” and Reason” columns with the Next/Back buttons

3.2 Design

The key element considered in the design of the app is *visualization*. Žakelj & Klančar (2022) defined visualization as the creation, application and reflection of diverse representations. In the process of visualization, the student creates, identifies or shapes visual representations and applies them to solve problems. In the design of TCP, geometric concepts can be visualized with constructed representations that are dynamic in nature. In particular TCP provides visual feedback for a particular line in the proof. Each line in the proof is detailed, in order to depict visually the necessary connections between each one. The visualization element is intended to guide the student in writing the mathematical statements, providing the

Table 1. *Visualization Representations in the Two Column Proof App*

Task	Visualization	Example from the app
To write mathematical statements in the “Statement” column where the justification provided in the “Reason” column is the definition of a geometric figure, such as a parallelogram, isosceles trapezoid or kite.	Relevant parts of the geometric figure are highlighted such as pairs of sides or angles. Color flashes are provided as visual hints.	In proving “The diagonal of a kite is an angle bisector of a pair of opposite angles,” in the “Reason” column the definition of kite is given. To visualize the properties that distinguish a kite from other quadrilaterals, the animation involves the first pair of sides AB and AD (repeated flashes in blue) (Figure 2(a)); followed by the second pair of sides, BC and DC (repeated flashes in red) (Figure 2(b)). Congruence of sides is hinted at by highlighting the congruence symbols.
To write mathematical statements in the “Statement” column where the justification provided in the “Reason” column is a geometric result a student has been previously taught.	An animation that will illustrate the theorem is shown.	In proving “Opposite sides of a parallelogram are congruent,” given in the “Reason” column is “If two parallel sides of a parallelogram are cut by a transversal then alternate interior angles are congruent”. The animation first shows one pair of parallel sides AD and BC (Figure 2(c)), followed by a transversal BD (Figure 2(d)) then the pair of alternate interior angles ADB, CBD (Figure 2(e)). The process is repeated for the other pair of parallel sides AB, DC, the transversal BD, then the pair of alternate interior angles ABD, CDB (Figure 2(f)).
To provide the correct triangle congruence theorem or postulate in the “Reason” column that justifies the given statement in the “Statement” column.	Animation is provided on the corresponding parts of both triangles; then the two triangles are highlighted.	For the Angle-Side-Angle (ASA) congruence used in proving a property of a parallelogram “Opposite sides are congruent,” an animation is provided in the following sequence: a pair of angles ADB then CBD from two different triangles (Figure 2(g)), an included side (Figure 2(h)), a second pair of angles ABD then CBD of the same two triangles (Figure 2(i)). Then the two triangles are highlighted, one after the other, first triangle ABD followed by triangle CDB (Figure 2(j)).

correct reasons to the given statements or understanding the logical order of the proof. We outline in Table 1 examples of visualization or visual representations used in the app.

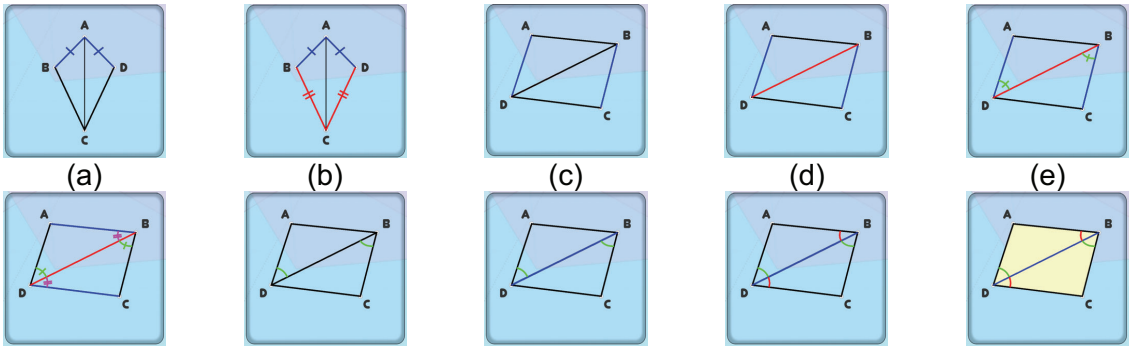


Figure 2. Animation for (a)-(b) definition of a kite; (c)-(f) "If two parallel sides of a parallelogram are cut by a transversal then alternate interior angles are congruent"; and (g)-(j) Angle-Side-Angle (ASA) congruence.

One of the features of TCP is that a student may save the answers to the lines of the proof. An answer key is provided by the app once he/she has finished drawing up a complete proof and saved the work; The student can rework the proof even after viewing the answer key and save a revised version of the proof.

4. Integration and Use of *Two Column Proof* (TCP) App

TCP can be used as a tool in teaching based on the RAT (Replacement, Amplification, Transformation) framework (Hughes, Thomas, & Scharber, 2006) where the app may replace the conventional method of teaching proofs without affecting the content or the learning objectives associated with proofs or the act of proving. It is common for teachers to write two columns on the board in the classroom to start the process of proving geometric statements. This practice is now replaced by the interface in the app. The task of completing the proof of a theorem remains the same but the static method of presenting them is now changed to the colorful and dynamic animation of geometric definitions and properties. Such features of the app can be used by the teacher as a scaffold to help students understand relationships between parts of the geometric figure.

Additionally, the app's integration can go beyond simple replacement. The app can amplify learning. The app offers a visualization and animation of angles and sides of geometric shapes that can communicate more effectively and clearly the connections between the angles and sides. The correct properties and definitions to employ as justifications in the proof are subsequently identified by the students with the help of these visual clues.

Last but not least, technology may transform teaching strategies and the learning process. The app may encourage teachers to shift from teacher-centric or teacher-controlled instruction to student-centered instruction. As student engagement is enhanced by the feedback and save-answer mechanisms, the learning process may change. To increase their exposure and practice, students can simply repeat and rework the proof. By doing this, they will be better able to comprehend the logical sequence of a proof. They can also compare kinds of proofs and proof strategies such as that of a theorem and its converse by repeatedly going through the lines of the proofs and their corresponding visual images. Also, the app can be utilized as a tool for group work in which students express their ideas and experiences while using it. This kind of activity also encourages collaboration among students and fosters a supportive learning atmosphere. Further, the app can be utilized in a face-to-face or distance learning set-up.

In terms of the advantage and disadvantage of using the Android and desktop versions of TCP: the desktop version has the advantage of letting the students type the proofs into the app with more precision and at times, more speed as the desktop device uses a mouse and

a keyboard. The animations would also be noticeably clearer as compared to portable devices with smaller screens. On the other hand, the primary advantage of using TCP in a mobile device is its portability.

In the implementation of the government funded projects (De Las Peñas et al., 2022, 2023) we observed the use of TCP in a Philippine high school in school years 2021-2022 (offsite because of the pandemic) and 2022-2023 (onsite) for selected Grade 9 classes. One of the ways the teachers use the app in class is when introducing the lesson in proving a geometric theorem or when discussing an assigned proving homework. The students found the animations from the app useful during synchronous classes via google classroom or in the classroom when the two column proof was taught for the first time. The app was also helpful for asynchronous classes that followed. The teachers were assured of a way they could check on how the students practiced proving, and they could assign several problems where the students could verify the proofs. A common feedback from the students is that the animations, when carried out repeatedly, were able to guide them in writing down sentences of the proof. Moreover, it was a big help to rely on the feedback of the app at the time when they were not with their teachers.

A study was conducted to assess the effectiveness of the app in the learning of Grade 9 students in a public high school in Quezon City, Philippines during the school year 2022-2023. Two pairs of control-experimental classes were given a pretest and a posttest on the competencies: (i) proves theorems on the different kinds of parallelograms (rectangle, rhombus, square), (ii) proves theorems on trapezoids and kites. The first pair's control class (38 students), where the app was not introduced and only traditional instruction was applied, recorded a mean gain score of 30.94%. On the other hand, the experimental class (40 students), where the app was used in synchronous lessons and in homework, got a mean gain score of 44.90%. The second pair of control and experimental classes recorded a wider gap between mean gain scores: 25.73% for the control (25 students) and 79.29% for the experimental (27 students). These results suggest the potential of the app to help students improve in the mentioned competencies.

5. Conclusion and Future Direction

The *Two Column Proof* app was designed as a technological tool to aid students in learning the process of proving geometric statements. Utilizing visualization features, the app shows definitions or geometric statements in a step-by-step animated format, actively involving students in an entire proving task. While the app focuses on specific learning competencies on parallelograms, kites and trapezoids it can also benefit students in other grades who require guidance in constructing geometric proofs. Studies have shown that using visualization tools helps pupils better understand geometric ideas and properties. In the Two-Column Proof app, students are assisted in developing mathematical arguments and constructing proofs in a clear and coherent manner. This support is provided through the app's visual components, which highlight the relationships between geometric objects such as lines, segments, angles, triangles, and quadrilaterals.

As a next step, a more detailed analysis is being carried out on the effectiveness of the app on student learning and competencies with geometry proofs. There are also plans to extend the app to include proofs of other geometric theorems. Adding more animations and interactive elements can help improve further the learning experience of students.

Acknowledgements

This paper is one of the outputs of the Ateneo Mathplus Resources Laboratory housed at the Department of Mathematics, School of Science and Engineering, Ateneo de Manila University. The authors thank the Department of Science and Technology-Philippine Council for Industry, Energy, and Emerging Technology Research and Development (DOST-PCIEERD) and the University Research Council (URC), Ateneo de Manila University for the support of the

development of the mathematical resources for Grades 1 to 11 Mathematics. Acknowledgement also goes to Dr. Ma. Mercedes T. Rodrigo and the Ateneo Laboratory for the Learning Sciences (ALLS) for providing technical assistance. We thank Mr. Nigel Benedict Cargo and Mr. Amiel Damian F. Justiniani for the development of the app and art assets, respectively.

References

Adams, C. (2017). *The 7 most important STEM skills we should be teaching our kids*. WeAreTeachers. https://www.weareteachers.com/important-stem-skills-teaching-kids/

Andreev, A. N., Emtseva, O. V., Babkina, E. S., Dolzhenkov, V. N., & Kudryashova, N. V. (2021). The effectiveness of using visualization tools and forms in distance learning. *Revista Tempos e Espaços Em Educação*, *14*(33). https://doi.org/10.20952/revtee.v14i33.16053

De Las Peñas, M. L. A. N., Aberin, M. A. Q., Garciano, A. D., Sarmiento, J. F., Tolentino, M. A. C., Verzosa, D. M. B. , Mallari, J. C. (2023). *Terminal report (Department of Science and Technology-Philippine Council for Industry, Energy and Emerging Technology Research and Development): Mathematical resources for distance learning utilizing community LTE networks and television frequencies*.

De Las Peñas, M. L. A. N., Verzosa, D.M.B., Aberin, M.A.Q., Tolentino, M.A.C., Sarmiento, J. F., Loyola, M. & Garces, L. P. D. M. (2022). *Terminal report (Department of Science and Technology-Philippine Council for Industry, Energy and Emerging Technology Research and Development): Technology interventions for mathematical reasoning, statistical thinking and assessment*.

De Villiers M. D. (2003). *Rethinking proof with Geometers Sketchpad*. Key Curriculum Press.

Department of Education. (2020). *K to 12 most essential learning competencies with corresponding CG codes*.

Hughes, J. E., Thomas, R., & Scharber, C. (2006). Assessing technology integration: The RAT – Replacement, Amplification, and Transformation – Framework. In C. M. Crawford, R. Carlsen, K. McFerrin, J. Price, R. Weber, & D. A. Willis (Eds.), *Proceedings of SITE 2006--Society for Information Technology & Teacher Education international conference* (pp. 1616-1620). Orlando, Florida, USA: Association for the Advancement of Computing in Education (AACE).

Kristiyajati, A., & Wijaya, A. (2019). The effectiveness of visualization of proofs in learning mathematics by using discovery learning viewed from conceptual understanding. *Southeast Asian Mathematics Education Journal*, *9*(1), 37-44.

Kunimune, S., Fujita, T., & Jones, K. (2010). Strengthening students' understanding of 'proof' in geometry in lower secondary school. In V. Durand-Guerrier, S. Soury-Lavergne, & F. Arzarello (Eds.), *Proceedings of the sixth congress of the European Society for Research in Mathematics Education* (pp. 756–765). Lyon: ERME.

Pantziara, M., Gagatsis, A., & Elia, I. (2009). Using diagrams as tools for the solution of non-routine mathematical problems. *Educational Studies in Mathematics, 72,* 39-60.

Presmeg, N. (2006). Research on visualization in learning and teaching mathematics. In A. Gutierrez & P. Boero (Eds.), *Handbook of research on the psychology of mathematics education: Past, present and future* (pp. 205-235). Rotterdam, the Netherlands: Sense Publishers.

Rivera, F. (2011). *Towards a visually-oriented school mathematics curriculum: Research, theory, practice, and issues*. Dordrecht, the Netherlands: Springer.

Rösken, B., & Rolka, K. (2006). A picture is worth a 1000 words – the role of visualization in mathematics learning. In J. Novotná, H. Moraová, M. Krátká, & N. Stehlíková (Eds.), *Proceedings 30th conference of the International Group for the Psychology of Mathematics Education Vol. 4* (pp. 457-464) Prague: PME.

Stylianou, D. A., Blanton, M. L., & Knuth, E. J. (2011). *Teaching and learning proof across the grades: a K-16 perspective*. Routledge.

Žakelj, A., & Klančar, A. (2022). The role of visual representations in geometry learning. *European Journal of Educational Research*, *11*(3), 1393–1411.

Zbiek, R. M. (2003). Using technology to foster mathematical meaning through problem solving. In H. L. Schoen, & R. I. Charles (Eds.), *Teaching mathematics through problem solving* (pp. 93–104). The National Council of Teachers of Mathematics, Inc.

App for Addition and Subtraction of Integers

By Debbie Marie B. Verzosa[1], Ma. Louise Antonette N. De Las Peñas[2], Maria Alva Q. Aberin[2] and Len Patrick Dominic M. Garces[2]

[1]Department of Mathematics and Statistics, University of Southern Mindanao, Kabacan, Cotabato, Philippines
dmbverzosa@usm.edu.ph

[2]Department of Mathematics, Ateneo de Manila University, Quezon City, Metro Manila 1108, Philippines
mdelaspenas@ateneo.edu
maberin@ateneo.edu
lgarces@ateneo.edu

Received: 31 May 2018 Revised: 1 October 2018 **DOI: 10.1564/tme_v25.4.02**

This paper presents a mobile app, AlgeOps, created to assist students in understanding addition and subtraction of integers. The design of the app amalgamated the neutralization model (based on cancelling integers of opposite signs) and the number line model to offer a more holistic representation of integers. Furthermore, since AlgeOps presents two objects, boxes and balloons, the learning objective may be extended to adding and subtracting polynomials. Pre- and post-assessments, student observations and interviews with teachers and students revealed the app can increase performance, facilitate conceptual development, and increase engagement in tasks involving integer addition and subtraction.

1 INTRODUCTION

Mastery of integer operations is a prerequisite skill for progressing in secondary school mathematics. Unfortunately, competence in integer operations remains elusive for some students. In the Philippines, it is quite common for students to reach their last year of high school without knowing how to perform operations on integers (Verzosa, Tulao-Fernando, & Vistro-Yu, 2017). This poses a major concern for teachers who are expected to teach more advanced topics which their students are not adequately prepared to learn.

Historically, integer operations are closely tied with keeping track of gains and losses. In *Brahmasphutasiddhanta* (The Opening of the Universe), written by Brahmagupta in 628 CE, negative integers represented debts and positive integers represented assets (Fierro, 2013). Real-life contexts have thus served as a natural starting point for developing a hypothetical learning trajectory and instructional model for learning integer addition and subtraction (Stephan & Akyuz, 2012).

Still, real-life contexts are not sufficient for representing the full range of possibilities involving integer operations. It may involve a stretch of the imagination to describe an authentic real-life situation involving subtraction of a negative integer. The reason is that rules for subtracting negative integers are based on more abstract mathematical properties such as the additive inverse of a real number. Thus, other integer models need to supplement the representations offered by real-life contexts.

Models for integer addition and subtraction can be classified into two broad categories: *neutralization* and *number line* models (Stephan & Akyuz, 2012). The neutralization method is based on cancelling integers of opposite sign. The use of red and black rods to represent positive and negative integers respectively, by the ancient Chinese, is one of the earliest examples of the neutralization method (Merzbach & Boyer, 2011). When ten black rods and ten red rods appear together on the counting board, these are considered to add to nothing, and are removed off the board.

Another neutralization model is proposed in Battista (1983). He used a jar of positive and negative charges as a model that can be used for all four operations, including subtraction of negative numbers. For example, consider the problem, $(+5) - (-3)$. If a jar initially contains 5 positive charges, it is physically impossible to remove 3 negative charges. The solution is to add 3 pairs of positive/negative charges to the jar, which does not affect any change in the collective charge in the jar. After removing the 3 negative charges, then 8 positive charges would remain, giving the answer of $+8$. This model was also utilized in the hands-on activity developed by Ponce (2007) which uses a regular deck of playing cards to facilitate class discussion about integers and adding and subtracting integers.

Integer addition and subtraction may also be modeled on a number line. Cemen (1993) proposed rules for adding and subtraction of integers. The model involves traveling along a number line. A person always starts at zero and faces towards the positive integers. For addition, the person walks forward for positive integers, and walks backward for negative integers. For subtraction, the person turns around before moving to represent the subtrahend. For example, to represent $(+5) - (-3)$, a person starts at zero and faces the positive direction (the right side of the number line). Then the person moves 5 spaces forward to land at $+5$. The person now turns around (i.e., faces the left) to represent subtraction. To represent -3, the person now moves backward to land at $+8$.

In light of earlier work, we present the researcher-designed mobile application *AlgeOps*, which incorporates both the neutralization and number line models for integers. This paper aims to describe the app's features and investigate the extent to which the app can increase student performance, increase engagement, and facilitate conceptual development in integer addition and subtraction through various data sources (pre- and post-assessments, student observations, and interviews). Whereas a number of previous studies focused on describing models for teaching integer operations (e.g., Chang, 1985) or a statistical analysis of their teaching approach (e.g., Smith, 1995), recent research has provided a more in-depth study of how the technology contributes to the learning of integer concepts (e.g., Bolyard & Moyer-Packenham, 2006; Stephan & Akyuz, 2012). Guided by these studies, this paper aims to utilize both quantitative and qualitative analyses to provide a richer account of the effects of using *AlgeOps.*

The succeeding discussion is organized as follows. In Section 2, we review extant models and technology on teaching integer operations. Section 3 contains a description of *AlgeOps*, its features, and its theoretical inspiration. We conclude this paper in Section 4 by evaluating the app, analyzing the results of a study that tests the effect of the app on students' performance in carrying out integer addition and subtraction, and discussing teachers' comments on the app.

2 TECHNOLOGY IN THE FIELD OF INTEGER ADDITION AND SUBTRACTION

Researchers have designed and tested various technology-based representational tools for integers. These representational tools are described below, and we organize the discussion according to whether the tool uses the neutralization and/or number line model as its mathematical basis.

Neutralization models often involve colored counters. For example, intervention lessons on integer addition and subtraction were carried out in a secondary school in Brunei (Sen et al., 2017). In one class, students (aged 11 to 13) used physical counters only, while in the other class, students used both physical counters and virtual manipulatives. Although both groups improved their performance after the intervention, students from the technology-assisted classes were more active and posed more questions than those who used physical counters only. They were also more motivated and less distracted by the logistics of using physical counters.

Another neutralization model can be found in computer microworld, *The Cy-Bee Chips* (Smith, 1995). The software allowed 6th grade and 8th grade students to place two-color counters on a screen work mat, manipulate the counter configuration, add or remove "zero-pairs", write answers to problems posed and receive feedback on correctness. Instruction using *The Cy-Bee Chips* was compared to a concrete manipulative approach and an approach that used both the concrete manipulative and the software. Analyses of quantitative data showed that students exposed to *The Cy-Bee Chips* approach performed significantly better on the post-test than those under the two other treatments. These students were also more successful in performing computational exercises.

A word processor may also be used for integer instruction. De los Santos (2013) passed on the task of creating representations to middle school students. Instead of providing ready-made representations of integers, students themselves were tasked to create their own representations of integer operations by using the Drawing tools in Microsoft Word. The representation became the basis for students' verbal explanations and problem-posing activities.

Objects other than counters can also be used for integer representation. The Balloons and Weights interactive software uses helium balloons to represent positive integers and weights to represent negative integers (Lamb & Thanheiser, 2006). Adding and removing, respectively, balloons or weights to a basket represent addition and subtraction of integers. Students who have used the software eventually learn to solve symbolic integer problems without the explicit use of the software by drawing on the imagery supported by the software. Lamb and Thanheiser (2006) claimed that because of the pulling down direction effect of the weight and the resulting final position, the rules associated with the actions are not as confusing as those of a typical horizontal number line.

Other research groups utilize both neutralization and number line models in instruction. Studies by Bolyard and Moyer-Packenham (2006, 2012) described three types of virtual manipulatives: Virtual Integer Chips, Virtual Integer Chips with Context, and Virtual Number Line. The Virtual Color Chips applets present the user with a problem statement in symbolic form (i.e.,$-2+3$), a color chip bank showing positive (black) and negative (red) chips, and a workspace where users enter their representation to the given problem. The Virtual Number Line applet is similar, except that the representation is done on a number line, with red and blue sliders to represent the first and second integers in the problem statement, respectively.

The general conclusion is that the virtual manipulative environments supported students' learning of these concepts. The age of the students ranged from 11 to 12 years. In addition, specific features shared by these three virtual manipulatives, including dynamic linked representations, interactive capabilities, multiple representations, and immediate feedback, appeared to be most important in supporting learning, and, in particular, enhanced student learning of the most difficult subtraction items. Interviews revealed that students were generally able to work flexibly within the written/metaphorical and picture representational forms for integer addition and subtraction. The same study also showed that students had more difficulty working with the symbolic representation for integer addition and subtraction. Specifically, students had much more difficulty creating representations and making connections when problems were presented in symbols or when a symbolic representation was requested, particularly for subtraction.

As indicated above, the use of several representational

© 2018 Research Information Ltd. All rights reserved.

tools for integer instruction appears to be effective. In another study, Rubin et al. (2012) analyzed the effect of various activities using virtual manipulatives of integers like the Target integer, Integer chips, the use of Damath (based on local checker game) and an online game Number Cruncher to 7th grade students. They then assessed students' conceptual understanding, procedural skills and perception. After the participants went through the various interventions using models of integers it was shown that these activities led to a greater increase in students' performance and conceptual understanding on integers. Moreover, the study of Reis (2010) examined the effect of teaching of integer operations with GeoGebra to 6th grade students. It involved two classes: one class was taught by presentation method or lecture type method while the other class was taught using pre-prepared GeoGebra materials published in the website http://www.emathforall.com/ where the number line illustrated addition and subtraction of integers. Students were assessed after the lesson and two weeks after. Results showed that the students who were taught with GeoGebra got higher scores in both tests. Reis claimed that GeoGebra engaged the students more in the activity, as more sensory organs were utilized in the process. This helped them conceptualize the addition and subtraction of integers.

The large number of technology-based tools (mostly software and virtual manipulatives) for teaching integer operations provides a rationale for developing mobile apps for integer learning. We designed the *AlgeOps* app for teaching integer addition and subtraction.

3 THE AlgeOps APPLICATION

3.1 Design Considerations

While *AlgeOps* was designed to focus on representations for adding and subtracting polynomials, it may also be used as a tool for teaching integer addition and subtraction. The application may be downloaded by accessing the website https://mathplusresources.wordpress.com/ (see resources for Grade 7 Algebra) on any Android-based mobile device.

Since addition and subtraction of polynomials is closely tied with integer operations, it was anticipated that the app can be used to strengthen integer instruction. In consideration of the literature in the previous section, the design amalgamated the neutralization and number line models to offer more holistic representation of integers. Broadly, the app presents problems that require users to first construct a pictorial representation. Neutralization (i.e., cancelling integers) is automated by the app for addition, and is required as a user input for subtraction. After the correct representation is constructed, a number line appears where students can enter their answers. More details are provided in Section 3.3.

In contrast to the web-based applets discussed in Bolyard and Moyer-Packenham (2012) or interactive software such as that by Lamb and Thanheiser (2006), *AlgeOps* is an app that may be used without an internet connection and designed to run on portable mobile devices, such as smart phones and tablet computers. These make it possible for use in a usual classroom without desktop computers and/or internet access. The *AlgeOps* app may also be downloaded for free and without any requisite subscription, unlike the virtual number line applet used in Bolyard and Moyer-Packenham (2012). Furthermore, the complementarity of the neutralization model and the number line representation is much clearer in *AlgeOps*, as both modes are presented to the user for each integer operation to be carried out in the app. As seen in Bolyard and Moyer-Packenham (2012), the neutralization and number line representations were delivered in three separate web-based applets.

3.2 Instrument as a Voice

A common approach to investigate the effect of the use of an app on mathematics performance is to provide a quantitative analysis, and indeed this will be employed in this study (see Section 4.1). A limitation is that this analysis involves a "black box" approach to understanding how the app contributes to learning. Under the assumption that causal processes can be directly observed, an in-depth study of one or a few cases can provide evidence of conceptual development resulting from the app (Maxwell, 2004).

The framework based on instrumental genesis (Guin & Trouche, 1999) and a dialogic relationship between voices (Kazak, Wegeriff, & Fujita, 2015) will be used in this study to guide the in-depth analysis of causal processes in conceptual development resulting from the app. Instrumental genesis presupposes a distinction between artefact and instrument (Verillon, 2000). An artefact is an object made for a specific task, but if it cannot be used, perhaps because it is unfamiliar or obsolete, then the artefact does not elicit an instrument. In other words, an instrument is a psychological construct that includes the user's knowledge, mental operations, motor skills, and more generally, personal schemes. An artefact becomes an instrument through the process of instrumental genesis. It may develop on two directions. The first relates to the artefact itself, such as when a user learns commands or relevant buttons in an app (instrumentalization process). A second genesis relates to the user, such as when the artefact is integrated within one's own schemes (instrumentation process). Instrumental genesis can enable a tool to be a "voice" that leads to a switch in perspective, which is a key process in conceptual development (Kazak et al., 2015).

3.3 Addition in *AlgeOps*

The "Add" mode of *AlgeOps* is split into two levels. In Level 1 of either mode, the given expression is represented using boxes and balloons, which may be colored green or red. Green objects are made to represent positive quantities, whereas red objects denote negative quantities. The use of color to represent positive and negative quantities is based on the use of colored counters in the neutralization model of integers. Level 2 of each mode is accessed after correctly answering ten problems in Level 1. In Level 2, given expressions are now written in the form $ax + b$, where a and b are constants. As such, boxes are now indicated to represent x and balloons are now made to denote 1.

Level 1. In Level 1 of "Add", the user is presented with two expressions represented by a certain number and a certain color of boxes and balloons (Figure 1). These pictorial expressions must then be added together.

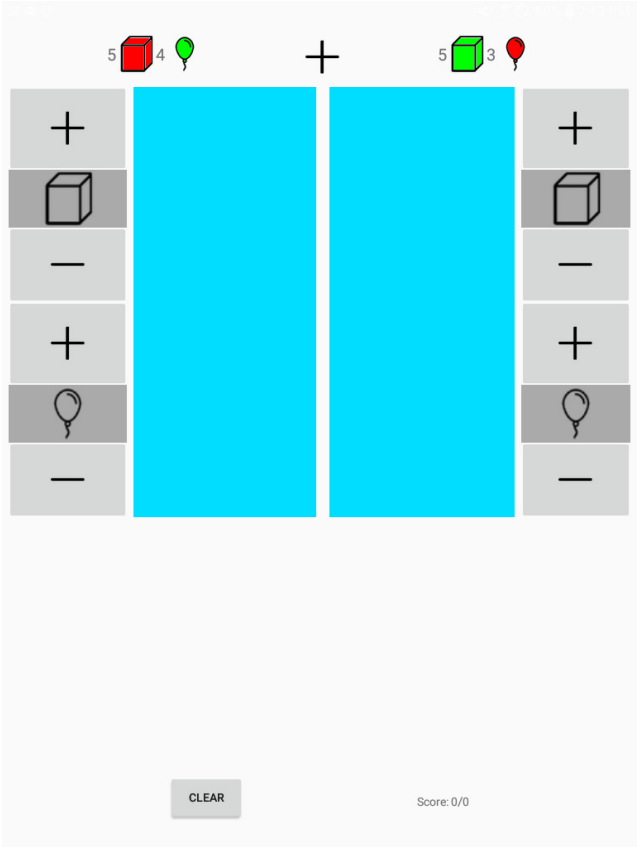


Figure 1 Primary interface of "Add" Level 1.

To facilitate the addition procedure, the user is first asked to construct the expressions in the panels on the left and right. From the neutral setting (no boxes and no balloons), green boxes or balloons are placed when the user clicks the "+" button for the symbol. To remove a green box, the user must press the "-" button. Likewise, red boxes or balloons are generated from the neutral setting by clicking the "-" symbol. After the appropriate figures have been added to the left and right panels, the application will deactivate the "+" and "-" buttons.

In *AlgeOps*, the neutralization method on integer operations is illustrated through an animation. Once the user correctly generates the visual representation, the application automatically checks for the presence of objects of the same shape, but with different colors, on the left and right panels. In Figure 2, there are 5 red boxes and 4 green balloons on the left panel and 5 green boxes and 3 red balloons in the right panel. One by one, objects of the same shape with different colors on the left and right panels disappear from the display until all such pairs have been eliminated. This is a direct illustration of the representation of the neutralization method as done by the ancient Chinese (Merzbach & Boyer, 2011). As such, the intended effect of this illustration is to show the operation

$$4 + (-3) = 1 + 3 + (-3) = 1 + 0 = 1$$

as is implied by the balloons.

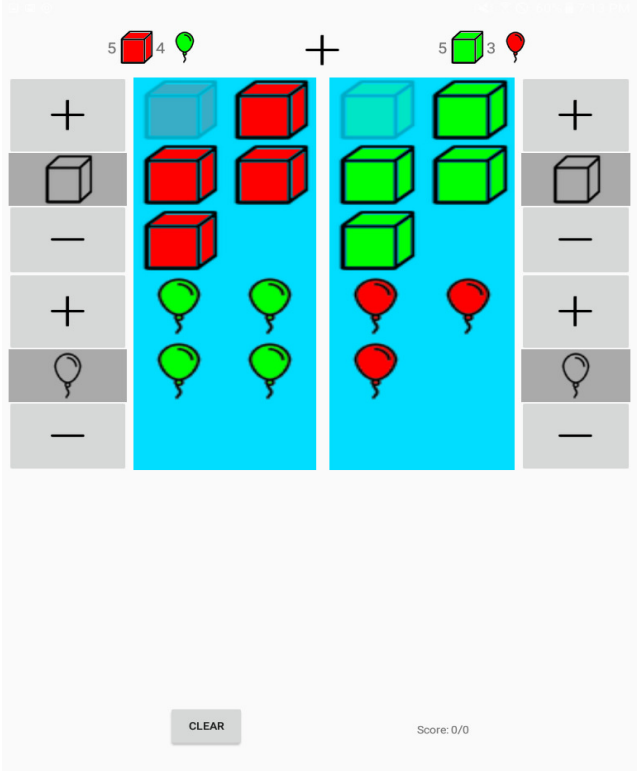


Figure 2 Animation of eliminating additive inverses in "Add" Level 1.

Once all of the additive inverses have been eliminated, two number lines (one for each object) will be shown below the panels. The user will then use the number lines to encode the answer by dragging the dot from the origin to the number representing the correct sum.

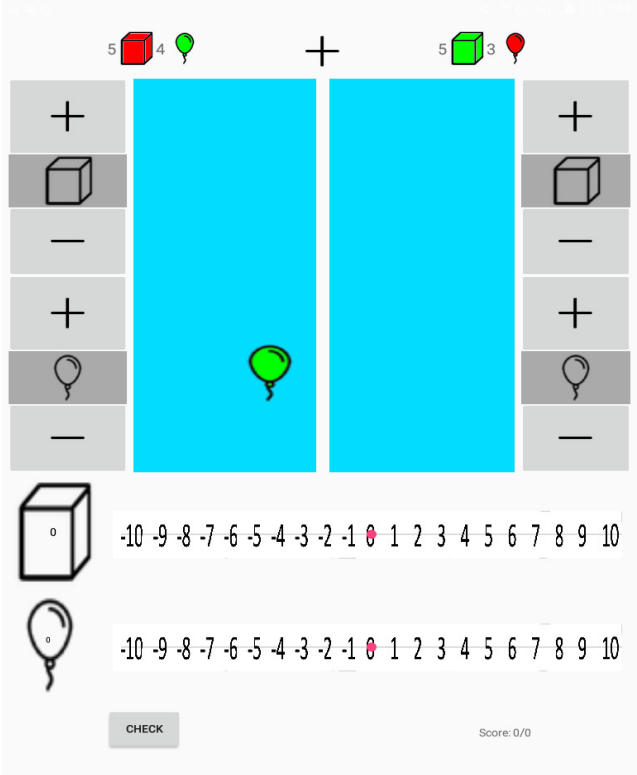
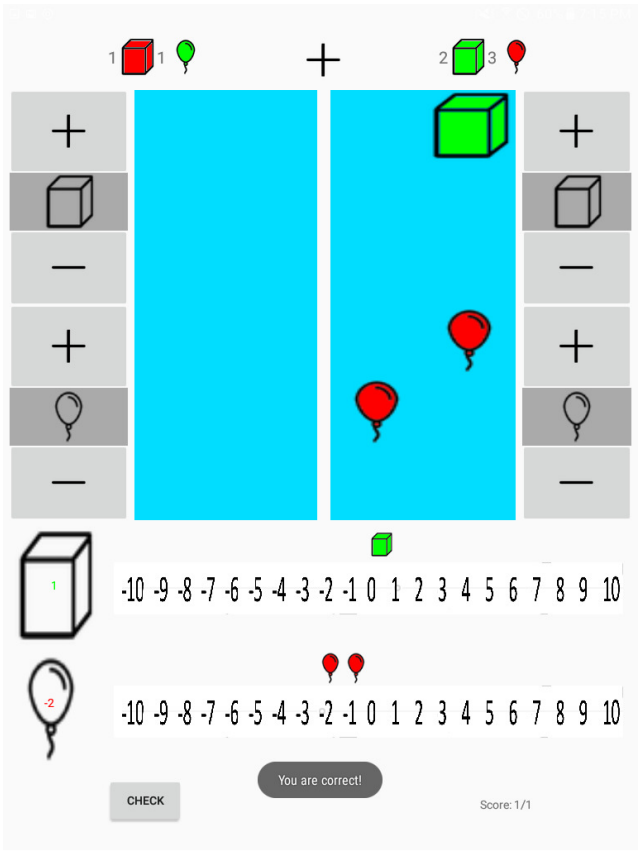


Figure 3 The number lines used to enter the user's answer in "Add" Level 1.

© 2018 Research Information Ltd. All rights reserved.

Note that at this point, the user only needs to map the final answer onto the number line. Thus, the user first needs to determine in which direction from the neutral position the marker must be dragged in order based on the color of the remaining objects to encode the correct answer. The app displays the corresponding numerical value inside the shape on the left of the number lines to assist the user in providing his or her response. While the marker is dragged, visual clues are also provided through the appearance of boxes and/or balloons with the corresponding color (Figure 4).

Once the user has entered the answer, the "Check" button is pressed to submit the answer. The app will indicate if the answer is correct or incorrect. If the answer is incorrect, the app will display the correct answer and will move on to the next problem.



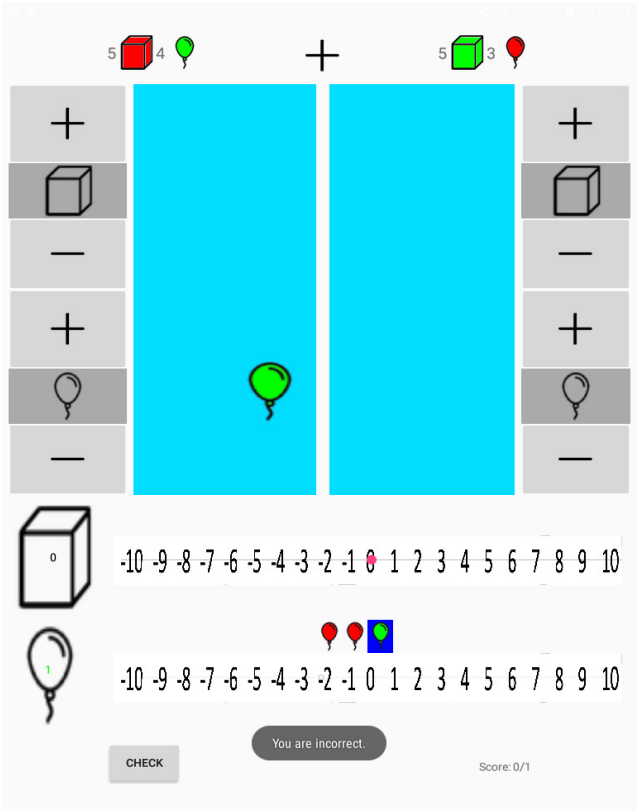


Figure 4 Confirming if the answer is correct (Figure 4a, based on a different problem) or if the answer is incorrect (Figure 4b) in "Add" Level 1.

Level 2. Level 2 consists of the same interface, except that the given expression is now written in the form $ax + b$. The user is still tasked to add the given expressions.

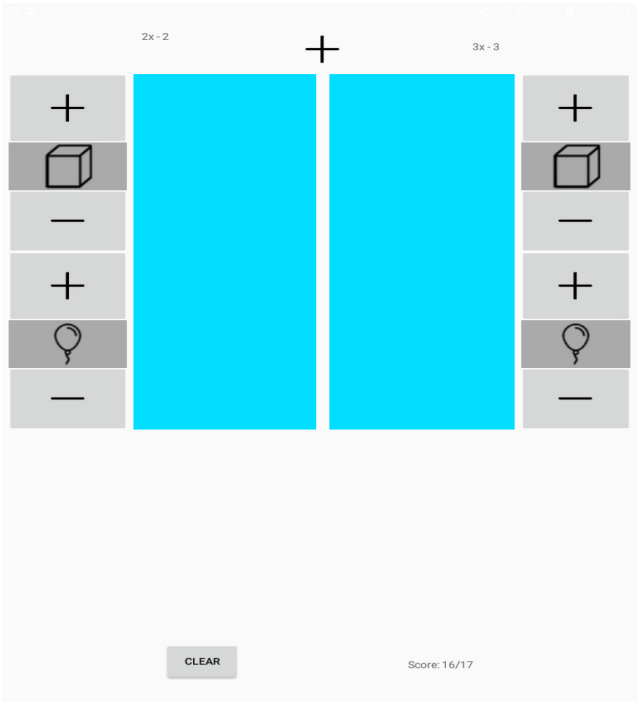
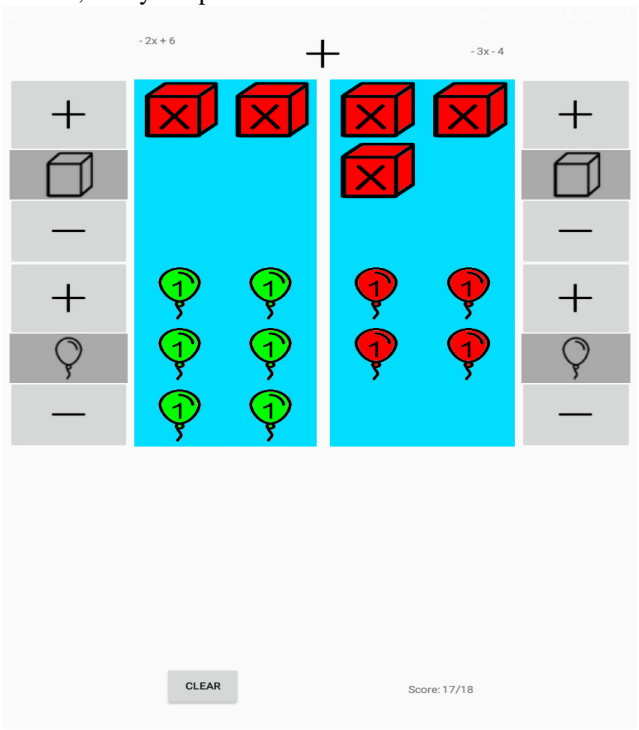


Figure 5 Primary interface of "Add" Level 2. The displayed problem is $(2x - 2) + (3x - 3)$.

In this level, boxes are made to represent the coefficient of x and balloons now denote the constant term b. To proceed, the user must construct the visual representation of the given expressions on the right and left panels by generating the appropriate number and color of boxes and balloons. The app still dynamically illustrates the elimination of additive inverses, if any are present.



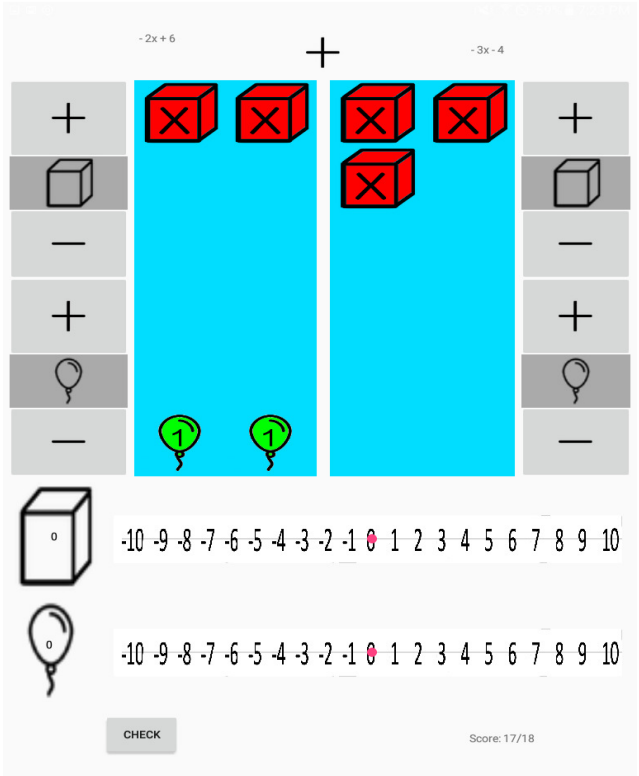
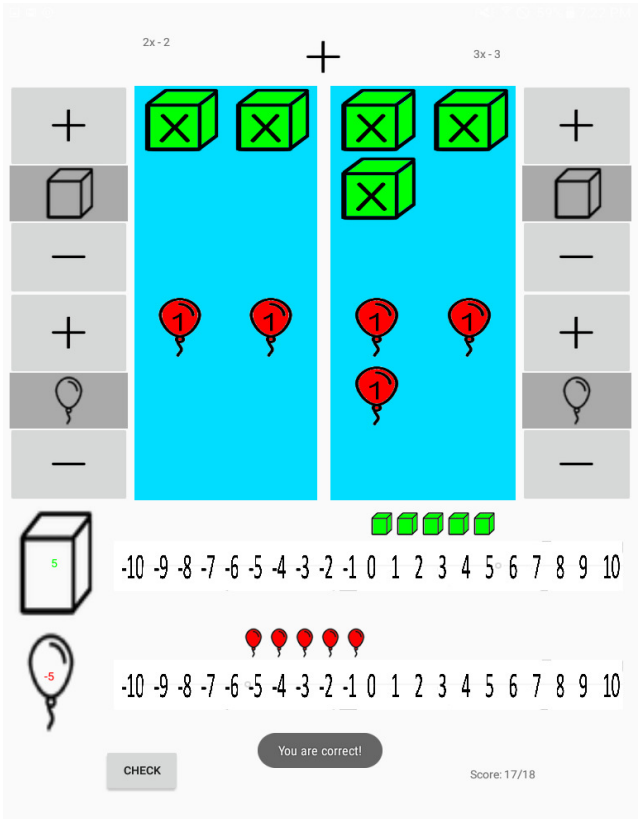


Figure 6 The interface before (Figure 6a) and after (Figure 6b) the elimination of additive inverses in "Add" Level 2. In both figures, the indicated problem is $(-2x + 6) + (-3x - 4)$.

As in Level 1, number lines will be provided for the user to encode his/her response. Similarly, the app provides the user the appropriate feedback upon clicking the "Check" button.



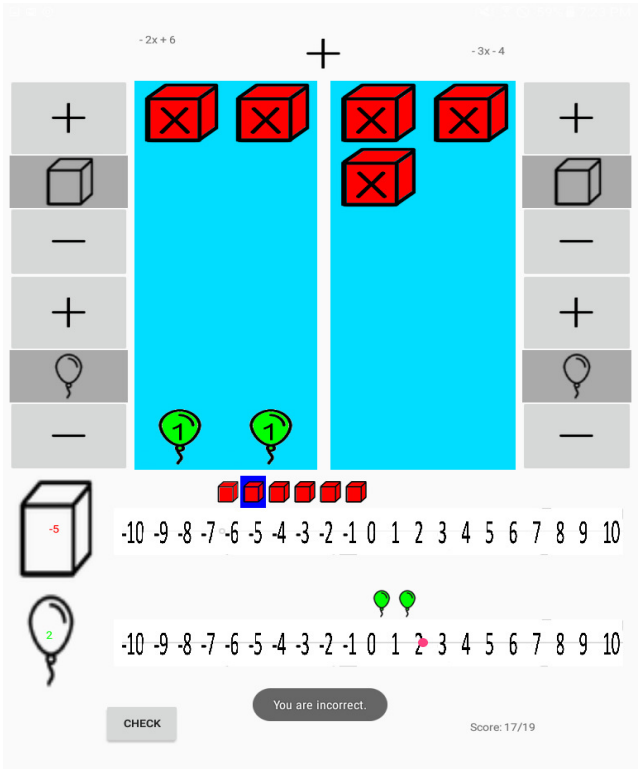


Figure 7 Confirming if the answer is correct (Figure 7a) or if the answer is incorrect (Figure 7b) in "Add" Level 2. In Figure 7a, the problem displayed is $(2x - 2) + (3x - 3)$, whereas for Figure 7b, the problem is $(-2x + 6) + (-3x - 4)$.

© 2018 Research Information Ltd. All rights reserved.

3.4 Subtraction in *AlgeOps*

There are two levels in the "Subtract" mode of *AlgeOps.Level 1*. In "Subtract" Level 1, the user is given a certain number of boxes and/or balloons in a certain color. The user is also prompted to "remove" a given number of boxes and/or balloons of a given color.

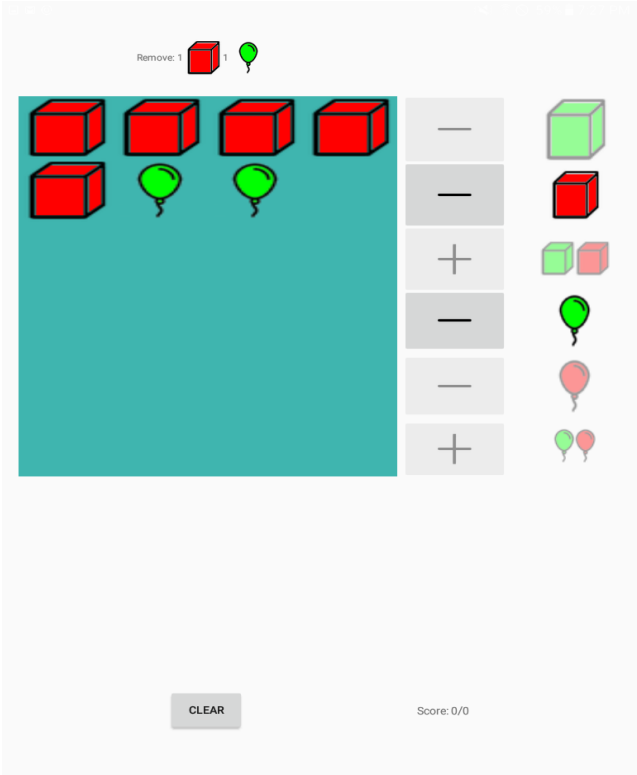


Figure 8 Primary interface of "Subtract" Level 1.

The removal process is done by clicking the "-" button next to the corresponding symbol. After correctly removing the required figures, number lines will be displayed at the bottom of the screen for the user to encode his/her answer.

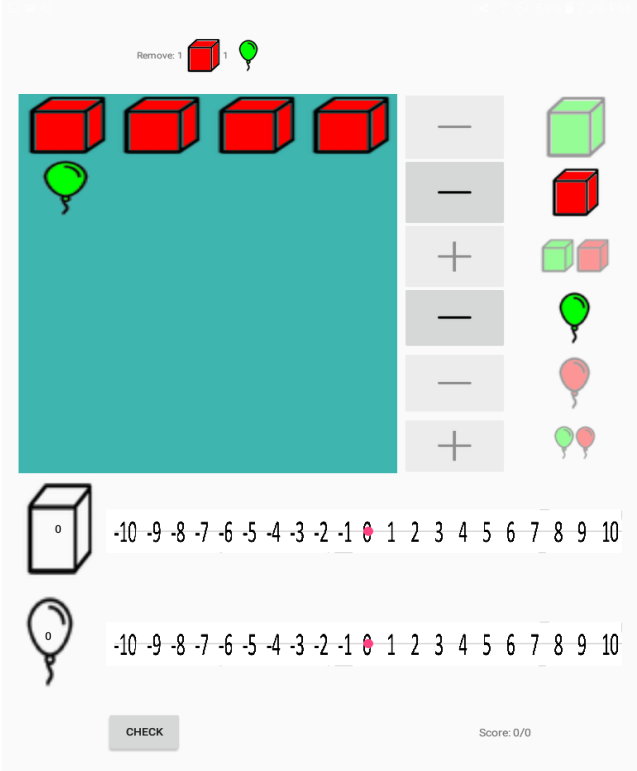


Figure 9 The number lines used to enter the user's answer in "Subtract" Level 1.

The app will provide the appropriate feedback (similar to the feedback given in "Add") upon clicking the "Check" button.

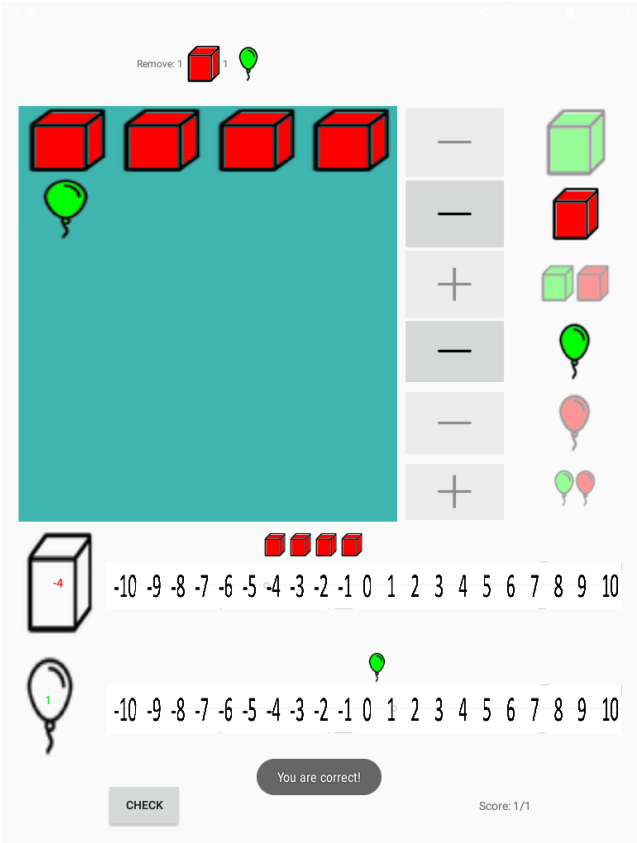


Figure 10 Confirming that the answer is correct in "Subtract" Level 1.

Note that it is possible, for example, that the app will display 3 green boxes and will ask to remove 2 red boxes. In this case, the user must press the "+" button next to the green and red box. Doing so will generate both red and green boxes in the visualization panel. This operation represents the addition of 0 to the expression using additive inverses. In this example, after the user adds 2 green and 2 red boxes, he/she may then proceed to removing 2 red boxes as required by the problem using the usual controls. It is noted that this feature is disabled unless it is required by the given problem.

Level 2. Like "Add" Level 2, "Subtract" Level 2 now features an algebraic representation of the expressions. In particular, Level 2 gives the user a certain number of boxes and balloons (labelled x and 1, respectively) in a certain color and requires the user to remove an expression in the form $ax + b$. As before, the boxes represent a and the balloons represent b.

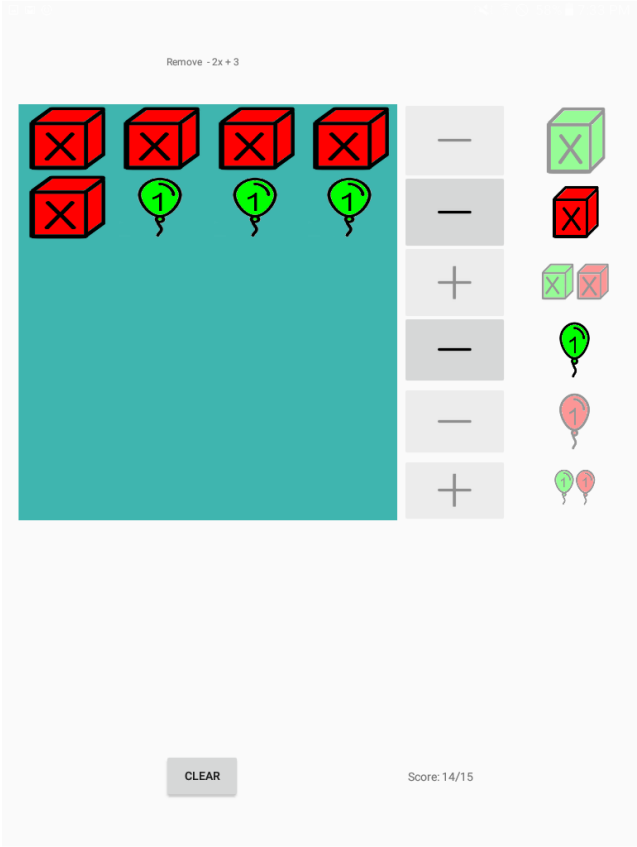


Figure 11 Primary interface in "Subtract" Level 2. The problem displayed is "Remove $(-2x + 3)$".

The buttons operate as they do in Level 1. After removing the required number of x's and 1's, number lines will be displayed at the bottom for the user to encode his/her answer.

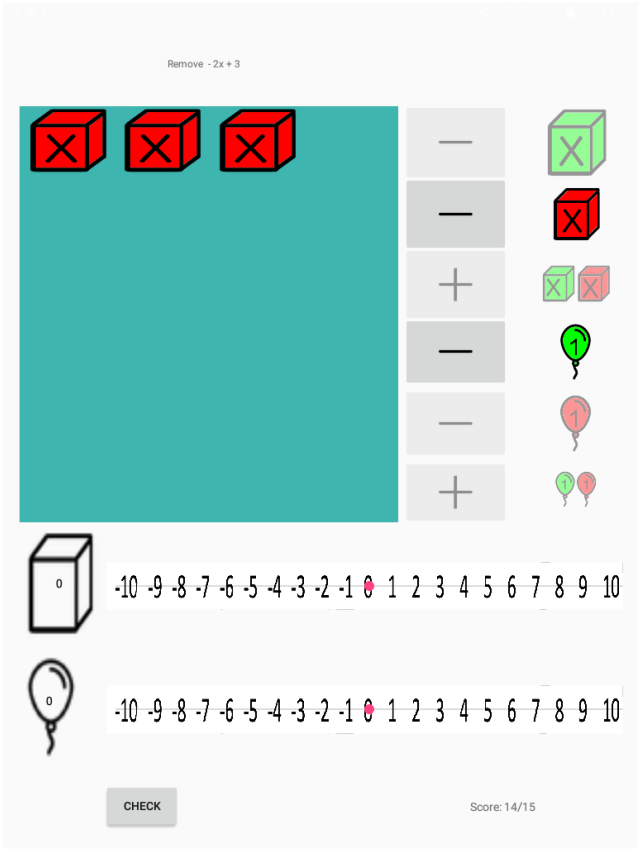
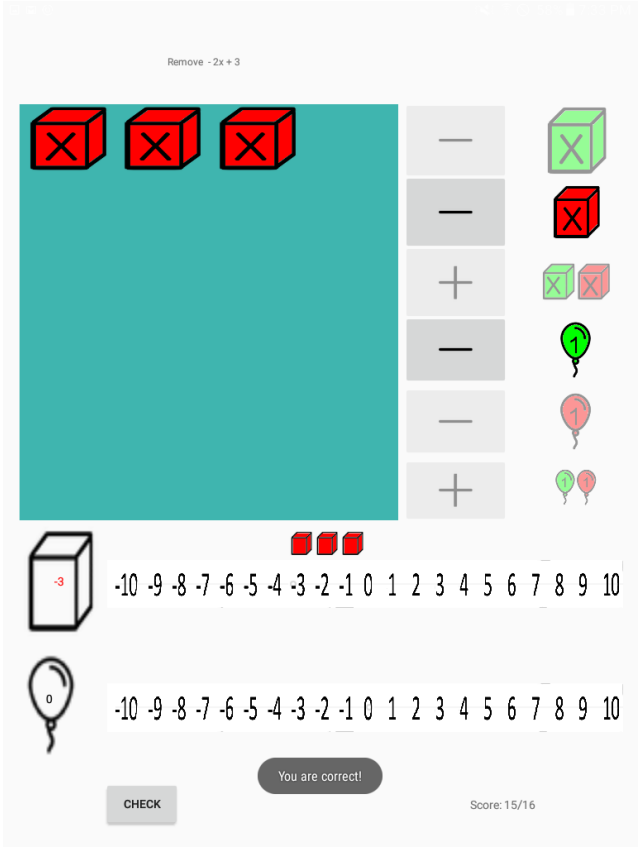


Figure 12 The number lines used to enter the user's answer in "Subtract" Level 2. The problem displayed is "Remove $(-2x + 3)$".

The app will provide the appropriate feedback (similar to the feedback given in "Subtract" Level 1) upon clicking the "Check" button.



© 2018 Research Information Ltd. All rights reserved.

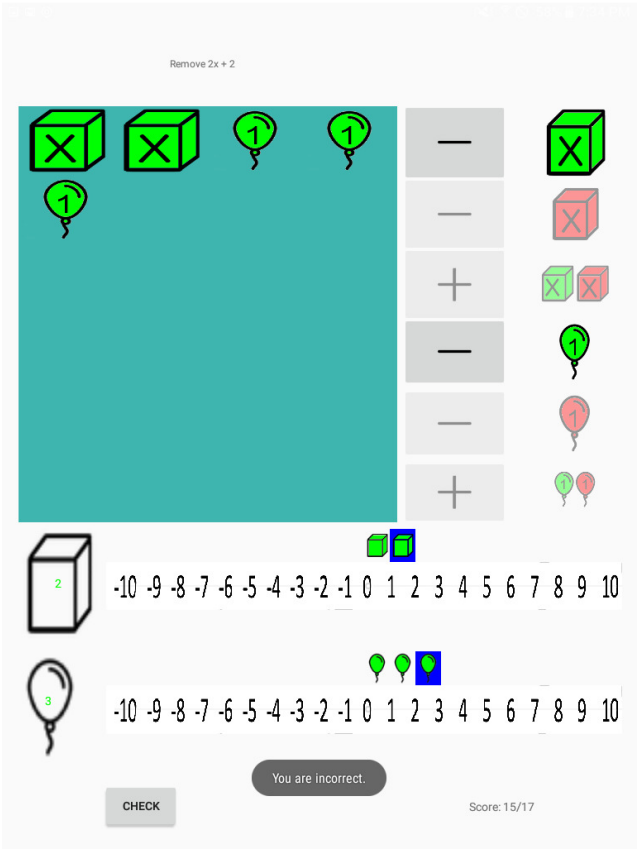


Figure 13 Confirming that the answer is correct (Figure 13a) or that the answer is incorrect (Figure 13b) in "Subtract" Level 2. In Figure 13a, the problem displayed is "Remove $(-2x + 3)$", whereas in Figure 13b, the problem is "Remove $(2x + 2)$".

4 EVALUATION AND DISCUSSION

AlgeOps can be distinguished from earlier software (see Section 2) in three ways. First, *AlgeOps* was designed to run on mobile devices, which increase access due to its portable nature (Norris et al., 2011). Secondly, *AlgeOps* utilizes both the neutralization and number line models in a single interface, whereas earlier software typically only show one kind of representation. Third, *AlgeOps* presents two objects (boxes and balloons), which may extend the concept of integer addition and subtraction to polynomial addition and subtraction.

One prominent disadvantage of *AlgeOps* is that the addition and subtraction problems are system-generated. At this stage, there is no way for the user to provide their own questions to the app. Another downside is that the Level 2 of the app uses the variable x in the statement of the problem. This may be confusing, particularly to students who are taking up integer operations but have not yet been exposed to basic algebra.

4.1 Pre- and Post-Test Results

An initial evaluation of the apps by elementary and secondary school students (aged 11 to 13 years) yielded promising results. The app was tested in two schools in Metro Manila. The first school (S1) is a public (government) high school with a population of 4,000 where majority come from middle to lower income families. The second school (S2) is a small private elementary school. Almost all students belong to middle income bracket families. Participants were 16 Grade 7 students (aged 12 to 13 years) from S1 and 10 Grade 5 and 6 students (aged 11 to 12 years) from S2.

Students took a 10-minute pretest that included 16 items on integer addition, and 16 items on integer subtraction. The pretest was designed by the researchers and included various cases of integer addition (e.g. positive plus positive, negative plus negative, negative plus positive equals negative (positive, zero)). Various cases were also considered for the pretest on subtraction. The first eight items involved addition or subtraction of one-digit integers and the next eight items consisted of two-digit integers. After the test, the *AlgeOps* app was introduced. A demonstration of its use was explained and how it can used to add and subtract integers. Afterwards, the students were allowed to explore and use the app for 15 minutes. A post-test on addition and subtraction of integers was then administered immediately after the intervention. The assessment included tasks involving one-digit or two-digit integers.

Table 1 shows the means scores and standard deviations in the Addition and Subtraction pre- and post-tests across the two schools. The reported *p*-values in the table were obtained from a one-tailed paired *t*-test to check for significant differences between the pre- and post-tests.

Addition		**Mean (SD)**		
School	***N***	***Pre***	***Post***	***p-value***
S1	16	9.13 (5.03)	12.25 (4.58)	0.009
S2	10	9.70 (5.46)	11.60 (4.20)	0.056
Subtraction		**Mean (SD)**		
School	***N***	***Pre***	***Post***	***p-value***
S1	16	9.50 (1.26)	9.94 (1.84)	0.110
S2	10	9.80 (4.57)	12.00 (3.56)	0.045

Table 1 Analysis of results from the pre-test and post-test on integer addition and subtraction.

Although significant improvements were only found in certain classes (S1/Addition and S2/Subtraction), scores were generally higher in the post-test. This result is promising given that the intervention was carried out within a very short amount of time. Further, there were improvements in items that involved two-digit integers even when the app (due to space restrictions) only represented one-digit integers. This

suggests that the conceptual structures underlying integer addition and subtraction had been developed after playing with the *AlgeOps* app. In fact, the visual representations provided by the apps seemed to help students organize their thinking. For example, to answer $9 - 14$ in the post-assessment, one student wrote $9G - 14G$, where G presumably represents the green boxes in the app (see Figure 14). Then to remove 14 green boxes, there is first a need to add 5 green and 5 red objects (in effect, this adds zero), so that the answer after removing 14 green objects would be 5 red boxes, or -5. From the perspective of instrumental genesis, *AlgeOps* had already become an "instrument" as it has become integrated within the student's conceptual schemes. The result is a relationship between two voices as the "voice" of the tool entered the dialogue and enabled a switch in perspective. This is evidenced using a different representation of integers - green and red colors for positive and negative integers, respectively. This student improved his score from 5 out of 16 in the pre-test to 13 out of 16 in the post-test.

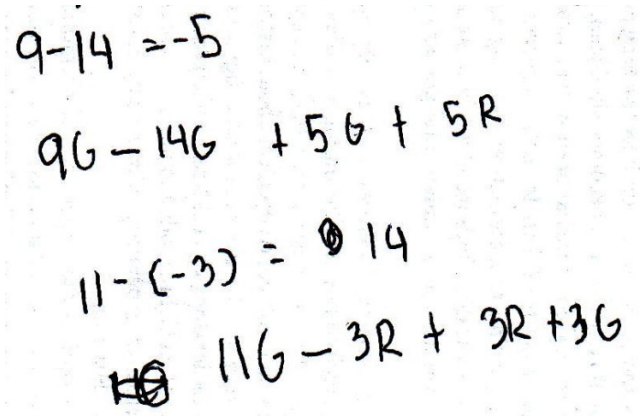


Figure 14 One student's solution to 9 – 14 and to 11 – (–3).

4.2 Student Observation Sessions

Validation of statistical results is obtained from the analysis of two episodes of 11-year old students who worked on the app. One co-author took on the role of a facilitator in both episodes. In the first episode, two students worked collaboratively, and in the second episode, one student worked with the facilitator. These students had not been exposed to any instruction of negative integers prior to the observation.

The facilitator presented the Addition mode of the app to the students and asked them to use the plus and minus buttons to copy the required number of objects shown at the top of the screen (for example, 3 red boxes and 5 green boxes plus 2 green boxes and 3 green boxes). No other instruction was given at the start.

In the first problem encountered, students had no difficulty in using the plus sign to add green objects to the panel. But they encountered difficulties when attempting to add red objects. When they clicked the plus sign, only green objects appeared so they undid this action by clicking the minus sign. However, they stopped clicking the minus sign when they reached zero objects. Apparently, they could not consider quantities less than zero, prompting one student to ask, "How do you change the color?"[1], and another to ask, "Why is it like that?" They tried clicking on other parts of the screen, such as the red box on top of the panel, but these were not buttons so no action was generated. At this point, instrumentalization (cf. Verillon 2000) was still incomplete. Students clicked on parts of the screen that did not correspond to buttons. Eventually, while one student was clicking on a picture of a box, he accidentally clicked the minus sign below the picture and a red box appeared. The child said, "That's it!" and laughs. However, because of the accidental nature of pressing the minus key, the students still had trouble with the next problem. They only pressed the minus button as many times as there are green objects on the screen. Eventually, one child clicked the plus sign three times, then the minus sign several times very quickly so that he ended up clicking it more than three times and a red box appeared. At this instant, both students shouted, "Whoa!" This marked the point when the app became an instrument that was integrated in their mental schemes. Using Verillon's (2000) terminology, the discovery of new properties, mediated by learning about buttons and constraints in the artefact (instrumentalization) has led to change in the user's cognitive schemes (instrumentation). By this point, the students adapted the newly learned concept of quantities less than zero so that right after this problem, they instantly knew what to do to produce red objects on the panel. At this point, it was now possible for them to consider and conceptualize subtraction even when there are zero objects on the screen.

A second cognitive insight occurred when students realized that green and red objects cancel each other. As mentioned in Section 3.3, once the students produce the correct representation, the app automatically checks for pairs of objects of the same shape, but with different colors, on the left and right panels. One by one, these pairs disappear from the display until all such pairs have been eliminated. Initially, the students could not predict when objects would disappear. For example, the third problem showed 4 red boxes on the left and 4 red boxes on the right. The app did not eliminate any of these boxes, prompting one student to ask, "Why wasn't anything taken away?" After several trials, though, they discovered the circumstances that led to the automatic eliminations performed by the app. The eleventh problem showed 2 green boxes and 4 green balloons plus 2 red boxes and 4 red balloons. Even before the app completed the automatic eliminations, one student claimed, "Everything will disappear." This insight occurred during the instrumented activity, specifically through the automatic elimination mechanism that afforded appropriation.

Connections between the objects' color (green or red) and the sign of the integer (positive or negative) is mediated by the number line that appears immediately after green/red pairs have been eliminated. When students enter the correct sum (for example, 2 red balloons) on the number line, red balloons appear just over the number line (see Figure 4a). From our observations, if students mistakenly move the cursor to the

[1] Students' comments are translated from Filipino to English.

© 2018 Research Information Ltd. All rights reserved.

wrong side of the number line, they would see that the objects above the number line would have a different color from that of the correct sum, and they would correct their answer accordingly. The effect is that students associate the red objects with negative integers and the green objects with positive integers.

After the observation sessions, the students were able to use their acquired knowledge that green/red (positive/negative) pairs disappear to solve problems involving objects (i.e., 8 red boxes plus 2 green boxes) or numbers (i.e., (–8) + 2) even without using the app. This was observed when students answered worksheets immediately after their interaction with the app. When the facilitator asked why the student answered "6 red" for the question 8 red boxes plus 2 green boxes, the child reasoned, "Because there are more red." And in another instance, the student answers 8 green plus 5 red correctly ("3 green"), and is able to justify his answer using the words positive and negative: "Because one box is positive, and the other is negative." These observations suggest that the student has acquired knowledge about the behavior of positive and negative integers, as mediated by the app.

To summarize, the initial difficulty that students had with the app (i.e., they had difficulties producing red objects on the screen) is due to their lack of conceptual knowledge about negative integers. For them, it was not possible to click on the minus sign when there were no objects on the screen. When serendipitous clicking produced the desired result (a red object on the screen), they experienced a "switch in perspective" (cf., Kazak et al., 2015). The "Whoa!" moment revealed a major shift in perspective as the app entered as another voice to challenge preconceived notions about subtraction. This proved to be a turning point - while it took several minutes to finish the first two problems, the rest of the problems were completed very quickly. Students knew what buttons to press, and as indicated above, eventually learned to predict the result and apply their acquired knowledge to pictorial and numeric tasks without needing to use the app.

4.3 Student Interviews

Interviews with the students of S1 and S2 after their exploratory sessions with the app revealed increases in engagement and motivation. The students' comments from S1 included the following: "our curiosity was enhanced investigating the concepts", "we understood the lesson easily" and "we found the app appealing and attractive". When asked what advantages the app had as compared to similar tasks in the textbook, their comments included: "it was easier to analyze with the app", "we liked the interactivity feature of the app", "the app modeled the idea whereas we simply read the concept in the textbook". One student mentioned that, "more of us students were getting excited and participating". Moreover, one student added that "studying with this type of app will make me and my classmates more motivated to study".

The students from S2 commented that the app was easy to use, and the app enabled them to understand the addition and subtraction of integers because they could visualize the operations as adding and subtracting boxes and balloons. The students mentioned that they had fun and enjoyed using the app.

These results indicate that the actions performed when playing with the app can help students conceptualize operations on integers. If students can be exposed to the app for an extended period of time, they can form meaningful representations of integer operations and improve their performance on these tasks, even those that involve larger numbers.

4.4 Teacher Evaluation

A full-day workshop was conducted to introduce various apps, including *AlgeOps*, for learning secondary school concepts. Teachers of Grade 7 to 10 Mathematics from public and private schools were invited to participate in the workshop. During the workshop, selected apps representative of the math strands (algebra, geometry, numbers and operations, statistics) were presented, alongside corresponding teaching guides. The teaching guides for particular grade levels were written based on mathematics competencies prescribed by the Philippine Department of Education and meant to optimize the use and benefits from the software. Guide questions are also provided in the teaching guides. For example, an overview of *AlgeOps* was presented with a brief discussion of the accompanying teaching guides for Grade 7 topics Additive Inverse, Addition and Subtraction of Polynomials. Teachers installed the app in their phone or laptop (which they were told to bring to the workshop), and hands-on demonstration followed were the teachers were taught key features of the app and given detailed instructions on how to use the app. The teachers were then given enough time to understand and play around with the apps. After using the apps, the teachers were asked to evaluate the apps through a survey.

During the workshop, participants expressed how the app can support mathematical learning, especially because it provides an opportunity for exploration and thinking. Some teachers stated that the app "is easy to understand and use. It provides the model for learning and teaching the lesson," and that "it is a timely intervention for us teachers, it can enhance our discussion because of its innovativeness and exploratory nature." The app also creates novelty that in turn increases student interest. As one teacher expressed, "it will help me make my students interested and challenged in the subject matter." The app can also be a convenient tool to use because "using the mobile application, I can prepare the lesson wherever I can go because I can access all the data needed before teaching. It saves time and money because there's no need to prepare visual aids." A caveat, though is that the students and teachers must have access to mobile devices, and that teachers themselves should know how to maximize the tool for student learning. As one teacher stated, "the app is innovative, however, it will of great help to students if teacher is skilled and knowledgeable in software."

Based on the preceding discussion, *AlgeOps* proves to be a promising tool that can be used to augment the instruction and learning of integer addition and subtraction, given that there is now a deliberate shift towards more interactive learning resources in the current educational landscape. Whereas previous technologies necessarily require computer laboratory set-ups (which may not be available especially in public schools) and/or subscription to exclusive materials, *AlgeOps* serves as a suitable alternative as it is usable in ubiquitous portable devices such as smart phones and computer tablets and does not require a subscription. Further, it has been shown that there is a considerable improvement in student performance even after a brief exposure to the app and that teachers and students alike appreciate the interactive and innovative elements of the software. It thus stands that more benefits, in both student performance and instructional delivery, may be reaped from a mathematics curriculum that incorporates mobile technologies such as *AlgeOps*.

AlgeOps, as an interactive visual tool, combines the neutralization model and the number line model - the two main models of integer addition and subtraction. The app allows students to explore visual representations of concepts, negative integers for example, that were otherwise difficult to understand via traditional instructional means. While using the app, students experienced a shift in perspective, which then facilitated the connection between the visualization and the mathematical problem and allowed the students to solve integer addition and subtraction problems with greater ease even without the app. Indeed, the discussion and evaluation of the app above appraise findings in literature that visual tools and virtual manipulatives, which include *AlgeOps*, contribute positively to students' performance in integer addition and subtraction.

As indicated earlier, *AlgeOps* may also be used in teaching and learning polynomial addition and subtraction. In future work, the researchers intend on studying ways to integrate *AlgeOps* in teaching these topics and measuring the impact the app may have on the performance of students.

AlgeOps is just one of the apps that the researchers developed. Other apps we have developed for other topics may also be downloaded from https://mathplusresources.wordpress.com. It is ultimately the goal of the researchers to produce an integrated suite of resources for mathematics that may be used in all levels of basic education.

ACKNOWLEDGEMENTS

The authors would like to thank the Department of Science and Technology (DOST) Philippine Council for Industry, Energy and Emerging Technology Research and Development (PCIEERD) for the funding support. We would like to acknowledge the technical assistance provided by SparkSoft Solutions Inc., Mr. Carlos Oppus, Mr. Daniel Lagazo and Mr. Winfer Tabares in the creation of the app. We are grateful to Fr. Aristotle Dy and Ms. Adelaida Bague of Xavier School, the administrators of Marikina High School, and the administrators of CBC Child Learning Center for the support during the testing phase of this study. Thanks also go to Mr. Edwin Morales, Mr. Edrian Neverio and Mr. Janree Gatpatan for the technical support.

REFERENCES

Battista, M.T. (1983) A complete model for operations on integers, *Arithmetic Teacher*, **30**(9), 26-31.

Bolyard, J. and Moyer-Packenham, P. (2006) The impact of virtual manipulatives on student achievement in integer addition and subtraction, in Alatorre, S., et al. (eds), *Proceedings of the 29th Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*, **2**, 879-880.

Bolyard, J. and Moyer-Packenham, P. (2012) Making sense of integer arithmetic: the effect of using virtual manipulatives on students' representational fluency, *Journal of Computers in Mathematics and Science Teaching*, **31**(2), 93-113.

Chang, L. (1985) Multiple methods of teaching the addition and subtraction of integers, *The Arithmetic Teacher.* **33**(4),14-19.

Cemen, P.B. (1993) Adding and subtracting integers on the number line, *Arithmetic Teacher,* **40**(7), 388-389.

De los Santos, E. (2013) Understanding integer addition and subtraction concepts using Microsoft Word illustrations, *Ohio Journal of School Mathematics*, **68**, 8-14.

Fierro, R.D. (2013) Mathematics for Elementary School Teachers, Belmont, CA: Brooks/Cole, Cengage Learning.

Guin, D. and Trouche, L. (1999) The complex process of converting tools into mathematical instruments: The case of calculators, *International Journal of Computers for Mathematical Learning,* **3**, 195-227.

Kazak, S., Wegerif, R. and Fujita, T. (2015) The importance of dialogic processes to conceptual development in mathematics, *Educational Studies in Mathematics,* **90**(2), 105-120.

Lamb, L.C. and Thanheiser, E. (2006) Understanding integers using balloons and weights software, in Alatorre, S., et al. (eds), *Proceedings of the 29th Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*, **2**, 163-164.

Maxwell, J.A. (2004) Using qualitative methods for causal explanation, *Fields Methods*, **16**(3), 243-264.

Merzbach, U.C. and Boyer, C. B. (2011) *A history of mathematics* (3rd ed). Hoboken, New Jersey: John Wiley & Sons.

Norris, C., Hossain A. and Soloway, E. (2011) Using smartphones as essential tools for learning: A call to place

© 2018 Research Information Ltd. All rights reserved.

schools on the right side of the 21st century, *Educational Technology*, **51**(3), 18-25.

Ponce, G. A. (2007) It's all in the cards: Adding and subtracting integers, *Mathematics Teaching in the Middle School,* **3**(1), 10-17.

Reis, Z. A. (2010) Computer supported mathematics with Geogebra, *Procedia Social and Behavioral Sciences,* **9**, 1449-1455.

Rubin, R.J., Marcelino, J., Mortel, R. and Lapinid, M.R.C. (2012) Activity-based teaching of integer concepts and its operations, Paper presentation, DLSU Research Congress, Manila, Philippines.

Sen, G.L., Tengah, K.A., Shahrill, M., and Leong, E. (2017) Teaching and learning of integers using hands-on versus virtual manipulatives, *Proceedings of the 3rd International Conference on Education,* **3**, 174-185.

Smith, Jeffrey Phillip. (1995) The effects of a computer microworld on middle school students' use and understanding of integers, (Doctoral dissertation).

Stephan, M. and Akyuz, D. (2012) A proposed instructional theory for integer addition and subtraction, *Journal of Research on Mathematics Education*, **43**, 428-464.

Verillon, P. (2000) Revisiting Piaget and Vygotsky: In search of a learning model for technology education, *Journal of Technology Studies,* **26**(1), 3-10.

Verzosa, D.M.B., Tulao-Fernando, M.T. and Vistro-Yu, C.P. (2017) Congruence between contest and opportunities for professional development of mathematics teachers in the Philippines, in Kaur B. and Kwon, O.N. (eds), *Professional Development of Mathematics Teachers: An Asian Perspective*, Singapore: Springer, 89-96.

BIOGRAPHICAL NOTES

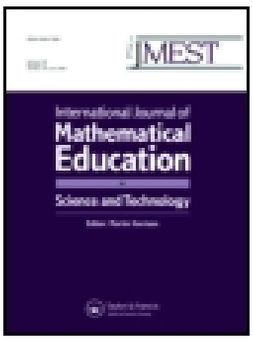
Debbie Marie B. Verzosa is a full-time faculty member of the University of Southern Mindanao. Her research interests include numeracy in elementary grades, mathematical learning in disadvantaged contexts, and technology in mathematics education.

Ma. Louise Antonette N. De Las Peñas is a Professor of Mathematics of the Department of Mathematics, Ateneo de Manila University. Her research interests include group theory, discrete geometry and technology in mathematics education. She led a research project funded by the Department of Science and Technology – Philippine Council for Industry, Energy, and Emerging Technology Research and Development (DOST-PCIEERD) on the development of interactive resources for Grades 7 to 10 mathematics which was intended to help support the implementation of the K-12 curriculum in the Philippine educational system.

Maria Alva Q. Aberin is an Assistant Professor of the Department of Mathematics, Ateneo de Manila University. Her research interests include metacognition and affect variables in learning mathematics and technology integration in mathematics education.

Len Patrick Dominic M. Garces, previously an Instructor of the Department of Mathematics, Ateneo de Manila University, is currently a PhD student of the School of Information Technology and Mathematical Sciences, University of South Australia. His research interests include financial mathematics, financial risk modelling, and mathematical economics and he has participated in research projects on the use of technology in mathematics education and on impact assessment of school-based feeding programs.

© 2018 Research Information Ltd. All rights reserved.



International Journal of Mathematical Education in Science and Technology

ISSN: 0020-739X (Print) 1464-5211 (Online) Journal homepage: http://www.tandfonline.com/loi/tmes20

App-based scaffolds for writing two-column proofs

Debbie Marie B. Verzosa, Ma. Louise Antonette N. De Las Peñas, Maria Alva Q. Aberin & Len Patrick Dominic M. Garces

To cite this article: Debbie Marie B. Verzosa, Ma. Louise Antonette N. De Las Peñas, Maria Alva Q. Aberin & Len Patrick Dominic M. Garces (2018): App-based scaffolds for writing two-column proofs, International Journal of Mathematical Education in Science and Technology, DOI: 10.1080/0020739X.2018.1500654

To link to this article: https://doi.org/10.1080/0020739X.2018.1500654

Published online: 23 Jul 2018.

Submit your article to this journal

View Crossmark data

Full Terms & Conditions of access and use can be found at
http://www.tandfonline.com/action/journalInformation?journalCode=tmes20

INTERNATIONAL JOURNAL OF MATHEMATICAL EDUCATION IN SCIENCE AND TECHNOLOGY
https://doi.org/10.1080/0020739X.2018.1500654

Taylor & Francis
Taylor & Francis Group

Check for updates

App-based scaffolds for writing two-column proofs

Debbie Marie B. Verzosa[a], Ma. Louise Antonette N. De Las Peñas[b], Maria Alva Q. Aberin[b] and Len Patrick Dominic M. Garces[b]

[a]Department of Mathematics and Statistics, University of Southern Mindanao, Kabacan, Philippines; [b]Department of Mathematics, Ateneo de Manila University, Quezon City, Philippines

ABSTRACT

This paper presents apps designed to assist students in understanding and developing proofs in geometric theorems. These technologies focus on triangle congruence, triangle similarity and properties of parallelograms. Focus group discussions and initial testing of the apps revealed that the apps offered a more engaging medium for learning proving and were capable of facilitating proof-writing skills in geometry.

ARTICLE HISTORY
Received 29 January 2018

KEYWORDS
Proofs; two-column proofs; mobile technologies

1. Introduction

Many mathematics educators are now advocating an increased role of proofs in the mathematics education of students at all levels [1]. Further, there are calls for rethinking the role of proofs from being mere exercises in logical thinking to providing opportunities for students to work like mathematicians – observing patterns, making conjectures and presenting justification [2]. Under this view, students are not just expected to know how to justify statements or theorems that are clearly known to be true. Instead, they are expected to explore, form their own conjectures, test whether such conjectures are true and justify their assertions.

Dynamic geometry has been a natural platform for students to test conjectures as it allows them to manipulate representations, observe the consequences and formulate conjectures. In fact, many examples of the affordances of dynamic geometry on structuring an authentic proving environment are presented in this journal [3,4] and in other journals as well [5].

This paper, however, will focus on the two-column proof, which was introduced in the early 1900s [6] and is still the most common proof format in many secondary school textbooks today, including those used in the Philippines where the authors of this paper are based. While some may argue that a two-column proof format is like a 'straitjacket' [7] that compels students to communicate their arguments along very stringent requirements, it has an enabling quality that facilitates greater flexibility in reasoning and proving [8].

CONTACT Ma. Louise Antonette N. De Las Peñas mdelaspenas@ateneo.edu Department of Mathematics, Ateneo de Manila University, Quezon City, Metro Manila 1108, Philippines

© 2018 Informa UK Limited, trading as Taylor & Francis Group

Although the two-column proof can provide a structure for students' reasoning, very few students can be said to have mastered proof writing, even of proofs involving elementary geometric concepts [9]. In fact, in disadvantaged areas in the Philippines, students enter secondary school without having mastered elementary mathematics concepts [10], with the inevitable result that proofs end up being skipped altogether in favour of easier topics.

2. Geometry proving software

Various software for facilitating proofs of geometry theorems have been designed in the past. Among these are the GPTutor, GeoCoq and Geometry Proof Tutor. More recently, mobile technologies have also been utilized for mathematics education.

2.1. GPTutor

Almost 30 years ago, Wertheimer [11] described the Geometry Proof Tutor (GPTutor), a software for individualizing proof instruction developed by John R. Anderson and C. Franklin Boyle. This software presents a proving task and requests students to enter a rule or reason (e.g. Reflexive Property) that can be used. After the rule is typed, the student now has to enter a conclusion (e.g. $\angle A \cong \angle A$). The student may also request the software to recall any rule learned thus far, or ask the software to provide a hint. Feedback is immediately provided.

2.2. GeoCoq

More recently, dynamic geometry software has become widespread in mathematics education. One such example is GeoGebra [12], which allows users to construct and manipulate mathematical representations through dragging objects on the screen. The interactive geometry proving tool GeoCoq that works alongside GeoGebra was developed by Pham and Bertot [13]. Within this environment, users can use GeoGebra to make observations and form conjectures. To prove these conjectures, users can identify statements that can be logically derived from the hypotheses and add these statements to their proof. Alternatively, users can work backward, starting from the goal and identify sufficient conditions for the conclusion to be reached.

2.3. Geometry Proof Tutor

The Geometry Proof Tutor (GPT) is another software that focuses on facilitating two-column proofs [14]. It consists of a bank of Geometry problem sets. Users are required to enter statements that are presented in both two-column and proof tree formats. The software can check these user inputs and provide immediate feedback.

3. Mobile technologies

The numerous advances in technology over the past 30 years led to more research into the potentials of digital technologies for mathematical learning. However, relatively fewer studies have been done on mobile technologies (e.g. iPads, iPhones and Android devices),

perhaps due to their more recent development [15]. A meta-analysis of the use of mobile technologies for mathematical learning found only 80 related studies and only 11 of these involved high school students.

Despite the small number of studies, mobile technologies have a clear advantage over computers or laptops. Mobile technologies such as smartphones are portable, easy to boot up, and can easily fit in one's hand. Students may thus tend to carry and use them constantly [16]. Well-designed apps can also promote deep learning, but most of these studies involve young children.

Not much is said in the literature on apps for proving designed for mobile technologies. In this paper, we report on the ProveIt app and web-based apps on proving, which were designed by the authors.

3.1. Proving apps

ProveIt is an Android-based mobile application that provides users a visual representation and interactive learning tool in aid of proving theorems and statements involving congruent triangles, similar triangles and parallelograms. These can be accessed through the Geometry icons in the MathPlus Resources website [17]. One of the app's most important features is that it allows the user to select parts of the geometric figures on the VMR itself, using the mobile device's touch-screen interface. As opposed to usual textbook exercises on these topics, ProveIt also provides immediate feedback, allowing the user to quickly amend any incorrect responses provided or to understand why the provided response is correct.

The application is subdivided into three main topics: (a) Triangle Congruence, (b) Triangle Similarity and (c) Quadrilaterals. As of this writing, two levels (Level 1 and Level 2) are available for Triangle Congruence, one level (Level 2) is available for Triangle Similarity and one level (Level 1) is available for Quadrilaterals.

3.1.1. Triangle congruence in the ProveIt app

Level 1. Triangle Congruence Level 1 dwells on the basic triangle congruence postulates and their visual representations. Upon initialization, the app presents tasks that require students to choose a triangle congruence postulate or theorem (if any) that guarantees that the two triangles presented are congruent (Figure 1).

When the user selects the correct response, the application will confirm the answer (Figure 2). If an incorrect response is given, the application will give the user one last chance to make a correct answer. The application will proceed as usual if the user provides the correct answer. Otherwise, the response will be marked incorrect, and the application will proceed to the next question.

At the end of the set, the application informs the user of his or her total score and provides the option to go back to the main menu or to retry the problem set. At any time during the use of the application, the user may use the 'Instruction' button to display the instructions for the level.

Level 2. While Level 1 requires the student to identify a triangle congruence criterion (if any) that guarantees that two triangles are congruent, Level 2 further requires users to identify three pairs of congruent parts that form the basis for the application of the triangle congruence criterion. These pairs of congruent parts are selected by first determining the reason why they are congruent (Figure 3).

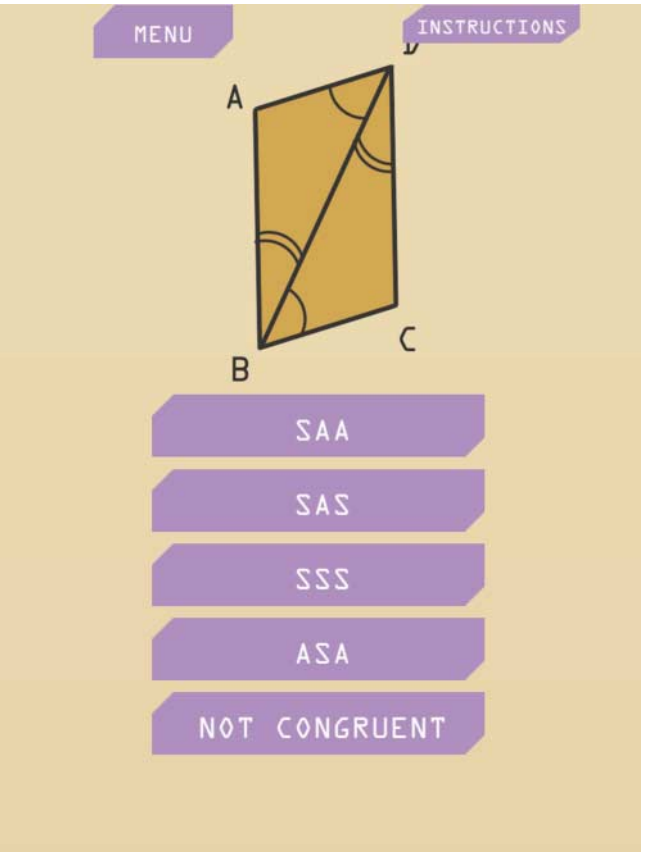


Figure 1. A sample task in Triangle Congruence Level 1.

In the given example, the user must use the SAS congruence postulate to prove that ΔABC and ΔEDC are congruent given that AE and BD bisect each other. At the start of the problem, the application instructs the user to use the buttons to construct the proof of congruence. These buttons correspond to the 'Reasons' column that is present in conventional two-column proofs in geometry.

In this example, clicking 'Vertical Angle Theorem' will require the user to select two vertical angles ($\angle ACB$ and $\angle ECD$) by dragging along its sides. Once these angles are selected, the 'A' in SAS turns green (Figure 4). This visual feedback indicates that the user has already identified one pair of congruent angles. To finish the proof, the user just needs to identify two pairs of congruent sides (as indicated by the 'S'). Ultimately, the user realizes that to complete the proof, the goal is to identify three pairs of congruent parts as indicated by an appropriate triangle congruence criterion. Immediate feedback is also provided, in the case that the user incorrectly selects the triangles' parts. In this situation, the user must repeat the selection process by clicking on a button and then correctly selecting the appropriate congruent parts.

At the end of the set, the application informs the user of his or her total score and provides the option to go back to the main menu or to retry the problem set. At any time

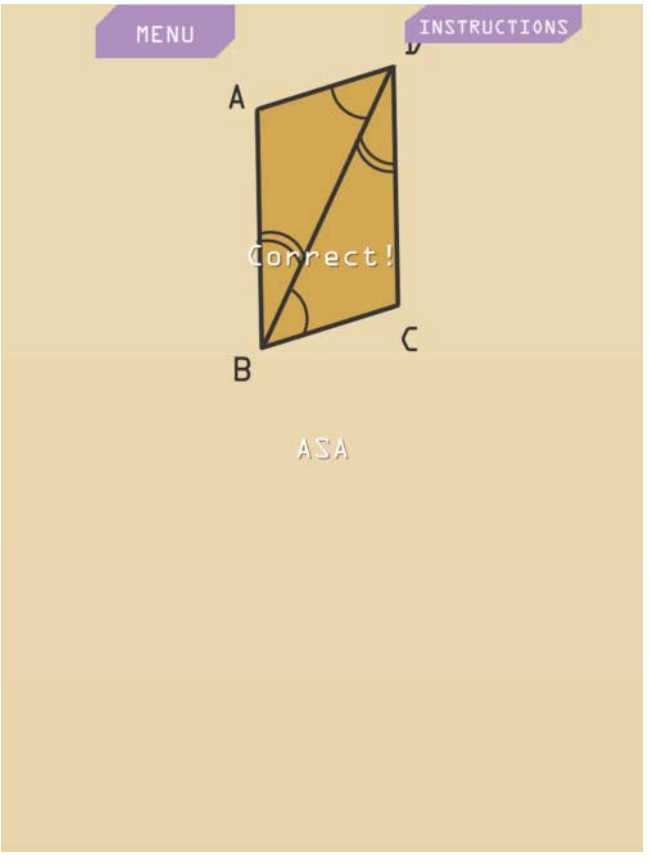


Figure 2. Confirmation of correct responses.

during the use of the application, the user may use the 'Instruction' button to display the instructions for the level.

By going through these activities, students are guided towards writing a two-column proof. Independently, they may be stuck at certain portions of the proof [9]. The goal (e.g. SAS) as well as the feedback provided by the ProveIt Level 2 app enables students to proceed with the proof by helping them identify pairs of congruent parts in the two indicated triangles. Through these opportunities, students can later work on proofs on their own.

3.1.2. Triangle similarity in the ProveIt app

The Triangle Similarity tasks involve proving two triangles are similar. The procedure is the same as for the triangle congruence app (see Figure 5). Unlike the tasks in the Triangle Congruence apps, some tasks in the Triangle Similarity apps first require the users to identify similar triangles before they can begin with the proof. The prompts and immediate feedback structure of the app should enable the students to accomplish the objective.

3.1.3. Quadrilaterals in the ProveIt app

This activity was designed for students to strengthen their understanding on properties of parallelograms and for identifying conditions for a quadrilateral to be a parallelogram. The

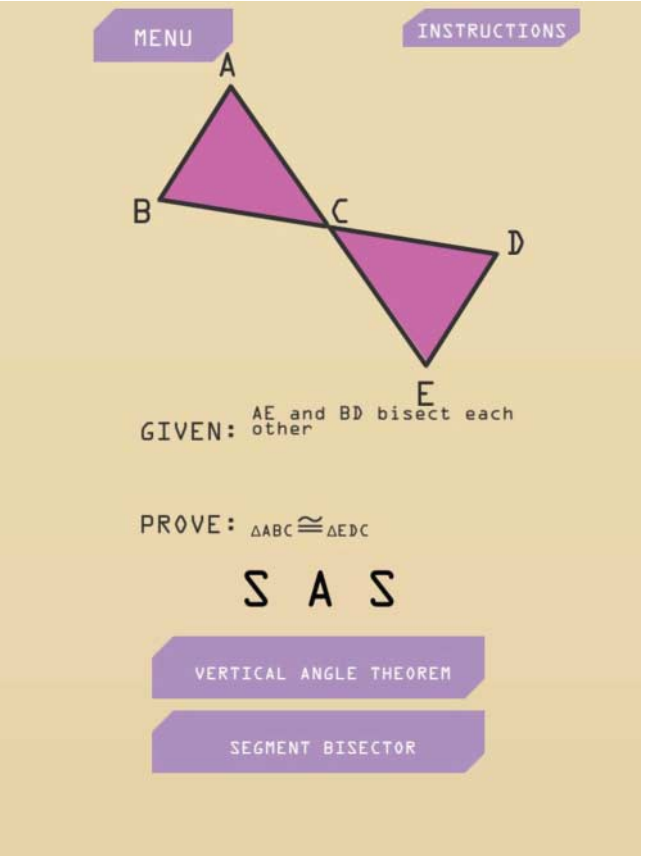


Figure 3. A sample task in Triangle Congruence Level 2.

intention is for the student to explore and learn from the situations made possible by the different quadrilaterals generated by the app. Firming their understanding on this concept will help them in formulating proofs on parallelograms.

The app generates different quadrilaterals and students pick the appropriate property/condition that can be used to prove that the quadrilateral is a parallelogram, given the information in the app (Figure 6). There are six fixed choices/conditions that are always given: two pairs of opposite sides are parallel, two pairs of opposite sides are congruent, two pairs of opposite angles are congruent, one pair of opposite sides are parallel and congruent, diagonals bisect each other, and not a parallelogram.

The app gives feedback accordingly. If the app is correct, the app produces the message 'Correct!' Otherwise, some parts of the parallelogram are highlighted to guide the user as to the appropriate response. For example, if the correct choice should be two pairs of opposite sides are congruent then the app highlights a pair of opposite sides at a time (Figure 7).

3.1.4. GeoGebra applets on two-column proof

We have written a number of applets on proving theorems using two-column proofs using the dynamic software GeoGebra. These applets provide students a framework for writing proofs by allowing them to visualize given statements of the proof. To navigate the proof,

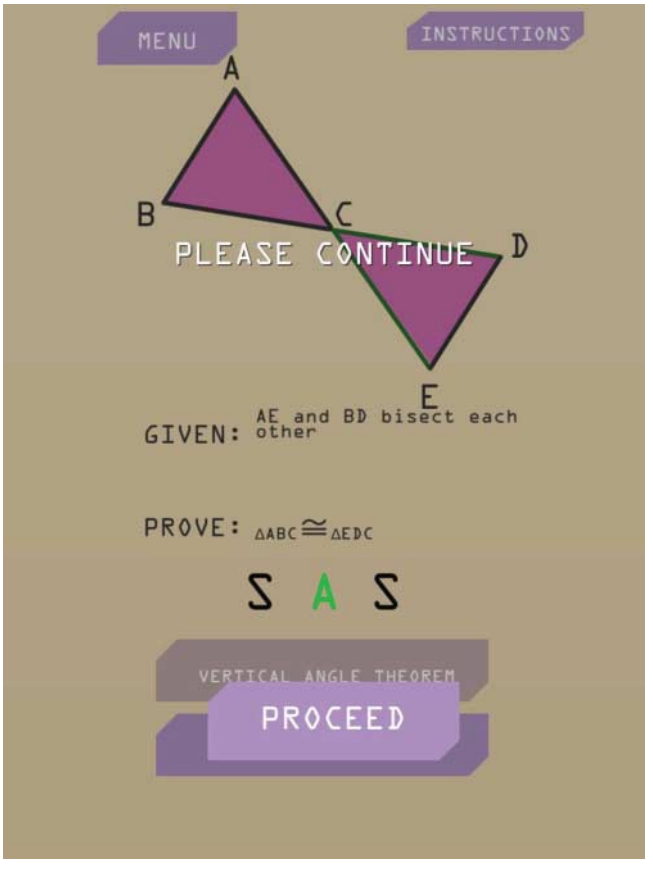


Figure 4. Visual feedback showing that one pair of congruent parts has been identified.

the student clicks the right arrow button. A line of the proof appears and the student writes the answer to the blank. To visualize the statement given, the box provided is ticked.

An example is proving the statement 'Opposite sides of a parallelogram are congruent' (Figure 8). Boxes on either side of the two-column proof are given and when a box is ticked, figures are shown or highlighted to guide the student as to the appropriate response. For example, when line 'Alternate interior angles are checked', the pair of angles (pair 1: $\angle$ ABD and $\angle$ BDC; pair 2: $\angle$ ADB and $\angle$ DBC) are animated (Figure 8). When 'Angle Side Angle Postulate' is checked, the first pair of corresponding angles is highlighted, then the side, and the second pair of angles, and then the two triangles one at a time are shaded (Figures 9 and 10). Note that Figure 9 was captured in the middle of the animation, whereas Figure 10 presents the finished outcome.

4. Evaluation and discussion

The apps in Section 3 designed by the authors can be distinguished from earlier software (see Section 2) in three ways. First, the ProveIt and GeoGebra apps were designed to run on mobile devices, which increase access due to its portable nature [16]. Second, our apps differ in the way by which users enter conclusions. The earlier software requires users to

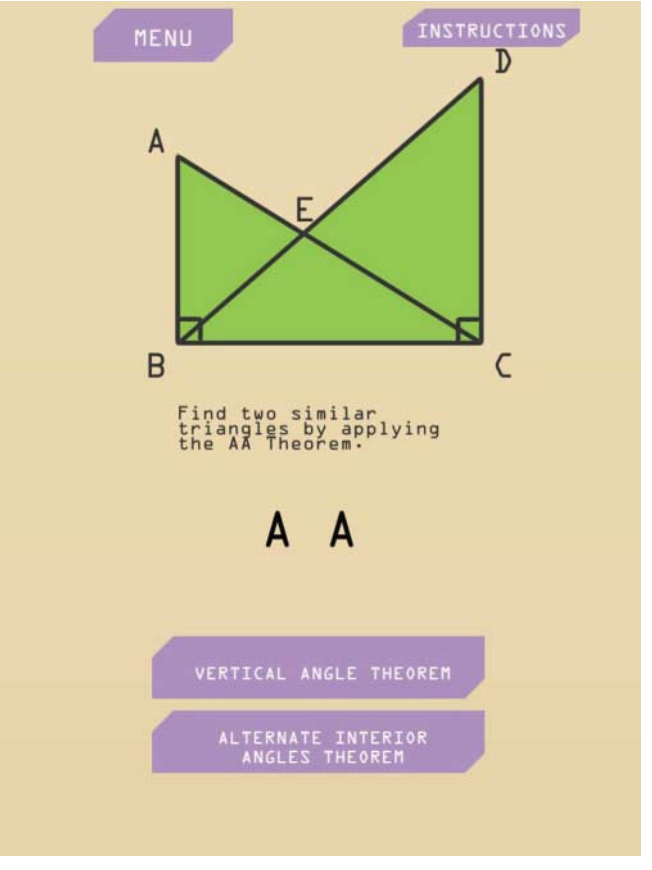


Figure 5. A sample task in Triangle Similarity.

enter conclusions and reasons manually or via a search function. In our apps, the reasons or properties are already indicated; users then need to select the objects (e.g. segments AB and CD) upon which the property may be applied. The design is towards scaffolding learning. Because properties are already indicated at the outset, users who are already at an impasse can readily select a property and focus on how to apply this property on objects on the screen. Further, the feedback can allow students to eventually figure out the correct conclusions. A third distinction between our apps and the existing software is that our apps highlight portions of the proof that have already been achieved. For example, to prove a statement involving SAS, suppose that a user identifies a pair of congruent sides. The app turns one 'S' green, enabling the student to see what has been done (one pair of sides has been shown to be congruent) and what still needs to be accomplished (to find one more pair of congruent sides and one pair of congruent angles).

An initial evaluation of the apps by secondary school students and their teachers yielded promising results. Focus group discussions with students and teachers revealed that students found 'the app is easier because it makes students highlight the parts', and that 'the app makes you think harder, allows you to think more about the figures'. One teacher indicated that the app 'helped them understand the basic concepts of congruence better, since

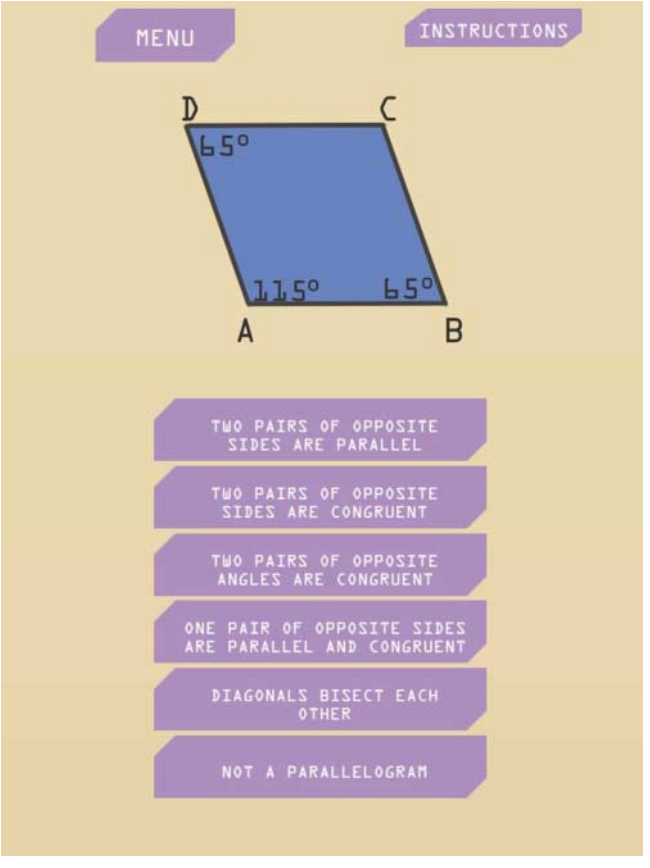


Figure 6. A sample task in Quadrilaterals.

they were able to visualize the angles'. The apps were also found to be different from textbook proof formats because the apps were 'more interactive', and that 'it makes proving not boring'. As one student remarked, 'Maybe when a student is asked to read a book, he may not be interested. But if that person is asked to turn on a tablet, it would be more interesting for him'. Further, the students also reported the app will make it 'easier for them to practice'.

To test the apps' effectiveness, a group of 12 students of varying achievement scores in mathematics were asked to use the triangle congruence app. They were first given a three-item test involving congruent triangles (15 minutes). Two items involved non-overlapping triangles, and the third item involved overlapping triangles. The students were then asked to use the app under the guidance of their teacher (30 minutes). The students were first guided to use Level 1 together for 10 minutes, and then were asked to independently work on Level 2 for 20 minutes. At the end of the session, they were again given a three-item test involving congruent triangles (15 minutes). The items were not the same as those in the pre-test, but were of the same level of difficulty, since they all required the same number of steps in the proof.

The pre-and post-tests were scored according to a rubric informed by Senk [9]. Four points were given for a complete proof. Three points were awarded for a proof with an

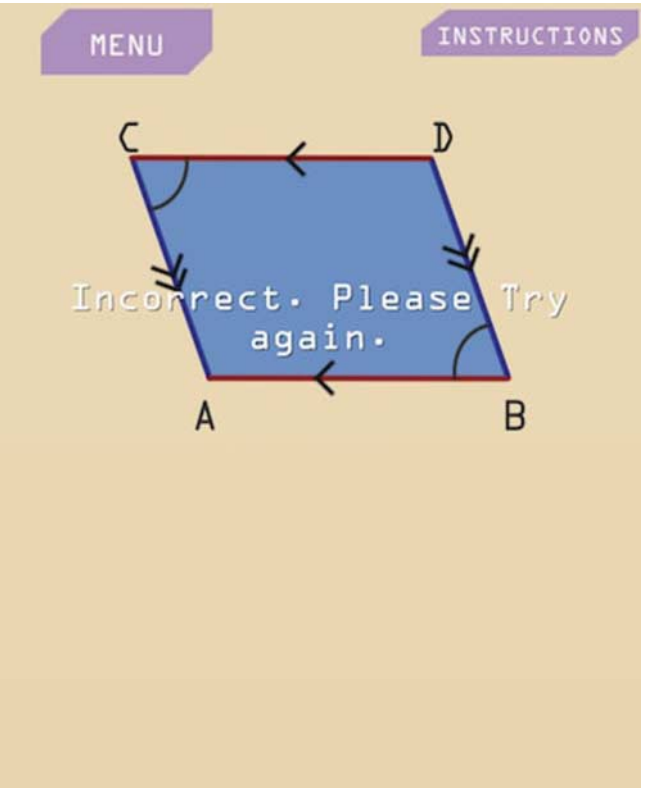


Figure 7. Visual feedback showing that an incorrect response was given.

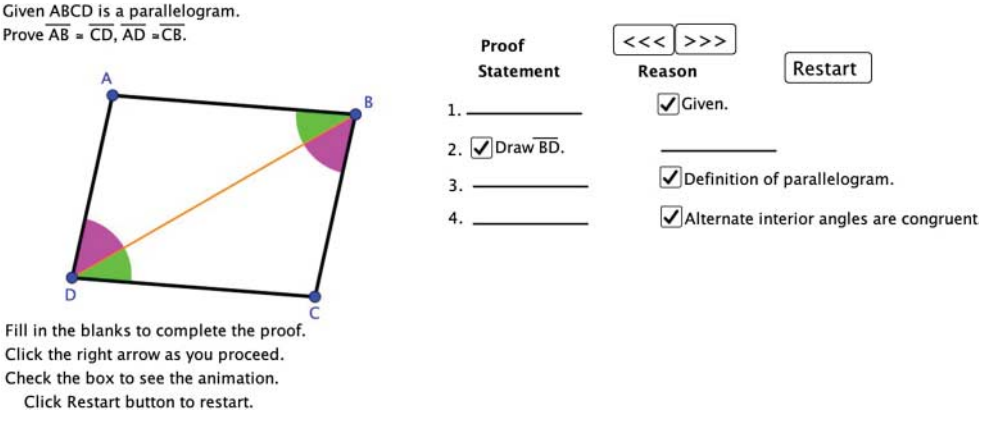


Figure 8. A sample task in the GeoGebra applet for two-column proofs.

error (e.g. typo error, wrong reason but correct conclusion in one step). Two points were awarded for an incorrect proof that contained two correct deductive conclusions related to the two triangles given in the task, and one point was given if only one correct deductive conclusion was made. Zero points were given if a proof only contained the premises or for any blank proof.

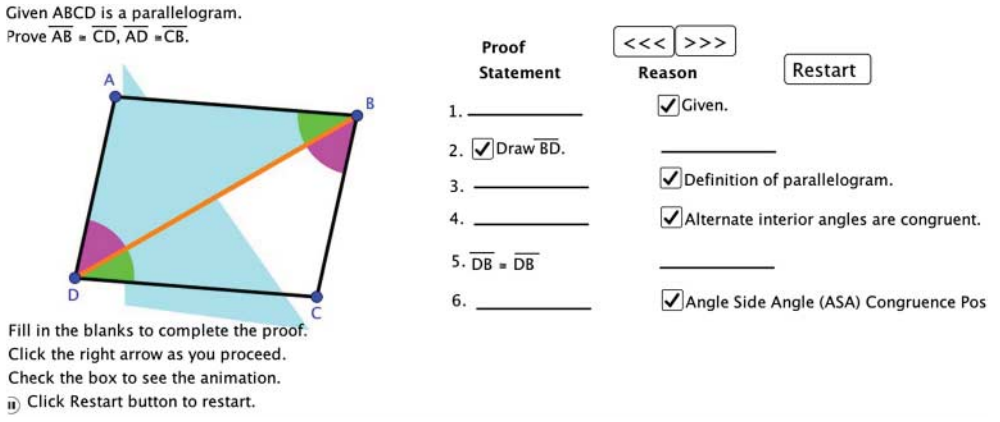


Figure 9. Visual feedback and assistance accompanying the two-column proof (captured mid-animation).

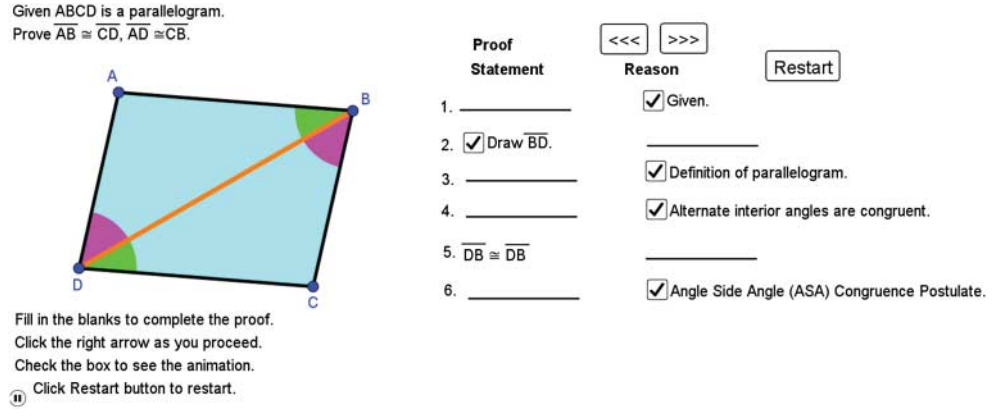


Figure 10. Visual feedback and assistance accompanying the two-column proof (final output).

A Friedman two-way analysis of variance was used to analyse the data. For each student, a rank of 1 was given to the test with a higher score, and a rank of 2 was given to the test with a lower score. If a student scored the same in both tests, the ranks for both tests were set at 1.5. After summing all ranks, the rank for the pre-test was 21.5 and the rank for the post-test was 14.5. This resulted in a p-value of 0.04331, which is greater than the critical value of 3.84 at the $\alpha = 0.05$ level of significance. That is, there is a significant difference in the pre- and post-test scores with the post-test scores being higher.

We also noted that the two students who produced the lowest scores in the pre-test (0 and 2 out of 12 points) did not improve their scores in the post-test. Even after using the app, they could not form any deductive conclusion in the post-test. This result may indicate that the ProveIt app may be more beneficial to students who already have attained some level of competence in proving. Further studies are required to substantiate this claim. Nevertheless, our initial evaluation has shown that even a short session on the use of the ProveIt app can produce some improvement in proving skills. Other apps we have developed for other topics may also be downloaded from https://mathplusresources.wordpress.com.

Acknowledgements

We would like to acknowledge the technical assistance provided by the Ateneo Java Wireless Competency Center (AJWCC), Dr. Marlene De Leon, Mr. Kyle Kristopher Garcia, Mr. Winfer Tabares and Dr. Eduard Taganap in the creation of the apps. We are grateful to Xavier School, Fr. Aristotle Dy and Ms. Adelaida Bague for the support during the testing phase of this study. Thanks also go to Mr. Edwin Morales, Mr. Edrian Neverio, Dr. Jude Buot and Mr. Janree Gatpatan, Mr. Mark Lester Garcia and Dr. Mark Anthony Tolentino for the technical support. The authors would also like to thank the Ateneo de Manila University School of Science and Engineering and the Department of Mathematics for their support towards the research project.

Disclosure statement

No potential conflict of interest was reported by the authors.

Funding

The authors would like to thank the Department of Science and Technology – Philippine Council for Industry, Energy, and Emerging Technologies Research and Development (DOST-PCIEERD) for the funding support.

References

[1] Stylianou DA, Blanton ML, Knuth EJ, editors. Teaching and learning proof across the grades: a K-16 perspective. New York: Routledge; 2009.
[2] De Villiers M. Rethinking proof with Geometer's Sketchpad. Berkeley (CA): Key Curriculum Press; 2003.
[3] Wares A. Using dynamic geometry to stimulate students to provide proofs. Int J Math Educ Sci Tech. 2007;38:599–608.
[4] Wares A. Dynamic geometry as a context for exploring conjectures. Int J Math Educ Sci Tech. 2018;49:153–159.
[5] Marrades R, Gutierrez A. Proofs produced by secondary school students learning geometry in a dynamic computer environment. Educ Stud Math. 2000;44:87–125.
[6] Herbst PG. Establishing a custom of proving in American school geometry: evolution of the two-column proof in the early twentieth century. Educ Stud Math. 2002;49:283–312.
[7] Schoenfeld AH. Series editor's foreword: the soul of mathematics. In: Stylianou DA, Blanton ML, Knuth EJ, editor. Teaching and learning proof across the grades: a K-16 perspective. New York (NY): Routeledge; 2009. p. xii–xvi.
[8] Weiss M, Herbst P, Chen C. Teachers' perspectives on 'authentic mathematics' and the two-column proof form. Educ Stud Math. 2009;70:275–293.
[9] Senk SL. How well do students write geometry proofs? Math Teach. 1985;78:448–456.
[10] Verzosa DMB, Tulao-Fernando MT, Vistro-Yu CP. Congruence between context and opportunities for professional development of mathematics teachers in the Philippines. In: Kaur B, Kwon ON, editors. Professional development of mathematics teachers: an Asian perspective. Singapore: Springer; 2017. p. 89–96.
[11] Wertheimer R. The geometry proof tutor: an 'intelligent' computer-based tutor in the classroom. Math Teach. 1990;83:308–317.
[12] GeoGebra [Internet]. International GeoGebra institute; [cited 2018 Jan 24]. Available from: https://www.geogebra.org.
[13] Pham TM, Bertot Y. A combination of a dynamic geometry software with a proof assistant for interactive formal proofs. Electron Notes Theor Comput Sci. 2012;285:43–55.
[14] Alicbusan RN, Foz LN, Merle PV, et al. GPT: A tutor for geometry proving. In: Chen W, et al., editors. Proceedings of the 25th international conference on computers in education; 2017 Dec 4–8; New Zealand. Asia-Pacific Society for Computers in Education; 2017. p. 68–73.

[15] Larkin K, Calder N. Mathematics education and mobile technologies. Math Educ Res J. 2016;28:1–7.
[16] Norris C, Hossain A, Soloway E. Using smartphones as essential tools for learning. Educ Tech. 2011;51(3):18–25.
[17] MathPlus Resources Page [Internet]. Quezon City, Philippines: Ateneo de Manila University; [cited 2018 Jan 24]. Available from: https://mathplusresources.wordpress.com/.

Ateneo de Manila University

Archīum Ateneo

Mathematics Faculty Publications | Mathematics Department

2023

Bridging the Mathematics Gap Through the Use of Mathematical Apps

Ma. Louise Antonette N. De las Penas

Debbie Marie B. Verzosa

Maria Alva Q. Aberin

Agnes D. Garciano

Juan Carlo F. Mallari

See next page for additional authors

Follow this and additional works at: https://archium.ateneo.edu/mathematics-faculty-pubs

 Part of the Algebra Commons, Algebraic Geometry Commons, Educational Technology Commons, and the Other Mathematics Commons

Authors

Ma. Louise Antonette N. De las Penas, Debbie Marie B. Verzosa, Maria Alva Q. Aberin, Agnes D. Garciano, Juan Carlo F. Mallari, Jumela F. Sarmiento, and Mark Anthony C. Tolentino

Bridging the Mathematics Gap Through the Use of Mathematical Apps

Ma. Louise Antonette N. De Las Peñas
mdelaspenas@ateneo.edu
Department of Mathematics
Ateneo de Manila University, Philippines

Debbie Marie B. Verzosa
dmbverzosa@usm.edu.ph
Department of Mathematics and Statistics
University of Southern Mindanao, Philippines

Maria Alva Q. Aberin
maberin@ateneo.edu
Department of Mathematics
Ateneo de Manila University, Philippines

Agnes D. Garciano
agarciano@ateneo.edu
Department of Mathematics
Ateneo de Manila University, Philippines

Juan Carlo F. Mallari
jmallari@ateneo.edu
Department of Mathematics
Ateneo de Manila University, Philippines

Jumela F. Sarmiento
jsarmiento@ateneo.edu
Department of Mathematics
Ateneo de Manila University, Philippines

Mark Anthony C. Tolentino
mtolentino@ateneo.edu
Department of Mathematics
Ateneo de Manila University, Philippines

Abstract: *During the COVID-19 pandemic, school campuses worldwide were forced to close, and students had to learn primarily from home. This sudden disruption is estimated to have caused significant learning loss among learners. This paper reports the use of mathematical applications (apps) to bridge the mathematical learning gaps in Grades 1 to 11 in the Philippines after the pandemic, as part of a project funded by a national government agency. The apps include those that strengthen foundational concepts in number and fraction sense in grade school mathematics, develop proving skills in geometry, promote mastery in algebraic and trigonometry through drill and practice, and facilitate statistical understanding and reasoning. The description of the apps, their design, and pedagogical basis are discussed. Challenges encountered in the implementation of the project are also presented.*

1. Introduction

The COVID-19 pandemic disrupted education worldwide due to the closure of schools and the shift to online and blended learning modalities. As classes have returned onsite post-pandemic, educators, teachers, and school administrators are assessing the extent of the learning loss in students, and finding means to address this. In the school year 2022-2023, the Department of Education (DepEd) in the Philippines developed the *Basic Education Learning Recovery Plan* [1] to guide schools in addressing and identifying learning gaps. This was followed by the adoption of the *National Learning Recovery Program* [2] to address these gaps and learning setbacks. The program is currently being implemented across all grade levels by local DepEd school division offices, with their external stakeholders and partners.

Under a government-funded project [3] our team collaborated with some DepEd school divisions in providing mathematical resources to partner schools. These resources (also called *Mathplus* resources) consist of mathematical applications (apps), teaching guides/instructional videos, and

performance tasks. The mathematical apps were designed to address the Most Essential Learning Competencies (MELCs) prescribed by DepEd [4] as well as to help narrow the existing learning gaps on competencies that were determined by the school administrators and teachers in our partner schools. The teaching guides and instructional videos were created as support to teachers towards the implementation of the apps in the curriculum. In partnership with the schools, capacity building seminars and webinars were given to teachers to guide them on the use of these resources.

The mathematical resources are made available through the *Mathplus* website https://mathplusresources.wordpress.com. Two new frameworks for sharing information are also used that are geared towards maximizing the reach of educational materials to locations with minimal internet connectivity. For selected schools, one way of distribution of the mathematical resources is through a community LTE network [5]. The community LTE (Long Term Evolution) network used is a mobile cloud network architecture called *Educloud* that facilitates a systematic and resilient way of distributing educational content that is ideal for distant and asynchronous learning. It is low cost, has a low technology requirement and is compatible with *Mathplus* resources for distribution [6]. A second way of distribution of instructional content is using a digital datacasting framework called *RuralCasting* [7]. Under this technology, a custom *set-top box* receives usual TV programming, along with the attached data content, providing multiple users access to the content thru Wi-Fi, and return information back using alternative transmission methodologies. The content can also be updated regularly using the datacasting transmission. The contents, namely, the mathematics applications, videos, text files, images are locally stored in the set-top box. A lightweight learning management system like Moodle and Canvas but specifically for the RuralCasting set-top box, named *Edukastv* was created. In *Edukastv*, the content creators or teachers can upload their subject outline, reading modules, quizzes, etc. for the students to access.

This paper discusses a selection of mathematical apps that were designed under [3] to bridge the mathematical learning gaps as communicated by partner schools to the *Mathplus* team. These apps are, for Grades 1-6: foundational concepts on place value, fraction and number sense; for Grades 7 to 10: algebra, geometry and trigonometry; and statistics for Grades 1 to 11. The design and pedagogical basis of the apps are discussed briefly. We also discuss the challenges in the implementation of the project.

2. General Design Considerations

The use of technology in mathematics education is at times criticized for perpetuating the "digital divide" or the gap between those with access to modern technology and those who do not [8]. However, the apps described in this paper were designed in consideration of under-resourced educational systems such as the Philippines where students and even teachers have limited access to the latest technology. First, the apps were meant to run on Android devices with basic specifications. Second, the apps were made freely available even for schools with little or no budget for purchasing software. Third, the portability and convenience of using mobile devices allow students to access educational materials more frequently [9], even when they are in their homes or when they are not under the guidance of their teachers.

The following key elements were incorporated in the design of the apps: the existing literature on mathematical learning, the benefits of technology in mathematics education and game-based learning principles.

An important strategy that has been adapted in the aforementioned project is the use of game-based learning. Thus, several of the apps developed in the project are games that have clearly defined learning outcomes. The use of games for learning may lead to increasing students' motivation and

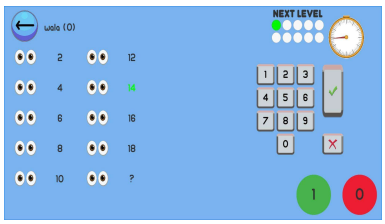
engagement while allowing them opportunities for graceful failures and to customize their playing experiences according to their own situation [10]. Thus, such games may have a stronger potential to help with bridging learning gaps in mathematics, which many students find difficult and/or unenjoyable.

The design of our game-based apps is guided by Shi and Shih's Game-based Learning (GBL) Design Model, which includes macro-design concepts and 11 interrelated game-design factors: game goals, game mechanism, game fantasy, game value, interaction, freedom, narrative, sensation, challenges, sociality, and mystery [11]. The central game-design factor is game goals, which naturally affect the other factors of the game's design. In the case of our apps, the game goals always involve the learning or development of different mathematics learning competencies. The game mechanism is then usually guided by pedagogical research that promotes a more effective achievement of the learning competencies. To increase engagement, the games are designed to be interactive and responsive while allowing some freedom for players to customize their playing experience. The game fantasy, which includes its narrative and sensation, is mostly influenced by the game mechanics and the target grade levels for the game. In most cases, simple narratives are used and are complemented with colorful graphics and appropriate music and sound effects. To enhance the game value, most of the games have different modes and levels that contribute to the game's mystery while also setting up a progression according to the challenge or difficulty level. Usually, this progression is also aligned to the progression of the learning content. Since the apps are also intended for independent learning, they are mostly single-player games; however, opportunities for social interactions through the game can be possible in the classroom or virtual meetings.

3. Apps for Grades 1 to 6

Remediation on the four fundamental operations has been continuously carried out in elementary schools in the Philippines even before the pandemic. Multiplication, for example, is one of the least learned competencies of students that has been emphasized by one of our partner school division offices. The *Multiplication Game* is one of the apps that we have designed to address this need.

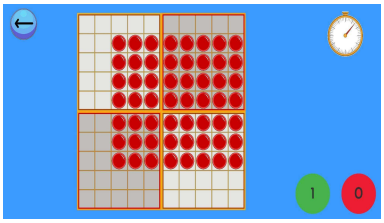
The *Multiplication Game* facilitates mastery of the multiplication table through research-based representations such as repeated addition (modeled by equal groups), arrays and skip counting (modeled by a number). The range of representations is necessary because some representations are more accessible than others in promoting multiplicative thinking [12]. Further, illustrating multiplication using these representations is one of the MELCs prescribed by the DepEd [4] in Grades 1 and 2, The *Multiplication Game* addresses this, starting with the *Beginner* level where students carry out repeated addition by 2's, 5's, and 10's (Figure 3.1(a)) and in the *Advanced I* and *II* levels where students perform skip counting by 3's, 4's, 6's, 7's, 8's and 9's. The sequence of levels follows the learning trajectory and the official DepEd curriculum [13] which starts with multiples of 1, 2, 5, and 10, in Grades 1 and 2, followed by the other multiples, starting in Grade 3.



(a) (b)



(c)



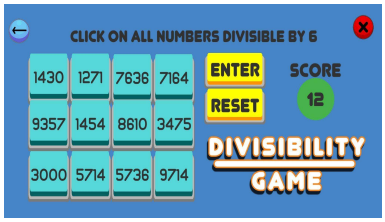
(d)

Figure 3.1 Repeated addition, skip counting and use of arrays in the *Multiplication Game*

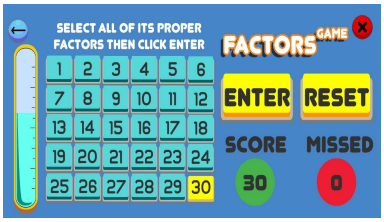
Skip counting is the recommended strategy by DepEd for Grade 4 learners [13] to identify the multiples of a given number up to 100. The game illustrates this using equal jumps in the number line. It is recommended that teachers first write multiplication equations on the board that match the repeated jumps on the number line. Using the *Multiplication Game*, students use skips of 5 (Figure 3.1(b)), for example, to help them multiply by 5. Students input the missing multiple in the number line. When students reach the last part of the *Beginner* and *Advanced* I and II levels, they write a multiplication equation using repeated addition and an array. As shown in Figure 3.1(c), students give the product of "9 x 3" and then the game will show that this is the same as "3 groups of 9". Consequently, the concept of commutativity is also reinforced by the *Multiplication* game. Lastly, in the *Expert I* and *II* levels (Figure 3.1(d)), students are shown red dots on an array and players must input the number of red dots. As the number of red dots becomes bigger, students will refrain from counting each dot one by one but will rather use a multiplication equation to get the number of dots.

After mastering the multiplication table by playing the *Multiplication Game*, students practice using divisibility and factoring rules by playing the *Divisibility Game* and *Factors Game*. Grade 5 learners are expected to use divisibility rules to find for 2, 5, and 10; 3, 6, and 9; 4,8,12, and 11 to find the common factors of numbers. In the *Divisibility Game* (Figure 3.2(a)), students can choose the dividends (3 or 4 digits), the divisors, for example 3, 6, 9, and the number of items (9, 12, or 16). Then the students click on the numbers that are divisible by the divisor. On the other hand, in the *Factors Game* (Figure 3.2(b)), the student first selects a number and then all its factors. The game helps students of varying grade levels in determining factors as well as multiples of a given number. This is helpful in providing them with a firm understanding of prime and composite numbers [14].

Some apps and games were also designed to study fractions, one of the least learned competencies for Grades 1 to 6. The learning of fractions may start with early experiences of 1/2 and 1/4, with the eventual aim of conceptualizing fractions as a number; that is, as a point on the number line [15].



(a)



(b)

Figure 3.2 The *Divisibility Game* and *Factors Game*

The apps designed for learning fractions provide various representations that are intended to help students connect fractions to the number line model. The first one we will mention here is the *Moving Fractions* game (Figure 3.3(a)). The following concepts can be visualized in a game-like setting: 1/2

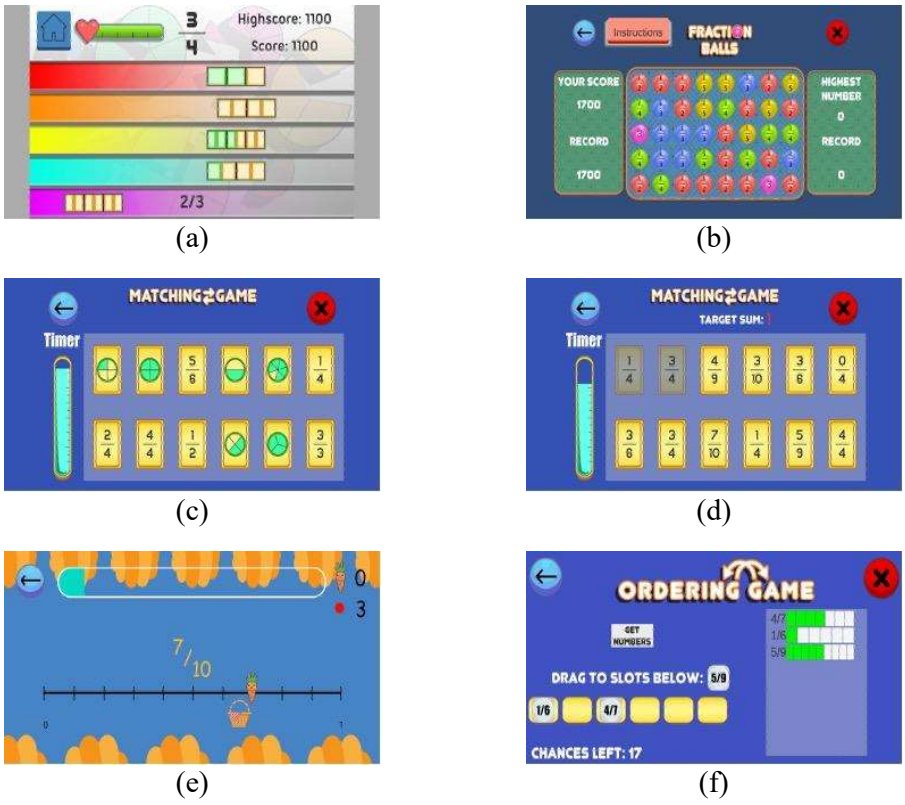


Figure 3.3 Apps and games focusing on fractions

and 1/4, unit fractions, proper fractions, improper fractions, and mixed numbers. Students can envision how to add (join) or subtract fractions. This game is discussed in detail in [16]. Another game is the *Fraction Balls* game (Figure 3.3(b)) which shows addition (through joining) and subtraction (through missing addend) of similar fractions with colored visual supports. The user has the option to work with sums that convert to the lowest terms or not. In the *Matching Game*, a set of cards is shown (Figure 3.3(c)) and the student must match fractions and their corresponding pictures. Another option of this game asks students to match two fractions that add up to a certain target sum (Figure 3.3(d)). Students also have an option to play with whole numbers or decimals in the *Matching Game*. In the *Catch the Carrot* game, a random fraction and a number line are shown on screen (Figure 3.3(e)). The student must estimate where the fraction is located on the number line. The student's aim is to situate as many numbers as possible within the time given. Other options available in the *Catch the Carrot* game are whole numbers, decimals, integers or irrationals. The article [17] presents the design of C*atch the Carrot*. It also discusses the pedagogy of the use of number lines in developing the estimation and number sense skills of students. In the *Ordering Game*, a fraction is generated at random (Figure 3.3(f)), and the learner drags it to a row. The goal of the game is to fill up the entire row, so that the fractions appear in an ascending order, within the allowed number of chances, indicated at the bottom part of the screen. The game becomes more difficult as it progresses because it becomes more likely that a randomly selected number cannot be correctly placed in any of the remaining slots [18].

To summarize, a list of apps that are used by learners in different grades levels, based on their math curricula, are given in the table below.

Table 1. Applicable apps for different grade levels

Level	Apps	Level	Apps
Grade 1	Multiplication Game, Matching Game, Moving Fractions, Catch the Carrot	Grade 4	Multiplication Game, Factors Game, Moving Fractions, Fraction Balls
Grade 2	Multiplication Game, Matching Game, Moving Fractions. Ordering Game, Catch the Carrot	Grade 5	Multiplication Game, Factors Game and Divisibility Game, Moving Fractions, Fraction Balls, Ordering Game, Catch the Carrot
Grade 3	Multiplication Game, Matching Game, Moving Fractions. Ordering Game, Fraction Balls, Catch the Carrot	Grade 6	Moving Fractions, Fraction Balls, Ordering Game, Catch the Carrot

4. Apps for Grades 7 to 10

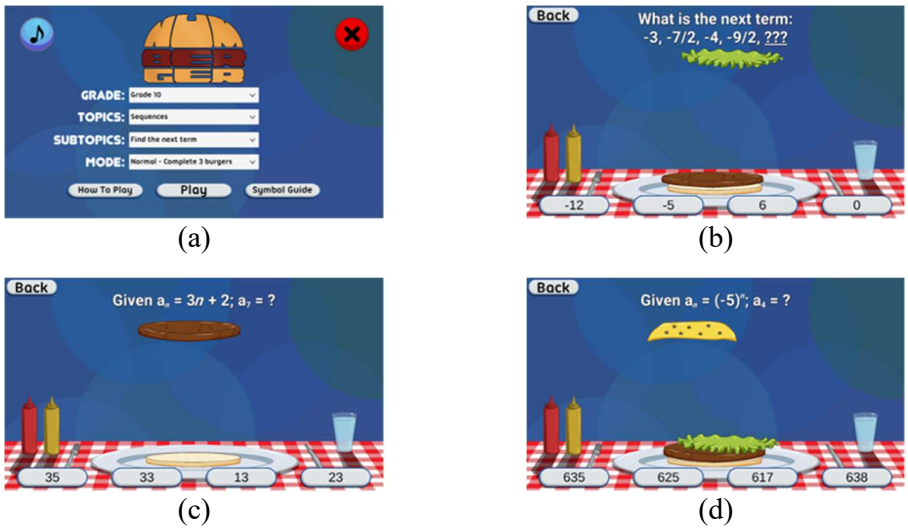
A selection of the mathematical apps developed for use in Grades 7 to 10 is meant to engage students in regular practice and drills on topics which the students have difficulty with. These apps, designed with game-like settings, are accessible, easy to play with and provide immediate feedback to students. This helps direct the students to either proceed to advanced levels in the app or simply give them opportunities to practice their math skills. Some mathematical skills can also be learned through deliberate practice [19]. Deliberate practice involves the learning of higher-order skills; drill and practice, meanwhile, is often associated with procedural skills [19]. Features of deliberate practice include well-defined goals to address weak points as determined for instance, by a teacher. Thus, the apps designed using the deliberate practice model can be used for remediation activities to provide students the opportunity to work on their weaknesses.

An important skill in mathematics is the ability to recognize number patterns. To this end, the authors designed an app called *Numberger* (Figure 4.1(a)). The app covers different topics: number identification (integers, rational numbers etc.), equivalent fractions, domain and range of functions and sequences. It aims to give opportunities for students to practice various math skills. The objective of *Numberger* is to let the user make burgers (hence the name "Numberger") by being able to choose the correct answer to a given question. One of the topics included in this app is "Sequences". There are two subtopics under "Sequences": "Find the Next Term" and "Find the Term". In the first subtopic, a student is shown some terms of a sequence and is asked what the next term is (Figure 4.1(b)). For this he needs to see the pattern or rule to be able to answer the question correctly. The questions are shown one at a time at the top part of the screen while an ingredient to make a burger is initially shown moving back and forth from left to right on the screen. Some options for the answer are shown at the bottom part of the screen and when a correct answer is chosen, the ingredient falls on the plate found at the bottom of the screen. In the second subtopic, an expression for the nth term, a_n, of a sequence is given and the student is asked to give the value of a particular term. Here the focus is on how to evaluate a term of a sequence, hence the sequence given may be an arithmetic sequence (Figure 4.1(c)) or a geometric sequence (Figure 4.1(d)).

There are two modes for playing the app: Normal Mode-Complete 3 Burgers or Endless-Play without end (Figure 4.2(a)). A player who chooses the Normal Mode receives a "Good Job" message after completing three burgers (Figure 4.2(b)) and can choose to go play again or end the game.

Another app that employs the deliberate practice model to hone algebraic skills is *Just Keep Solving.* The app focuses on helping students become more proficient in solving linear equations and

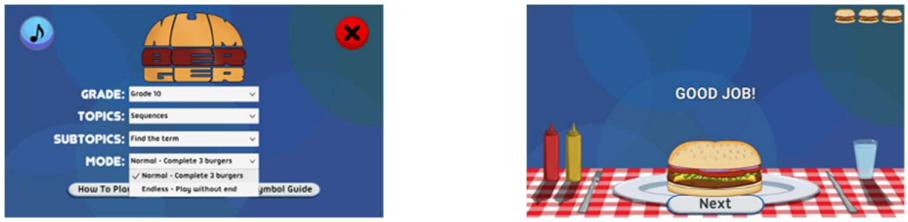
inequalities. There are two topics: *Linear Equations* and *Linear Inequalities*. A sample opening screen of this app is shown in Figure 4.3(a) where the topic chosen is *Linear Inequalities*.



(a) (b)

(c) (d)

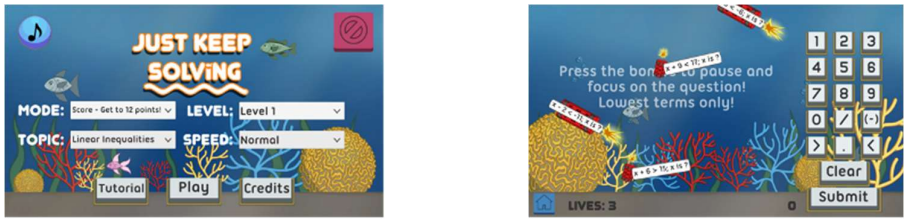
Figure 4.1 (a) Numberger Topic: Sequences; (b) Finding the Next term; (c) Finding a term (arithmetic sequence); (d) Finding a term (geometric sequence)



(a) (b)

Figure 4.2 (a) Modes: Normal or Endless; (b) Message after completing 3 burgers

When a student presses the play button, a screen such as the one shown in Figure 4.3(b) appears. The display is an underwater scene and bombs containing inequalities to be solved fall from the top of the screen. The goal is to be able to solve the inequalities before the bombs reach the ocean floor and cause destruction of the corrals. The mechanics for the topic of *Linear Equations* are like that of *Linear Inequalities*. More details about this app are given in the paper [20].



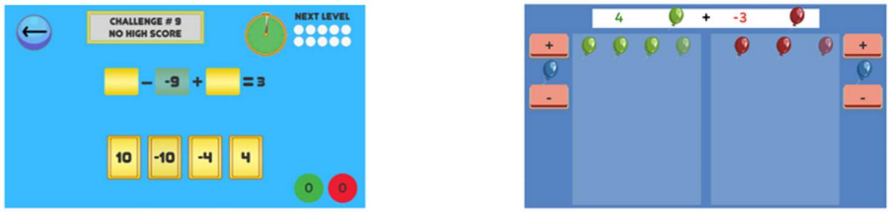
(a) (b)

Figure 4.3 The *Just Keep Solving* app

Disruption to education systems in the pandemic has led to many students experiencing learning loss and learning gaps as they enter high school. For instance, Grade 7 students have not mastered numeracy skills involving fractions, decimals, and integers. Some of the apps developed during the

project were helpful in a high school remediation program to address this problem. The *Number puzzles* app, for instance, provides numerous opportunities for practice involving addition, subtraction, and multiplication. *Algeops* incorporates three models: objects, numerals, and number line in a visualization of integer operations. The app incorporates both the neutralization and number line models [21] which offer a more holistic representation of integers. *Moving Fractions and Fraction Balls* were also very helpful in addressing the difficulties of the students in fractions.

Students and teachers were given a survey through Google Form with the goal of evaluating the apps used in the remediation program in three (3) different categories: (a) usability; (b) game-like experience; and (c) alignment with learning objectives. Results indicate a positive response from students and teachers. Most of the students (73%) had no difficulty using the apps. The apps helped 74% of the students comprehend the lesson and its solutions. The apps also helped them find the answers easily and quickly. About 70% of the students noted that the apps provided a lot of opportunities for practice and expressed that with continued use, their math skills would improve. Moreover, the teachers found the apps useful in helping the students stay motivated and engaged throughout the lesson. They also commented that the numerous exercises provided by the apps helped the students master the foundational skills in mathematics.

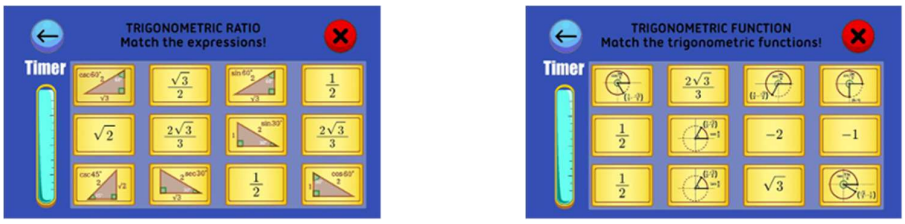


(a) (b)

Figure 4.4 The (a) *Number Puzzles* and (b) *AlgeOps* apps

One of the high school math subjects that requires the development of algebra and geometry abilities is trigonometry. These abilities are crucial for studying pre-calculus and calculus. In the course of our work in [3], teachers reiterated the need for technological tools involving trigonometry.

Trigomatch employs an interactive game with interactive feedback to help users master basic trigonometric concepts like reference angles, trigonometric ratios and trigonometric functions. It presents visual representations of these concepts to allow students to extend their understanding of trigonometry from the right triangle to the unit circle approach. When visual representations of trigonometry are presented using interactive technology, they are more accurate and allow the learners to proceed at their own pace [22]. Additionally, the accuracy of these presentations can reinforce images in a person's memory. This is not the case for hand-drawn images which tend to be distorted. Figure 4.3(a) and 4.3(b) display the *Trigonometric Ratio* and *Trigonometric Function* set-up, respectively. The objective is for the learners to match two equivalent representations of the trigonometric ratio and function within the allotted time.

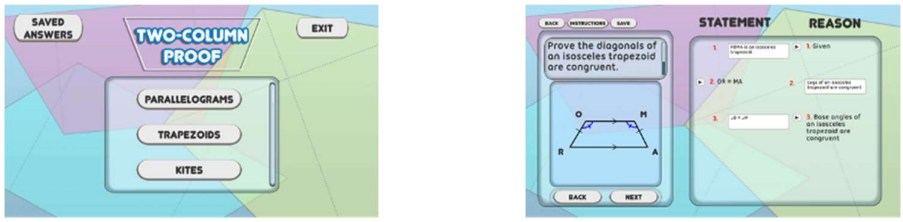


(a) (b)

Figure 4.5 (a) *Trigonometric Ratio* Level; (b) *Trigonometric Function* Level of *Trigomatch*

In high school geometry, one of the topics teachers find very challenging to teach is proving geometric statements. Students struggle with writing proofs and find the process abstract and uninteresting. This situation was exacerbated during the time of the pandemic when it was much more difficult to teach proving remotely. When teaching proofs, teachers rely on textbooks or modules, students' opportunities to learn proofs are limited to what is provided in these instructional materials. Two apps to meet the needs of teachers and students on proving are *Two Column Proof* and *ProveIt.* The design of both apps is grounded on the use of visualization and imagery in mathematics [23].

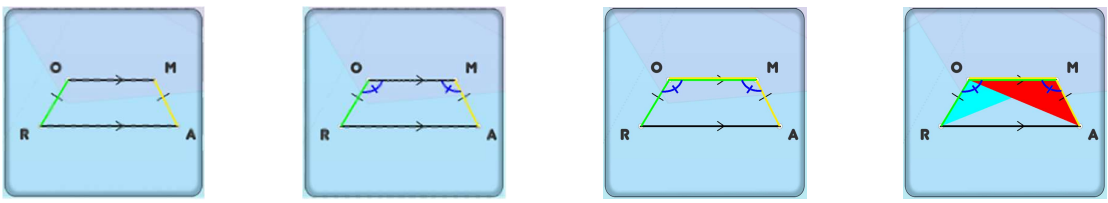
The *Two Column Proof* app is structured to support the students in writing a two-column proof, where a statement or reason in the proof is supported by a visualization element. The intent is to highlight properties and congruence relations of sides, angles and triangles through animation which will serve as visual clues to the students when completing the line of a proof. The visualization elements not only strengthen the student's understanding of the proof process but also stimulate their interest. In the Philippines, Grade 9 lessons on writing two column proof begin with properties of parallelograms. Much time is utilized discussing the first proving topic on parallelograms so oftentimes proving properties of other quadrilaterals such as kites and trapezoids are left for the students to work on individually. We designed the *Two Column Proof* app to include proving geometric concepts on parallelograms, kites, and trapezoids (Figure 4.6(a)).



(a) (b)

Figure 4.6 (a) Main Menu of *Two Column Proof;* (b) Proving exercise on trapezoids

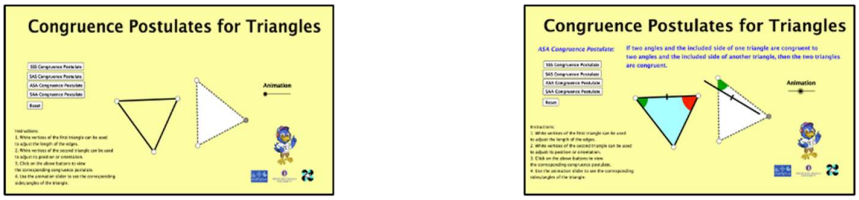
One proving exercise given in the *Two Column Proof* app is to prove the diagonal isosceles trapezoid congruency (Figure 4.6(b)). An example of a task is to write a mathematical statement in the "Statement" column where the justification in the "Reason" column is given. To guide the student, an animation is provided to visualize the property of a trapezoid or a congruence property in the "Reason" column. For instance, to visualize "Base angles of an isosceles trapezoid are congruent", the congruent base angles (highlighted in blue) of the isosceles trapezoid are flashed repeatedly (Figure 4.7(b)). Next, to visualize the Side-Angle-Side (SAS) congruence, an animation is provided in the following sequence: a pair of sides OR and MA from two different triangles (Figure 4.7(a)), a pair of congruent angles ROM and AMO (Figure 4.7(b)), the common side OM (Figure 4.7(c)). Then the two triangles are highlighted: triangle ROM (light green) followed by triangle AMO (red) (Figure 4.7(d)).



(a) (b) (c) (d)

Figure 4.7 Animations in the *Two Column Proof* app

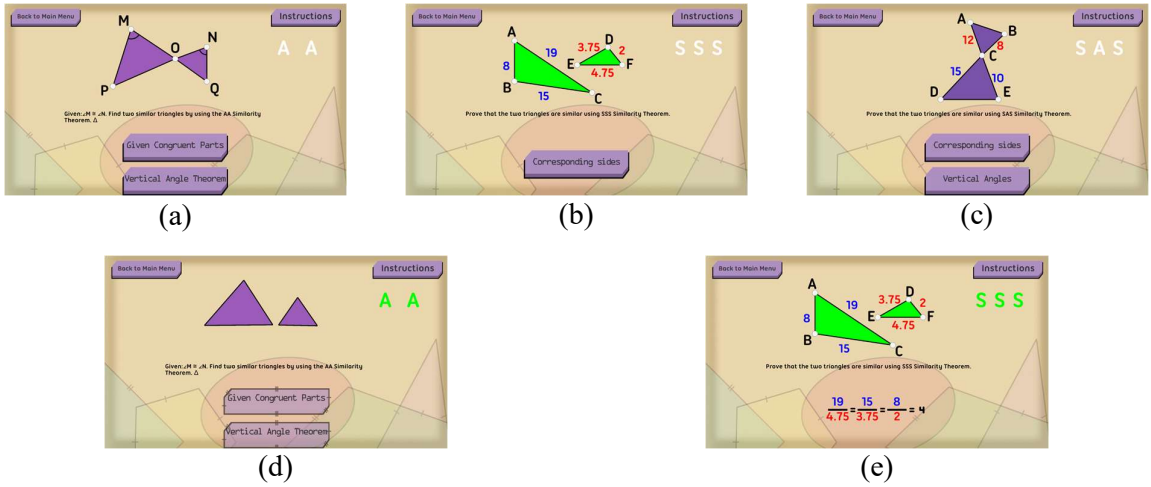
One of the essential tools when formulating proofs is knowledge of previously taught mathematical concepts that are needed in formulating the proof. For instance, in proving concepts on parallelograms, kites or trapezoids, one needs a firm understanding of the congruence postulates for triangles. Unfortunately, students either forget these concepts that have been taught earlier in high school, or these were not taught at all in the time of Covid. We also designed *GeoGebra* apps on triangle congruence for this purpose (Figures 4.8(a)-(b)). One advantage of *GeoGebra* is one can create applications in a much shorter time (than math apps that require some extensive programming) to fill in the learning gap, when necessary.



(a) (b)

Figure 4.8 GeoGebra applets on Triangle Congruence

Prove It is another app that aims to facilitate students' proving competency. The first version of *Prove It* for the topic of triangle congruence is given in [24]. In [3], the app was extended to include the topic of triangle similarity, as a need to bridge the learning gap. A task requiring the Angle-Angle (AA) Similarity Theorem (Figure 4.9(a)), Side-Side-Side (SSS) Similarity Postulate (Figure 4.9(b)), or SAS Similarity Postulate (Figure 4.9(c)) is initially shown. The corresponding text AA, SSS, and SAS on the screen provides guidance to help students draw their attention to the relevant parts of the triangles. For AA, students need to select two pairs of congruent angles. For SSS, students need to select three pairs of proportional sides. For SAS, students need to select two pairs of proportional sides and one pair of congruent angles (which should be the included angle between the two proportional sides). Feedback is also provided to guide students towards selecting the correct pair. Once the student successfully identifies the correct pairs, an animation shows that the two triangles are similar (Figure 4.9(d)); for SSS and SAS, a corresponding ratio is shown (Figure 4.9(e)).



(a) (b) (c)

(d) (e)

Figure 4.9 The app *ProveIt* featuring tasks on Similarity

In the third quarter of the school year 2022-23, a short study was conducted to assess the effectiveness of the *Two Column Proof* app. Two pairs of control-experimental classes in a public high school in Quezon City, Philippines were given pretests and posttests covering the following competencies: (i) proves theorems on the different kinds of a parallelogram (rectangle, rhombus, square), (ii) proves theorems on trapezoids and kites. For the first pair of classes, the control class (38 students) recorded a mean gain score of 30.94% while the experimental class (40 students) recorded a mean gain score of 44.90%. The gap between mean gain scores was wider for the second pair of classes: 25.73% for the control (25 students) and 79.29% for the experimental (27 students). These results indicate the potential of the *Two Column Proof* app to help students learn geometry, particularly proving topics, better.

5. Probability and Statistics Apps

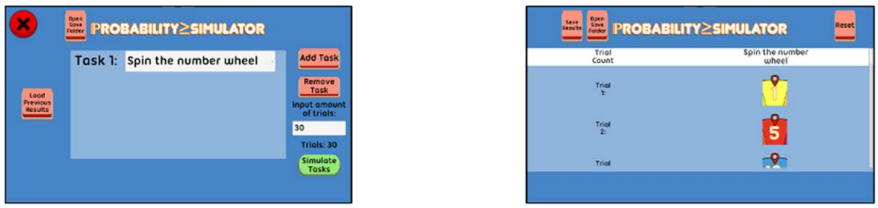
Probability and Statistics is identified as one of the five content strands in the Philippines' K to 12 Basic Education Curriculum [13]. The focus of the strand is on "developing skills in collecting and organizing data using charts, tables, and graphs; understanding, analyzing and interpreting data; dealing with uncertainty; and making predictions about outcomes" [13, p.5]. Competencies in probability and statistics primarily comprise the fourth quarter of the mathematics subject in Grades 1 to 10, except 9. Additionally, Grade 11 students take the core subject *Statistics and Probability* during the third and fourth quarters of the school year [25].

Even before the pandemic, there have already been some challenges in ensuring that students develop the learning competencies in probability and statistics. As the corresponding topics are scheduled to be covered during the last quarter of each school year, class delays or cancellations (e.g., due to typhoons, other disasters, or unexpected holidays) usually result in some of these topics being taught quickly or neglected altogether [26, pp. 300-301]. Another challenge in statistics education in the Philippines is the lack of resources [27]. Particularly, there is a need for accessible and available materials that can support or enhance the teaching and learning of probability and statistics that are also compatible with independent learning.

To bridge the learning gap in probability and statistics, we have developed different apps as part of [3]. These apps have been deliberately designed so that these can be used not only in actual classroom settings but also for remote or independent learning activities.

As emphasized in the Pre- K-12 Guidelines for Assessment and Instruction in Statistics Education II (GAISE II), "the focus on variability in data, the importance of context associated with the data, and the questioning of data, sets statistics apart from other mathematical sciences and makes it particularly relevant for all fields of study" [28, p.8]. Thus, it is important to have different data sets with clear contexts and with which learners can deal with variability. Further, a classroom study by Pfannkuch [29] has found that the contexts of data sets have the potential to assist students, up to a certain extent, in developing informal statistical inferential reasoning. Rossman and Chance [30], in their proposed introductory course for statistics, included activities where students are tasked to do probabilistic experiments such as flipping a coin. This is in alignment with Cobb [31], who has suggested using simulation- and randomization-based methods to develop statistical inference.

Given the above, we have developed the Android/PC app *Probability Simulator* and the web-based app *Senso Eskwela Pilipinas* (*SEP*). The *Probability Simulator* app (Figure 5.1) allows for the simulation of different probabilistic experiments (e.g., rolling a fair or unfair die, drawing a card from a deck). The number of trials of the experiments can be set in the app. The outcomes are displayed in the app or can be saved as a spreadsheet file for further processing or analysis.



(a) (b)

Figure 5.1 Screenshots of *Probability Simulator*'s (a) home screen and (b) outcomes screen

SEP [32, 33] is an online platform that is based on the Census-at-schools program [34]. Students get access to *SEP* through their teachers and, once logged in, they can anonymously answer a 31-question survey that consists of varied questions that can appeal to young Filipinos' interests and experiences. The students' responses then become part of the *SEP* database that is accessible to *SEP* users. Teachers and students may access their own class data while any *SEP* user may access random samples from the database. Thus, *SEP* becomes a readily accessible source of authentic data that is relevant and relatable to the students. Initial data on the potential benefits of *SEP* with respect to students' learning of statistics are presented in [33].

With *Probability Simulator* and *SEP*, students have ready and easy access to data sets that are otherwise difficult, timely, or tedious to obtain. Moreover, these contextually clear data sets exhibit the variability that students need to handle when learning probability and statistics. Finally, both apps can be used with minimal supervision so these are ideal for independent learning and can be repeatedly used for learning activities across different competencies in probability and statistics.

Student worksheets have also been developed to complement the apps mentioned above. Each student worksheet corresponds to a learning activity; the materials needed, and step-by-step instructions are provided in the worksheet. The worksheets are also formatted in a way that students can readily write their answers on the worksheets, whether on a printout or electronically. For example, in the Grade 3 worksheet *Likely and Unlikely* (Figure 5.2), students are asked to use the

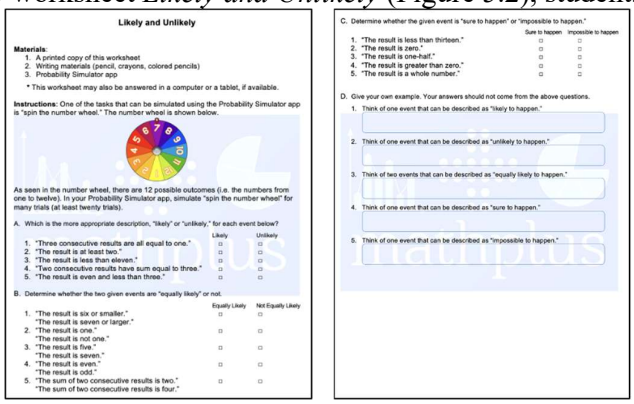


Figure 5.2 Screenshots from the Grade 3 student worksheet *Likely and Unlikely*

Probability Simulator so that they can develop their sense of events that are likely, unlikely, sure to happen, or impossible to happen. Another example is a Grade 11 worksheet *Discrete Random Variables,* which outlines an activity in which students use an *SEP* question and data set to define a discrete random variable then construct its probability distribution and histogram. The student worksheets can be used for in-classroom activities and are also ideal for independent learning activities. Thus, together with the apps, these can be used to continue or to reinforce the learning of

probability and statistics competencies that can no longer be fully accommodated within the last quarter of the school year.

6. Challenges in Implementation

Several challenges were encountered during the project [3] in the implementation of the use of technological tools in schools to bridge the learning gaps in mathematics.

Students' access to technology (smartphones, tablets, computers, etc.) remains one of the most challenging problems in the Philippines. Some schools under DepEd school divisions get support from the local government where each student is provided with a tablet, but this is not true for all schools. In this respect, it was difficult for students who did not have their own gadgets to have access to the apps and to use these to help in their math learning. Other students had to rely on the gadgets of their parents or older siblings but could only use these at certain times during the week.

Lack of teachers has also been a main concern. With the lack of teachers there are missed opportunities for a more focused and nurturing approach in addressing the least learned competencies of students. In a class for example, the learning gaps are different for each student, and the choice of mathematical app to remedy a particular learning competency may vary. Moreover, even if school administrators would want to conduct remediation programs outside of class hours, there are not enough teachers to carry out these programs.

Classroom shortage has also been a concern. There are insufficient classrooms that could be venues for remedial classes or learning spaces where students work on the mathematical apps during their free time in school.

Finally, families play a critical role in successfully integrating technology into learning. It has not been easy, for example, for teachers to collaborate with parents with regards to knowledge on the mathematical apps adopted in class so that their contribution at home could be more effective. For one, there is difficulty on the part of the parents understanding how the gadget and the mathematical apps work. Particularly for low-income families, there is really no time left for parents to learn the technology, as they must prioritize work to support their families. This can enhance the learning inequality further, particularly for young learners. For some students, there are no opportunities to carry out learning activities with the mathematical apps as parents must rely on them to carry out tasks at home.

7. Conclusion and Future Direction

The use of technological tools to help address the issue of major learning gaps in students after the pandemic is not unique to the Philippines. In India for example, the World Economic Forum launched the new 'Education 4.0 India Report' where digital tools will be utilized to enhance foundational literacy and numeracy skills [35]. The implementation of our project [3] to schools in collaboration with DepEd school division offices, is one step towards addressing this problem in the Philippines. In this paper, we showed the different mathematical apps that we have developed to address learning gaps in several content areas in the DepEd curriculum that have been identified by teachers and administrators in our partner schools. The design of the apps has been based on pedagogical principles and game design elements, as well as what is informed in the literature as benefits of technology in mathematical learning. The apps are free and run primarily on low-end mobile devices for accessibility. It is hoped that other educators can find these apps helpful in their work on the use of technological tools in bridging gaps in mathematics learning outcomes.

There is a lot of work to be done in terms of the mathematical apps-more updates can still be done in terms of adding questions and levels of difficulty. Collaboration is still ongoing with DepEd school division offices in terms of assessing the effectiveness of these apps in student learning. Furthermore, there are ongoing efforts soliciting support from NGOs and private sectors to bring the mathematical resources to more schools in the country.

Acknowledgments This paper is one of the outputs of the Ateneo *Mathplus* Resources Laboratory housed at the Department of Mathematics, School of Science and Engineering, Ateneo de Manila University. The authors would like to thank the Department of Science and Technology- Philippine Council for Industry, Energy, and Emerging Technology Research and Development (DOST-PCIEERD) and the University Research Council (URC), Ateneo de Manila University for the support of the project [3]. We thank our app developers Mr. Victor Antonio M. Ortega, Mr. Nigel Benedict Cargo, Mr. Jose Teodoro Lacson, Sparksoft Solutions, Inc and our artist Mr. Amiel Damian F. Justiniani.

References

[1] Department of Education (DepEd) Region I. (2022). *Regional Memorandum No. 664, s. 2022: Basic Education Learning Recovery Plan in light of the COVID 19 Pandemic*. Retrieved 7/20/2023 from depedro1.com/wp-content/uploads/2022/06/rm0664s2022.pdf

[2] Department of Education. (2023). *Adoption of the National Learning Recovery Program in the Department of Education.* Retrieved 7/3/2023 from www.deped.gov.ph/wp-content/uploads/DO_s2023_013.pdf

[3] De Las Peñas, M. L. A. N., Aberin, M. A. Q., Garciano, A. D., Sarmiento, J. F., Tolentino, M. A. C., Verzosa, D. M. B., and Mallari, J. C. (2023). *Terminal Report (Department of Science and Technology-Philippine Council for Industry, Energy and Emerging Technology Research and Development): Mathematical Resources for Distance Learning Utilizing Community LTE Networks and Television Frequencies*.

[4] Department of Education. (2020). *K to 12 Most Essential Learning Competencies with Corresponding CG Codes*.

[5] De Las Peñas, M.L.A.N., Aberin, M.A.Q., Garciano, A.D., Mallari, J.C.F., Sarmiento, J.F., Tolentino, M.A.C., and Verzosa, D.M.B. (2022). Deployment of Mathematical Resources to a Philippine High School through a Community LTE Network. In W.-C. Yang, M. Majewski, D. Meade, and W.K. Ho (Eds.), *Proceedings of the 27th Asian Technology Conference in Mathematics*. Mathematics and Technology, LLC.

[6] Villanueva, J.C.P.M., Melendres, M.A.V., Lagunzad, C.G.B., and Libatique, N.J.C. (2021). Design and Deployment of a Mobile Learning Cloud Network to Facilitate Open Educational Resources for Asynchronous Learning. In M.M.T. Rodrigo, S. Iyer, A. Mitrovic (Eds.), *Proceedings of the 29th International Conference on Computers in Education*. Asia-Pacific Society for Computers in Education.

[7] De Las Peñas, M. L. A. N., Kwong, J. C. T., Banzon, P. A. B., Martinez, P. A., Lapada, W. I. G., Eballa, J. N., Sebastian, J. E. C., Asido, M. D., San Juan, J. D. M., Verzosa, D. M. B., Sarmiento, J. F., Garciano, A. D., Tolentino, M. A. C., Aberin, M. A. Q., and Mallari, J. C. F. (2023). Mathematical Mobile Apps via Rural Casting. In P. Kommers, I. A. Sánchez, and P. Isaías (Eds.), *Proceedings of the International Conferences on e-Society 2023 and Mobile Learning 2023*. International Association for Development of the Information Society.

[8] Sacristan, A.I., Parada, S.E., and Miranda, L. (2011). The Problem of the Digital Divide for Mathematics Teachers in Developing Countries. In M. Joubert, A. Clark-Wilson, and M.

McCabe (Eds.), *Proceedings of the 10th International Conference for Technology in Mathematics Teaching*.

[9] Norris, C., Hossain, A., and Soloway, E. (2011). Using Smartphones as Essential Tools for Learning. *Education Technology*, 51(3), 18-25.

[10] Plass, J.L., Homer, B.D., and Kinzer, C.K. (2015). Foundations of Game-Based Learning, *Educational Psychologist*, 50(4), 258-283.

[11] Shi, Y.-R. and Shih, J.-L. (2015). Game Factors and Game-based Learning Design Model. *International Journal of Computer Games Technology*, 2015, 1-11.

[12] Bolden, D., Barmby, P., Raine, S., and Gardner, M. (2015). How Young Children View Mathematical Representations: a Study Using Eye-tracking Technology. *Educational Research*, 57, 59-79.

[13] Department of Education (2016). *K to 12 Curriculum Guide: Mathematics*. Retrieved 7/20/2023 from www.deped.gov.ph/wp-content/uploads/2019/01/Math-CG_with-tagged-math-equipment.pdf

[14] De Las Peñas, M.L.A.N., Verzosa, D.M.B., Aberin, M.A.Q., Garciano, A.D., Sarmiento, J.F., and Tolentino, M.A.C. (2021) Designing Performance Tasks in Mathematics Using Technological Tools. In W.-C. Yang, D. Meade, and M. Majewski (Eds.), *Electronic Proceedings of the 26th Asian Technology Conference in Mathematics*. Mathematics and Technology, LLC.

[15] Wu, H., (2011). *Understanding Numbers in Elementary School Mathematics.* Providence, RI: American Mathematical Society.

[16] Verzosa, D.M.B., De Las Peñas, M.L.A.N., Aberin, M.A.Q., Garciano, A.D., Sarmiento, J.F., Mallari, J.C.F., and Tolentino, M.A.C. (2022) Development of an App and Videos to Support the Fraction Learning Trajectory from Grades 1-7. In Iyer, S. et al. (Eds.), *Proceedings of the 30th International Conference on Computers in Education. Asia-Pacific Society for Computers in Education*. Asia-Pacific Society for Computers in Education.

[17] Verzosa, D.M.B., Aberin, M.A.Q., De Las Peñas, M.L.A.N., Garciano, A.D., Sarmiento, J.F., and Tolentino, M.A.C. (2021). Development of a Gamified Number Line App for Teaching Estimate and Number Sense in Grades 1 to 7. In M. M. T. Rodrigo, S. Iyer, and A. Mitrovic (Eds.), *Proceedings of the 29th International Conference on Computers in Education*. Asia-Pacific Society for Computers in Education.

[18] Verzosa, D.M.B., De Las Peñas, M.L.A.N., Sarmiento, J.F., Aberin, M.A.Q., Tolentino, M.A.C., and Loyola, M.L. (2021). Using Mobile Technology to Promote Higher-order Thinking Skills in Elementary Mathematics. In I.A. Sánchez, P. Kommers, T. Issa, and P. Isaías (Eds.), *Proceedings of the International Conferences on Mobile Learning 2021 and Educational Technologies 2021*. International Association for Development of the Information Society.

[19] Lehtinen, E., Hannula-Sormunen, M., McMullen, J., and Gruber, H. (2017). Cultivating Mathematical Skills: from Drill-and-practice to Deliberate Practice. *ZDM Mathematics Education*, 49, 625-636.

[20] Garciano, A.D., Verzosa, D.M.B., De Las Peñas, M.L.A.N., Aberin, M.A.Q., Mallari, J.C.F., Sarmiento, J.F., and Tolentino, M.A.C. (2023) Practice through Play Using Mobile Technology. In P. Kommers, I. A. Sánchez, and P. Isaías (Eds.), *Proceedings of the International Conferences on e-Society 2023 and Mobile Learning 2023*. International Association for Development of the Information Society.

[21] Stephan, M. and Akyuz, D. (2012). A Proposed Instructional Theory for Integer Addition and Subtraction. *Journal of Research on Mathematics Education,* 43, 428-464.

[22] Jenkins, R. (2022). *Impact of Interactive Computer-aided Instruction in Learning Trigonometry in a High School Precalculus Course.* [Doctoral dissertation, Franklin University]. https://fuse.franklin.edu/ docpub/71
[23] Presmeg, N. (2006). Research on Visualization in Learning and Teaching Mathematics. In A. Gutierrez and P. Boero (Eds.), *Handbook of Research on the Psychology of Mathematics Education: Past, Present and Future* (pp. 205-235). Rotterdam, the Netherlands: Sense Publishers.
[24] Verzosa, D.M.B., De Las Peñas, M. L.A. N., Aberin, M.A.Q., and Garces, L.P.D. (2018). App-based Scaffolds for Writing Two-Column Proofs. *International Journal of Mathematical Education in Science and Technology*, 50(5), 1-13.
[25] Albert, J.R.G. et al. (2016). *Teaching Guide for Senior High School: Statistics and Probability*. Commission on Higher Education (Philippines).
[26] Batilantes, S.P. (2021). Project VLOGI (Video Lectures on Giving Instructions): Effects on Learners' Performance in Probability and Statistics. *International Journal of Educational Studies in Mathematics*, 8(4), 299-315.
[27] Reston, E. (2023). Statistics Education in the Philippines: Curricular Context and Challenges of Implementation. In G.F. Burrill, L. de Oliveria Souza, and E. Reston, E. (Eds.), *Research on Reasoning with Data and Statistical Thinking: International Perspectives. Advances in Mathematics Education.* Springer, Cham.
[28] Bargagliotti, A., Franklin, C., Arnold, P., and Gould, R. (2020). *Pre-K-12 Guidelines for Assessment and Instruction in Statistics Education II (GAISE II).* American Statistical Association.
[29] Pfannkuch, M. (2011). The Role of Context in Developing Informal Statistical Inferential Reasoning: A Classroom Study. *Mathematical Thinking and Learning*. 13, 27-46.
[30] Rossman, A. J. and Chance, B. L. (2014). Using Simulation-based Inference for Learning Introductory Statistics. *Wiley Interdisciplinary Reviews. Computational Statistics*, 6(4), 211-221.
[31] Cobb, G.W. (2007). The Introductory Statistics Course: a Ptolemaic Curriculum? *Technology Innovations in Statistics Education*, 1, 1-15.
[32] De Las Peñas, M.L.A.N, Verzosa, D.M.B., Sarmiento, J.F., Tolentino, M.A.C., and Loyola, M.L. (2020). Designing Mobile Apps to Promote Numeracy and Statistical Reasoning. In W.-C. Yang and D. Meade (Eds.), *Electronic Proceedings of the 25th Asian Technology Conference in Mathematics*. Mathematics and Technology, LLC.
[33] Tolentino, M.A.C., De Las Peñas, M.L.A.N., Aberin, M.A.Q., Garciano, A.D., Loyola, M.L., Sarmiento, J.F., Mallari, J.C.F., and Verzosa, D.M.B (2022). Engaging Learners through Data: Senso Eskwela Pilipinas. In W.-C. Yang, M. Majewski, D. Meade, and W.K. Ho (Eds.), *Proceedings of the 27th Asian Technology Conference in Mathematics*. Mathematics and Technology, LLC.
[34] Connor, D. (2002). CensusAtSchool 2000: Creation to Collation to Classroom. In *Proceedings of the Sixth International Conference on Teaching of Statistics*. International Statistical Institute.
[35] Poddar et al. (2022). *Education 4.0 India Insight Report*. World Economic Forum.

© 2024 International Commission on Mathematical Instruction
https://doi.org/10.1142/9789811287183_0046

46

Challenging Deficit Perspectives in Developing Countries: Teachers' Explanations of Fraction Concepts

Debbie Marie Verzosa[1]

Proceedings of the 14th International Congress on Mathematical Education Downloaded from www.worldscientific.com by 203.177.19.234 on 07/06/25. Re-use and distribution is strictly not permitted, except for Open Access articles.

ABSTRACT Dominant discourses in teacher development often posit teachers as being lacking in knowledge, beliefs, or skills, thus justifying the "need" for further development and for educational reforms. This perspective shaped the analysis of Filipino teachers' explanations of fraction concepts using the constructs of content knowledge and pedagogical content knowledge, leading to an interpretation that reinforced deficit narratives about teachers. However, there are increasing contestations of these deficit research narratives (Adiredja, 2019) that neither acknowledge the larger context that contributes to the ways teachers perform nor highlight the productive resources that teachers may draw upon in their teaching. This paper aims to illustrate a reconceptualization of the research away from focusing on what teachers lack towards identifying the ways by which teachers' fractional explanations reflect their constructed perception of ideal mathematics teaching as shaped by the broader system where education takes place. This is my attempt to acknowledge my own participation in the deficit perspective and challenge the narrative about education in a developing country.

Keywords: Developing countries; Preservice teachers; Fractions; Anti-deficit.

The scholarly literature is replete with reports of teachers' knowledge of fractions. Olanoff et al. (2014) found at least 43 papers that focused specifically on teachers' fraction knowledge. Since then, further studies on teachers' fraction knowledge were published (Bansilal and Ubah, 2020; Chinnapan and Forrester, 2014; Depaepe et al., 2015; Depaepe et al., 2018; Lee, 2017; Lemonidis et al., 2018; Van Steenbrugge et al., 2014). Majority of the studies were carried out in developed countries or in countries with average to above average mathematics achievement based on international benchmarks such as PISA (OECD, 2019).

The initial objective of the current study was to perform a similar assessment of preservice teachers' content and pedagogical content knowledge of fractions in the context of a developing country (the Philippines). An implicit assumption was that knowledge gaps of teachers in developing countries may be more serious than those in developed countries, which is why this study needed to be conducted. This perspective

[1] Department of Mathematics and Statistics, University of Southern Mindanao, Kabacan, Cotabato, Philippines. E-mail: dmbverzosa@usm.edu.ph

This is an open access article published by World Scientific Publishing Company.
It is distributed under the terms of the Creative Commons Attribution-NonCommercial 4.0 (CC BY-NC) License.

typified the "fixit" approach (Graven, 2012) to professional development. Breen described the prevailing mindset regarding teacher training.

> There is something wrong with mathematics teaching world-wide, and that we, as mathematics educators, must fix it... Mathematics teachers need someone to fix them, and mathematics educators need someone to fix … This culture is based on judging what is right and wrong, paying little attention to what mathematics teachers are actually doing (since it is wrong anyway) in their classrooms, and looking outside themselves for the "right" way, the newest "fix". (Breen, 1999, p. 42)

Deficit master narratives about preservice teachers are much entrenched in the scholarly literature. They exist even in reputable mathematics education journals, and even in carefully designed studies that focused on the development of understanding (Adiredja, 2019). Specifically, deficit master narratives also abound in studies that focus on fraction understanding. Based on teachers' performance in various fraction assessments, they are described in terms of what they "could not" do; their knowledge is often described as "inadequate", "weak", "insufficient", "limited", or as having "gaps" and "misconceptions" (Bansilal and Ubah, 2020; Chinnapan and Forrester, 2014; Depaepe et al., 2015; Klemer et al., 2017; Putra and Winsløw, 2018; Rosli et al., 2020; Şahin et al., 2016; Van Steenbrugge et al., 2014). I also acknowledge my own contribution to these deficit master narratives through claims of teachers' poor reasoning strategies (Verzosa, 2020). Were it not for a wise suggestion and for a further review of the literature, I would probably remain unaware and continue to be entrenched in a deficit view.

Deficit research narratives neither acknowledge the larger context that contributes to the ways teachers perform nor highlight the productive resources that teachers may draw upon in their teaching (Adiredja, 2019). This paper aims to illustrate a reconceptualization of the research away from focusing on what teachers lack towards identifying the ways by which teachers' fractional explanations reflect their constructed perception of ideal mathematics teaching as shaped by the broader system where education takes place. This is my attempt to challenge the narrative about education in the Philippines.

1. Anti-deficit Perspectives

It can be argued that claims of the teachers' weak or inadequate knowledge in fractions in the previous section are based on data. However, the interpretation of the data was shaped by a deficit master-narrative and failed to recognize the teachers' sense making. Previous studies have demonstrated how the same data can be analyzed through a deficit or anti-deficit lens. For example, Lewis (2014) offered two contrasting explanations for the persistent understandings found in her case studies of two children. In the first approach, the two children's persistent understandings could be thought of as an indication of their inability to mentally represent and manipulate numbers. Lewis

Proceedings of the 14th International Congress on Mathematical Education Downloaded from www.worldscientific.com by 203.177.19.234 on 07/06/25. Re-use and distribution is strictly not permitted, except for Open Access articles.

Proceedings of the 14th International Congress on Mathematical Education Downloaded from www.worldscientific.com by 203.177.19.234 on 07/06/25. Re-use and distribution is strictly not permitted, except for Open Access articles.

argues that this interpretation does not provide much value to the identification or remediation of students with mathematical learning disabilities. In the alternative anti-deficit approach, Lewis offered a Vygotskian perspective and interpreted the children's understandings as resulting from the inaccessibility of mediational tools. Within this interpretation, the reason for the two children's persistent understandings was not because they could not mentally represent and manipulate numbers but because they understood the representation of a fractional quantity in unconventional and atypical ways.

In another study, Adiredja (2021), students' claims about the temporal order of epsilon and delta in the formal definition of the limit was investigated. In this study, four questions regarding the temporal order of epsilon and delta were asked—what depends on what? Which one do you think comes first? Which one do you think is set first? How would you arranged the four variables, epsilon, delta, x and $f(x)$ in order? Only one student correctly answered "epsilon first" on all questions. Half the students answered "epsilon first" in at least one question. A deficit perspective would immediately lead to an interpretation that most students did not understand the temporal order. However, the study demonstrated that a single context/question cannot capture students' understanding. The fact that some students answered epsilon first on some, but not all, questions revealed that the students' justifications were context sensitive. Different questions influenced the cueing priority of certain knowledge elements (e.g., functional dependence). These knowledge elements (which are useful in other contexts) were found to be resources that students possessed during their sense-making of the temporal order of the limit definition.

These studies demonstrate that a deficit perspective focuses on what individuals lack (including misconceptions), rather than the resources that they draw upon (Adiredja, 2019). It locates students' knowledge as a problem to be fixed, and not as a resource for learning. Further, a deficit perspective views certain individuals as if they possessed deficiencies, which may be biological or cultural (Settlage, 2011). Some individuals may be thought to be "biologically inferior" or deficient by virtue of being raised in a certain culture. A deficit perspective does not consider that education occurs within a sociopolitical context (Adiredja, 2019) where forces may systematically undo the efforts of teachers or schools (Settlage, 2011).

2. The Philippine Context and Research Questions

Nebres (2009) stated that problems of mathematics education in the Philippines include two types: micro problems or problems internal to mathematical education (curriculum, teacher training, textbooks, etc), and macro problems or issues arising from pressures from other sectors of society. These macro problems very much exist in the Philippines.

In the Philippines, teaching is not an attractive career choice, and entry standards are typically lower than in other degree programs (Tatto et al., 2012). In some cases, education is chosen as a field of study because it is relatively cheap (no equipment is required), or because it accepts students who are unqualified in more attractive

programs. Additionally, teachers are known to be overworked and underpaid. Public school teachers teach a maximum of six hours per day, and are also expected to write very detailed lesson plans, fill up official forms, and complete reports (Bautista et al., 2008). They may be asked to perform other non-teaching related duties in a highly centralized environment where information, opinions, and teachers' options are tightly controlled (Bernardo and Garcia, 2006; Nebres, 2009).

Teaching is generally not an option among graduates of the top universities. To encourage the best students to teach, even just for a fixed term such as two years, initiatives such as Teach for the Philippines have been devised (Sodusta and De Leon, 2019). The words of a promising graduate turned public school teacher, sums up the aversion to teaching as a career:

> During my first year as a public school teacher, ahhh, there are three reactions that I always get: surprise, amazement, dismay. It's like, "What? Why are you teaching there?" "Oh you come from a prestigious university but you're a public teacher? What a waste". It's always like that. (Sabrina Ongkiko, quoted by Sodusta and De Leon, 2019, p. 12)

The study reported in this paper is grounded in the assumption that the macro problems emerging from the contextual realities of the educational system can contribute to teachers' training and preparation. It asks the following research questions. How do preservice teachers reason about fraction comparison and operation tasks? What forms of reasoning are associated with correct responses?

3. Method

3.1. Sample

The participants were 405 preservice elementary and 157 preservice secondary teachers. The preservice teachers were enrolled in six universities, spread across the northern, central, southern, and capital region of the Philippines. Moreover, the universities had varied ranking in terms of the passing rate in the September licensure examination for teachers (ranging from 40 to 93% for elementary teachers, and from 54 to 94% for secondary teachers). The preservice teachers were all in their third or fourth year, and had completed all mathematics courses within their program of study. No personal details were collected, but it can be assumed that most preservice teachers in the sample were between 19 and 21 years old.

3.2. Materials

The instrument consisted of four explanatory tasks, three of which involved fractions, and were included in this paper. For each of the three tasks, the respondents were asked to provide an answer and describe their solution to a student. The tasks were as follows: (1) Which is larger, 1/5 or 1/8? (2) What is 2/3 + 1/2? (3) What is 1 ÷ 2/3? For Tasks

Proceedings of the 14th International Congress on Mathematical Education Downloaded from www.worldscientific.com by 203.177.19.234 on 07/06/25. Re-use and distribution is strictly not permitted, except for Open Access articles.

Proceedings of the 14th International Congress on Mathematical Education Downloaded from www.worldscientific.com by 203.177.19.234 on 07/06/25. Re-use and distribution is strictly not permitted, except for Open Access articles.

2 and 3, the respondents were specifically asked to explain their solutions using a drawing. The concepts (comparison, addition, and division) were chosen because they represented basic knowledge in elementary mathematics, and there was sufficient literature to provide task-specific frameworks to guide this study's analysis (Bansilal and Ubah, 2020; Chinnapan and Forrester, 2014; Geller et al., 2017; Kaasila et al., 2010; Lee, 2017). Although the medium of instruction in mathematics from Grade 4 onwards is English, the teachers could write whole or part of their explanations in the regional language to lessen obstacles brought about by using a second language to explain academic content. Participants were given 30 minutes to complete the tasks, but most were finished before the time allotment.

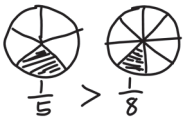
3.3. Data Analysis

Responses were initially coded as correct or incorrect. The preservice teachers' explanation strategies were derived inductively from the responses. From the raw data, repeating responses were noted and assigned codes. Similarities among codes were identified, which resulted in an intermediate list of codes. The data was coded and re-coded until the codes were finalized. With the final set of codes, the data were coded a second time as a manner of checking.

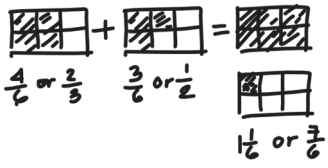
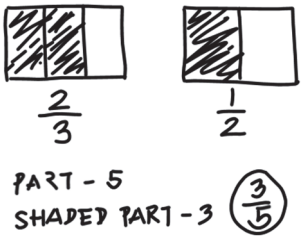
Because I wanted to capture the all the reasoning strategies provided by each preservice teacher, I coded all strategies. If the preservice teacher gave more than one reasoning strategy, responses were coded more than once.

Tab. 1, Tab. 2, and Tab. 3 show the codes for each of the three tasks in this study. Sample responses for each code are also provided.

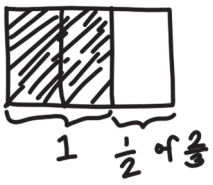
Tab. 1. Response codes for task 1 (which is larger, 1/5 or 1/8?)

Correct picture	Incorrect picture	LCD/decimal
		"8/40 > 5/40; 1/5 is larger."
		1/5=0.2; 1/8=0.125; 1/5 is larger
Cross multiply	**Denominator**	**Real life**
To know the larger fraction, the two denominators will be multiplied to the opposite numerator. 	As long as the denominator increases, the value decreases. Simply compare the denominator. The larger fraction is the one with smaller denominator.	In our quiz everyday, if we get 1/2 the half of paper is what we need and in 1/4 the half of one-half is what we need then I conclude that the smaller the denominator, the closer is the fraction to the whole so 1/5 is larger.

Tab. 2. Response codes for task 2 (what is 2/3 + 1/2?)

Correct fraction relations	Incorrect fraction relations	Procedure
		First, get the LCD of the two denominators. Divide it from the original ones. 6 divided by 3 is equal to 2. Then 2 multiply by 2 is equal to 4. So the answer is 4/6. Do the same for the other, then add.

Tab. 3. Response codes for task 3 (what is 1 ÷ 2/3?)

Measurement Division	Procedure	Commutativity
	In dividing fractions, the whole number always has 1 in the denominator so the division symbol will be replaced with multiplication. In dividing, we have what we call a reciprocal so reverse 2/3, making it 3/2 then cross multiply.	Any number divided by 1, the answer is also the number.

4. Results

Fig. 1 shows the results in task 1 (which is larger, 1/5 or 1/8?). The white regions represent correct answers. Results show that correct answers were produced through different kinds of explanatory approaches. The use of real-life situations was more prevalent among the preservice elementary teachers, and the use of the least common denominator (LCD) or decimals was more prevalent among the preservice secondary teachers. The strategies LCD/decimal, cross multiply and denominator, considered "non-conceptual" in other studies (Geller et al., 2017), were found to produce correct answers, especially for the preservice secondary group. Most strategies underlying incorrect responses (represented by the dark regions) were incorrect drawings. However, there were 78 preservice teachers who produced an incorrect drawing and a correct answer, implying that an incorrect drawing did not automatically result in an incorrect solution. Further, there were 43 preservice teachers who produced an incorrect drawing but had other resources or strategies from which to draw upon (Fig. 2).

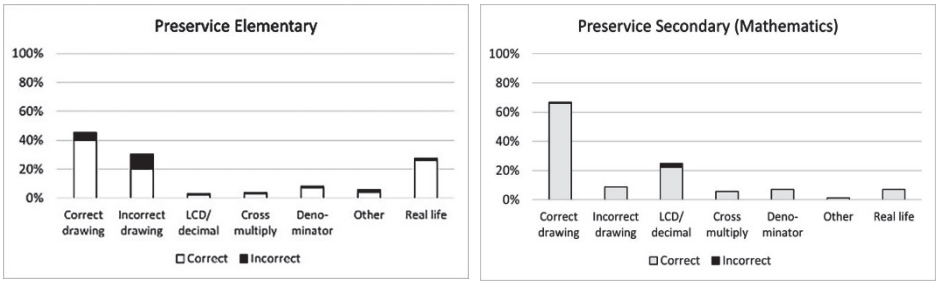


Fig. 1. Explanatory approaches in task 1 (which is larger, 1/5 or 1/8?)

Proceedings of the 14th International Congress on Mathematical Education Downloaded from www.worldscientific.com by 203.177.19.234 on 07/06/25. Re-use and distribution is strictly not permitted, except for Open Access articles.

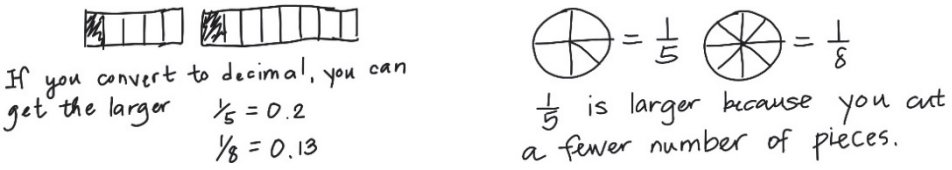


Fig. 2. Sample response with more than one strategy

Fig. 3 shows the results in task 2 (what is 2/3+1/2?). The most common explanation approach for preservice elementary teachers was based on procedures. This approach was effective for the preservice secondary teachers, but had around a 50% success rate among the preservice elementary teachers. For the preservice secondary teachers, the most common explanatory approaches were based on correct fraction relations and procedures. Among those who used correct and incorrect fraction relations, all produced a correct solution. The only exception was one preservice teacher who did not write an answer. The explanation approach based on incorrect fraction relations was only found among the preservice elementary teachers. This strategy of "count the shaded and count the parts" may have been productive in other contexts, such as when recognizing the fraction represented by a drawing, but was incorrectly applied here. Teaching students to understand the contexts where certain strategies are productive remains a learning goal.

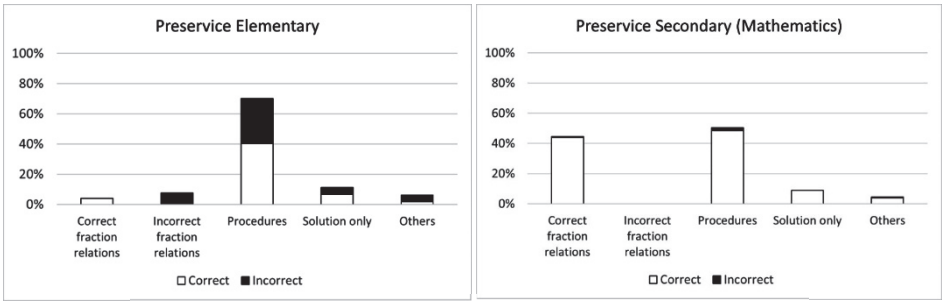


Fig. 3. Explanatory approaches in task 2 (what is 2/3 + 1/2?)

Fig. 4 shows the results in task 3 (what is 1÷2/3?). The most common explanatory approaches for this task mirrored those in the second task. For both cohorts, procedural explanatory approaches dominated. As in the previous task, this approach was effective for the preservice secondary teachers, but had a rather 50% success rate among the preservice elementary teachers. Except for one preservice teacher, everyone who used measurement division as an explanatory approach gave correct answers. Thus, this offers a potential goal for learning. Procedures were not as reliable, as almost half of preservice elementary teachers used an incorrect rule to solve the task. The rules they provided also resembled some of the rules they might have encountered in their mathematical experiences. The explanation approach based on commutativity was only

Proceedings of the 14th International Congress on Mathematical Education Downloaded from www.worldscientific.com by 203.177.19.234 on 07/06/25. Re-use and distribution is strictly not permitted, except for Open Access articles.

found among the preservice secondary teachers. For these teachers, the strategy of "interchanging the numbers in an operation", which is productive in addition or multiplication contexts, was incorrectly applied.

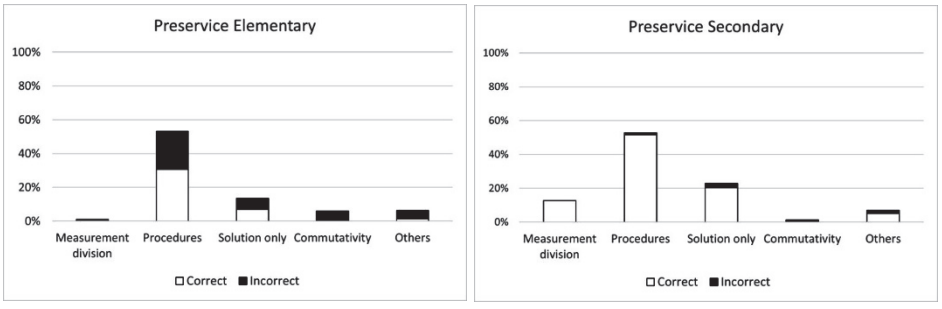


Fig. 4. Explanatory approaches in task 3 (what is 1 ÷ 2/3?)

Although procedures led to correct responses among half of the preservice elementary teachers, most of their explanatory approaches resembled standard rules. To compute 1÷2/3, they utilized elements of the correct solution (such as getting the reciprocal at some point). For example, one preservice teacher said, "We will make 1 the numerator, and then 2/3 the denominator. So that 1 becomes 3/1."

4.1. Repeating strategies

As indicated in the Data Analysis section, the explanatory approaches were searched for repeating ideas as a means to finalize codes. One repeating pattern in the responses was the use of words such as "just", "simply" or "easily". For example, 33 (7%) respondents included these words in their explanatory approaches. These words convey encouragement to students by suggesting that the task is doable.

Several explanations made use of a stated fact. These did not require much thought — these can be directly conveyed to any student who listens attentively. For example, in task 3, 64 preservice teachers explained that to divide 1 by 2/3, one must write 1 as 1/1. They explained that it is "understood" or "automatic" that a whole number has 1 in the denominator, that there is an "imaginary" or "invisible" 1 in the denominator, or that any whole number has 1 in the denominator.

5. Discussion

The data suggests that procedures are reasonably reliable, particularly for the preservice secondary teachers. Most preservice secondary teachers remembered the procedure while around half of the preservice elementary teachers did not. The preservice secondary teachers had more math courses in their training, which may have increased their opportunities to learn. Another possible explanation is that preservice teachers who enrolled in preservice secondary mathematics teaching are themselves already predisposed to math to begin with. Thus, it appears that procedural

Proceedings of the 14th International Congress on Mathematical Education Downloaded from www.worldscientific.com by 203.177.19.234 on 07/06/25. Re-use and distribution is strictly not permitted, except for Open Access articles.

Proceedings of the 14th International Congress on Mathematical Education Downloaded from www.worldscientific.com by 203.177.19.234 on 07/06/25. Re-use and distribution is strictly not permitted, except for Open Access articles.

explanations are reliable only for students who have reached some threshold of exposure to mathematics.

Knowledge of the fundamental fraction concept of equivalence (Smith, 1995) provides a reliable gateway towards a correct solution. Those who reasoned correctly about quantities were able to provide correct answers. The implication is that without the resources offered by thinking about quantities, procedural thinking can be a hit-or-miss — it is either you remember the rule or not. Interviews may reveal the reason why some preservice teachers remembered, while some did not.

The large proportion of explanatory approaches found in this study is consistent with the beliefs that Philippine teachers and teacher educators hold about mathematics learning. In the Teacher Education and Development Study in Mathematics (TEDS-M) study (Tatto et al., 2012), Filipino teachers and teacher educators, on average, believed 90% of the statements related to mathematics as a set of rules and procedures. Bergqvist and Lithner (2012) used Brousseau's (2002) theory of didactic situations to interpret that giving the students a procedure relieves the student of his/her mental work. Whether this is indeed the case for the preservice teachers in this study is up for further investigation. Certainly, the less-than-ideal educational context where students are crammed 80 to rooms built for 50, or where teachers are burdened by too much work makes it reasonable to ensure that rules sounded clear and friendly and to promote the easiest and most straightforward explanatory approach possible.

To highlight the difference and affordances of an anti-deficit approach in the interpretation of the results, I would like to offer two deficit conclusions form the same data set. Admittedly, this deficit model informed my initial analytic framework and the interpretation of the results. The first conclusion stemming from a deficit model is that the preservice teachers, particularly the preservice elementary teachers, lacked content knowledge and pedagogical content knowledge in fractions, given that more than half the teachers did not give a correct answer to Tasks 2 and 3. However, a more detailed look at their explanatory approaches revealed some potential resources from which the preservice teachers drew upon. These included "count the shaded", the notion of "commutativity", and the production of rules that resembled standard rules. Teacher educators may build upon these and elucidate the contexts where such knowledge is reliable. Teacher educators may also design assessments so that these contexts can be clarified. One example to facilitate a discussion of an "equal pieces" model is to ask preservice teachers to draw 1/5 and 1/8 given (a) two similar-sized objects, and (b) given two different-sized objects. These tasks may be a springboard for discussion whether it is possible to compare 1/5 and 1/8 using these models.

A second possible conclusion that perpetuates a deficit master narrative is that preservice teachers promoted learning by rote because most of their strategies relied on procedures. However, this explanation does not recognize the classroom contexts or the broader educational system as major influencers of teachers' actions. Johnson et al. (2000) argued that teachers know more strategies than what they use in the classroom. They argued,

> Whereas the wisdom gained from northern/western contexts suggests it is the teacher that does the selecting, we wish to reverse this wisdom, and suggest that for science teachers who are in educational systems at anything other than the professional stage, it is the environment in which the teacher works that creates the selection. (Johnson et al., 2000, p. 186)

These teacher constraints were expressed by a South African teacher during the time when an innovative assessment was being piloted in South Africa (Bansilal, 2011). This teacher said,

> I found out that you can't do any of the innovative tasks with them, you have to teach the concepts. Because if you don't teach these concepts, then they can't do the activity. You saw for yourself how difficult it was — there are 50 learners in each of my classes. So we have to stand in the front and do as much as we can, even though we ourselves know at the end of the day it doesn't make much of a difference. (Bansilal, 2011, pp. 104–105)

This quote demonstrates how contextual factors such as students' non-readiness for the task or the physical setup of the classroom can constrain teachers' actions. In the Philippines, the same contexts and macro-structures are also present. An interview with a Filipino teacher revealed such pressures (Verzosa et al., 2017). In relation to a mandated focus on critical thinking and exploration which was considerably different from the "spoonfeeding" method that their students were exposed to in their elementary school years, one Filipino teacher mentioned (p. 93), "ikaw na nga gagawa ng activity, ikaw din ang sasagot (you designed the activity but you end up answering it as well)." Further, for the recommended strategies to work, teachers are compelled to provide worksheets and other materials for students at their own expense.

6. Limitations and Future Research

A three-item written assessment cannot access students' full understanding. Interviews with the preservice teachers would have provided the information needed to understand how their choices were shaped by their own experiences. Most studies that focused on students' productions rather than misconceptions utilized interviews in their research design (Adiredja, 2021; Adiredja et al., 2020; Hunt and Empson, 2014; Lewis, 2014; Lewis and Lynn, 2018). As was stated earlier in this paper, this study was initially not designed with an anti-deficit perspective as a supporting framework. If not for the wise guidance of experts in the field, I would have persisted with my initial analysis that focused on errors and downplayed the resources from which the preservice teachers drew upon. By doing so, I would have contributed even further to deficit master narratives about teachers. While I was reconceptualizing my analysis, I began to recognize that many of my initial conclusions produced a deficit story about preservice teachers.

Much of the research framed by anti-deficit perspectives involve students of color or students with mathematical learning disabilities. There are few, if any, published research from non-Western contexts. Thus, I will end this paper by also pushing for

Proceedings of the 14th International Congress on Mathematical Education Downloaded from www.worldscientific.com by 203.177.19.234 on 07/06/25. Re-use and distribution is strictly not permitted, except for Open Access articles.

the use of anti-deficit perspectives in designing, implementing, and interpreting research, especially in developing countries.

Acknowledgments

The author thanks the University of Southern Mindanao for funding this research under Fund 101. The author also thanks the universities and preservice teachers who participated in this study.

Proceedings of the 14th International Congress on Mathematical Education Downloaded from www.worldscientific.com by 203.177.19.234 on 07/06/25. Re-use and distribution is strictly not permitted, except for Open Access articles.

References

A. P. Adiredja (2019). Anti-deficit narratives: Engaging the politics of research on mathematical sense making. *Journal for Research in Mathematics Education*, 50*,* 401–435.

A. P. Adiredja (2021). Students' struggles with temporal order in the limit definition: uncovering resources using knowledge in pieces. *International Journal of Mathematical Education in Science and Technology*, 52, 1295–1321.

A. P. Adiredja, R. Bélanger-Rioux, and M. Zandieh (2020). Everyday examples about basis from students: An anti-deficit approach in the classroom. *PRIMUS*, 30, 520–538.

S. Bansilal (2011). Assessment reform in South Africa: opening up or closing spaces for teachers? *Educational Studies in Mathematics*, 78, 91–107.

S. Bansilal and I. Ubah (2020). The use of cross multiplication and other mal-rules in fraction operations by pre-service teachers. *Journal of Mathematical Behavior*, 58, 100781.

C. R. B. Bautista, A. B. I. Bernardo, and D. Ocampo (2008). When reforms don't transform: Reflections on institutional reforms in the Department of Education. HDN discussion paper series (2008/2009). Retrieved from http://hdn.org.ph/wp-content/uploads/2008/10/dp02_bautista_etal.pdf

T. Bergqvist and J. Lithner (2012). Mathematical reasoning in teachers' presentations. *The Journal of Mathematical Behavior*, 31, 252–269.

A. B. I. Bernardo and J. A. S. Garcia (2006). School improvement in a centralized educational system: The case of the Philippine basic educational system. In: J. C.-K. Lee and M. Williams (eds.) *School Improvement: International Perspectives.* New York: Nova Science Publishers.

C. Breen (1999). Concerning mathematics teacher development and the challenges of the new millennium. *Pythagoras: Journal of the Association for Mathematics Education of South Africa,* 49*,* 42–48.

G. Brousseau (2002). *Theory of Didactical Situations in Mathematics*. New York: Kluwer Academic Publishers.

M. Chinnapan and T. Forrester (2014). Generating conceptual and procedural knowledge of fractions by pre-service teachers. *Mathematics Education Research Journal*, 26, 871–896.

F. Depaepe, J. Torbeyns, N. Vermeersch, D. Janssens, R. Janssen, G. Kelchtermans, L. Verschaffel, and W. V. Dooren (2015). Teachers' content and pedagogical content knowledge on rational numbers: A comparison of prospective elementary and lower secondary school teachers. *Teaching and Teacher Education*, 47, 82–92.

F. Depaepe, P. Van Roy, J. Torbeyns, T. Kleickmann, W. Van Dooren, and L. Verschaffel (2018). Stimulating pre-service teachers' content and pedagogical content knowledge on rational numbers. *Educational Studies in Mathematics*, 99, 197–216.

E. H. Geller, J. Y. Son and J. W. Stigler (2017). Conceptual explanations and understanding fraction comparisons. *Learning and Instruction*, 52, 122–129.

M. Graven (2012). Changing the story: Teacher education through re-authoring their narratives. In: C. Day (ed.), *The Routledge International Handbook of Teacher and School Development*. Oxon: Routledge.

J. H. Hunt and S. B. Empson (2014). Exploratory study of informal strategies for equal sharing problems of students with learning disabilities. *Learning Disability Quarterly*, 38, 208–220.

R. Kaasila, E. Pekhonen, and A. Hellinen (2010). Finnish pre-service teachers' and upper secondary students' understanding of division and reasoning strategies used. *Educational Studies in Mathematics*, 73, 247–261.

A. Klemer, S. Rapoport, and H. Lev-Zamir (2018). The missing link in teachers' knowledge about common fraction division. *International Journal of Mathematical Education in Science and Technology*, 50*,* 1256–1272.

M. Y. Lee (2017). Pre-service teachers' flexibility with referent units in solving a fraction division problem. *Educational Studies in Mathematics*, 96, 327–348.

C. Lemonidis, H. Tsakiridou, and I. Meliopoulou (2018). In-service teachers' content and pedagogical content knowledge in mental calculations with rational numbers. *International Journal of Science and Mathematics Education*, 16, 1127–1145.

K. E. Lewis (2014). Difference not deficit: Reconceptualizing mathematical learning disabilities. *Journal for Research in Mathematics Education*, 45, 351–396.

K. E. Lewis and D. M. Lynn (2018). Access through compensation: Emancipatory view of a mathematical learning disability. *Cognition and Instruction*, 36, 424–459.

S. Johnson, M. Monk, and M. Hodges (2000). Teacher development and change in South Africa: A critique of the appropriateness of transfer of northern/western practice. *COMPARE,* 30(2), 179–192.

B. F. Nebres (2009). Engaging the community, targeted interventions: Achieving scale in basic education reform. *Education Research for Policy and Practice*, 8, 231–245.

OECD (2019). *PISA 2018 results (Volume I): What students know and can do*. Paris: PISA, OECD Publishing. https://doi.org/10.1787/5f07c754-en.

D. Olanoff, J. J. Lo, and J. M. Tobias (2014). Mathematical content knowledge for teaching elementary mathematics: A focus on fractions. *The Mathematics Enthusiast*, 11(2), 257–310.

Z. H. Putra and C. Winsløw (2018). Teachers' collective knowledge: The case of equivalent fractions. *Journal of Physics: Conference Series,* 1088*,* 012003.

R. Rosli, D. Goldsby, A. J. Onwuegbuzie, M. M. Capraro, R. M. Capraro and E. G. Y. Gonzalez (2020). Elementary preservice teachers' knowledge, perceptions and attitudes towards fractions: A mixed analysis. *Journal on Mathematics Education,* 11, 59–76.

Ö. Şahin, B. Gökkurt, and Y. Soylu (2016). Examining prospective mathematics teachers' pedagogical content knowledge on fractions in terms of students' mistakes. *International Journal of Mathematical Education in Science and Technology,* 47*,* 531–551.

J. Settlage (2011). Counterstories from White mainstream preservice teachers: Resisting the master narrative of deficit by default. *Cultural Studies of Science Education*, 6, 803–836.

J. P. Smith (1995). Competent reasoning with rational numbers. *Cognition and Instruction*, 13, 3–50.

D. J. P. Sodusta and R. C. A. De Leon (2019). Teaching in disadvantaged schools: A cross-cultural exploration of millennial American and Filipino teachers. *Research, Society, and Development*, 8(11), 1–33.

Proceedings of the 14th International Congress on Mathematical Education Downloaded from www.worldscientific.com by 203.177.19.234 on 07/06/25. Re-use and distribution is strictly not permitted, except for Open Access articles.

M. T. Tatto, J. Schwille, S. L. Senk, L. Ingvarson, G. Rowley, R. Peck, K. Bankov, M. Rodriguez, and M. Reckase (2012). *Policy, Practice, and Readiness to Teach Primary and Secondary Mathematics in 17 countries: Findings from the IEA Teacher Education and Development Study in Mathematics (TEDS-M)*. Amsterdam: IEA.

H. Van Steenbrugge, E. Lesage, M. Valcke, and A. Desoete (2014). Preservice elementary school teachers' knowledge of fractions: A mirror of students' knowledge? *Journal of Curriculum Studies*, 46(1), 138–161.

D. M. B. Verzosa (2020). Pre-service elementary teachers' reasoning in introducing multidigit subtraction. *International Journal of Mathematical Education in Science and Technology*, 51, 876–892.

D. M. B. Verzosa, M. T. Tulao-Fernando, and C. P. Vistro-Yu (2017). Congruence between context and opportunities for professional development of mathematics teachers in the Philippines. In: B. Kaur, O. Kwon, and Y. Leong (eds.), *Professional Development of Mathematics Teachers. Mathematics Education: An Asian Perspective*. Singapore: Springer.

Proceedings of the 14th International Congress on Mathematical Education Downloaded from www.worldscientific.com by 203.177.19.234 on 07/06/25. Re-use and distribution is strictly not permitted, except for Open Access articles.

Color Symmetry in the Hand Woven Mats of the Jama Mapun

Ma. Louise Antonette N. De las Peñas*, Agnes D. Garciano and Debbie Marie Verzosa
Department of Mathematics
Ateneo de Manila University
Loyola Heights, Quezon City
E-mail: mdelaspenas@ateneo.edu

Abstract

The Jama Mapun is an indigenous Filipino community known for their hand-woven mats. Their technique is impressive given that their designs are directly woven into the mat, and are not produced by simply painting or inserting colored leaves in completed mats. In this paper, we highlight concepts in color symmetry evident in the Jama Mapun mats. We discuss perfect and non-perfect colored designs including a finite, frieze and wallpaper pattern.

Jama Mapun Mats

The beauty and quality of Jama Mapun mats is well known [2]. This Filipino community primarily lives in the Sulu archipelago and in Southern Palawan. The intricate designs of a *tepo* (mat) attest to the artistry as well as the geometric skill of the weavers, a skill that has been passed over several generations. Their preference for colorful patterns is inherent in their culture. As described in [2], quarrelling families may display a number of colorful mats to cool off and appease each other.

The Jama Mapun use a variety of pandan (*Pandanus tectorius* or screw pine) for the mats, and these leaves are boiled and dyed. Their weaving technique is unique. While it is customary for mat weavers to begin at the edge or at the corners, the Jama Mapun weave along the center of the mat, continuing downward along a line or axis, until the central panel is completed (see Figure 1). Once this central panel is completed, the leftover portions of the leaves on either side of the panel are woven to complete the entire mat. No additional straws are inserted at this point.



Figure 1: *The mat's central panel*



Figure 2: *A tupi or fold*

One impressive feature of a Jama Mapun mat is that the design is directly woven into the mat. That is, the designs do not arise from simply painting or inserting colored leaves in completed mats. To facilitate the creation of colored patterns, the Jama Mapun employ a mat-weaving technique called the

tupi, or fold. Without a *tupi*, each leaf necessarily follows a single direction, so that color changes would not be possible along that direction. However, the *tupi* or folding technique allows the weaver to change the direction of a colored leaf and create a wider range of designs. These *tupi* can be seen along borders where leaves of different colors meet (Figure 2).

Because designs are directly woven into the mat, a Jama Mapun weaver should have a mental picture of the mat's overall design, including the motifs and the interplay of the colors within the mat when they start weaving the central panel [4]. For example, Figure 3a shows how the leaves within the mat are cleverly organized and laid out. Each colored line drawn in the figure shows the path of one pandan leaf from the central panel. Clearly, weavers must have foresight. They must determine beforehand how many leaves of a certain color are needed. They also need to know when to utilize a *tupi* so that their planned design is realized, while at the same time ensuring that the weaving still results in a rectangular mat.



(a) (b)

Figure 3: *(a) A Jama Mapun mat (arrows signifying the paths of some pandan leaves); (b) a larger view of a portion of the mat*

This paper highlights color symmetry concepts [3,5] found in Jama Mapun mats, and demonstrates the mathematical insight necessary for weaving complex geometric patterns directly into these mats.

Color Symmetry Theory

Colored handwoven Jama Mapun mats are a physical manifestation of color symmetry theory [3], where a basic problem addressed is the assignment of colors to designs or patterns to arrive at a colored symmetrical pattern. In the construction and analysis of such a pattern, there are two groups that are usually taken into consideration: the group G consisting of symmetries of the uncolored pattern [1]; and the group H consisting of elements in G that effect a permutation of the colors. The elements of H are called the color symmetries of the pattern. If all symmetries in G are color symmetries, that is, $H = G$, then we say the pattern is *perfectly colored*.

Perfectly colored patterns and designs can be found in Jama Mapun mats. An example is the mat shown in Figure 3a exhibiting one of their typical designs: the *katam* (crab), which consists of five colors. The central panel in this case is along a vertical line passing through the center of the mat. There is a $180°$ rotational symmetry, which is a natural consequence of standard Jama Mapun weaving, which is to weave the design on one side of the central panel and turning it around to weave the same design in other side (this is referred to as *limbang*). Although there seems to be a reflection along a horizontal line passing through the center of the mat, the direction of the weaves (Figure 3b) shows otherwise. A similar analysis will show that there is also no reflection along a vertical line through the center of the mat. The $180°$

rotation is the only non-trivial symmetry of this finite pattern (with colors disregarded). The symmetry group of the pattern is a cyclic group C_2 of order 2. This rotation is a color symmetry of the pattern and fixes the given colors.

A mat consisting of a repeating pattern of five colors is cut and created into a lady's handbag shown in Figure 4. If we consider this as a planar repeating pattern, its symmetry group (disregarding colors) is a plane crystallographic group of type $p2$ (2222 in orbifold notation), generated by three distinct 180° rotations with centers that are not collinear as labeled in the figure. This is also a perfectly colored pattern. Observe that the 180° rotation with center located at the pink square motif fixes the colors pink, purple and white and interchanges the colors blue and red. On the other hand, the 180° rotation with center located at the center of the white parallelogram interchanges blue and pink; purple and red, and fixes white. Finally, the 180° rotation about the point where four parallelograms meet fixes white and interchanges blue and pink, and purple and red.

Aside from perfectly colored patterns, we have also found non-perfectly colored patterns in the Jama Mapun mats. The mat in Figure 5 shows the weaver's rendering of five pairs of leaves (*gumamela*). In weaving this mat, the intention was to balance the vibrant colors (blue, pink, orange, green), with white and this color is used more dominantly than the other colors to arrive at the colored pattern. The total area of all the white regions exceeds the area of each of the regions of another color. Assuming that the pattern repeats indefinitely to the right or left, the symmetry group of the pattern with colors disregarded is a frieze group of type $p112$ (22∞ in orbifold notation) which is generated by two distinct 180° rotations with centers spaced at half a translation distance. Note that if we consider the 180° rotation at the center of the mat, a white triangle is brought to a pink triangle and a green triangle, so that this rotation does not effect a permutation of the colors. Thus the pattern is not perfectly colored.



Figure 4: *A perfectly colored pattern*



Figure 5: *A non-perfectly colored mat*

The symmetry group of the uncolored pattern found in Figure 6(a) is a plane crystallographic group of type $p2$ (2222), with generators three distinct 180° rotations with centers (black, white, and ringed dots) shown. Note that, the 180° rotation centered at the white dot does not effect a permutation of the colors. The white triangular regions are sent to pink, blue, green and orange regions. This is a non-perfectly colored pattern. Just like the pattern in Figure 4, here the white leaves are used more often than the other colored leaves.

The Jama Mapun use motifs that form an integral part of the mat's design. These motifs may possess local symmetry that is not a symmetry of the overall design. Consider for instance the motif in Figure 6(b), consisting of two colors coming from the mat in Figure 6(a). Here, we see that the symmetry group of the uncolored motif is a cyclic group of order 4 generated by a 90° rotation. This rotation is not a global symmetry of the mat; that is, it does not send the overall pattern to itself. However, the 180°

rotation on this motif is a local symmetry that is also a global symmetry. In Figure 6(a), we identify the center of this rotation to be at the ringed dot. This rotation fixes the colors pink and white in the motif. Although the mat is not properly stretched out, we can see that this rotation also fixes the other regions colored pink and white in the overall design. In color symmetry theory, this symmetry is called both a local and global color symmetry. Interestingly, this coordination between the colors of the motif and that of the overall pattern is a taken into consideration by the Jama Mapun in their weaving.



(a) (b)

Figure 6: *(a) A non-perfectly colored pattern; (b) a motif taken from (a)*

Conclusion

What we have discussed in this short note is a glimpse of the variety of colored symmetrical patterns found in the mats created by the Jama Mapun. Without formal or advanced training in mathematics, it is impressive how they can organize and arrive at these colored designs in the mat that are concrete realizations of ideas from color symmetry theory, without use of template, pencil or paper. We have shown that the patterns and motifs have inherent symmetries and color symmetries as a result of their weaving technique (e.g. the existence of 180° rotation due to *limbang*). The manner in which the leaves are woven in place such as that shown in Figure 6(b) disqualifies some symmetries. There is no vertical reflection with axis passing through the center of the motif that is a symmetry of the given motif. A future direction in our research is to discuss the existence or non-existence of symmetries or color symmetries in these mats while also considering particular combinations of weaving strokes.

References

[1] D. W. Crowe, *Introduction to the plane symmetries*, Symmetry Comes of Age: The Role of Pattern in Culture, eds. D. K. Washburn and D. W. Crowe, University of Washington Press, pp. 3-18, 2004.
[2] L. Fernando-Amilbangsa, *Visual arts of the Sulu archipelago*, Ateneo de Manila University Press, Quezon City, 2005.
[3] M. Senechal, *Color symmetry*, Comp & Maths with Appls. Vol. 16, pp. 545-553. 1988.
[4] M.L.A.N. De las Peñas, A. Garciano, & D. Verzosa, *Weaving mat(h)s*, Mathematical Intelligencer (in press).
[5] R. R. Roth, *Local color symmetry*, Geom. Dedicata, Vol. 17, pp. 99-108. 1994.

Ateneo de Manila University

Archīum Ateneo

Mathematics Faculty Publications | Mathematics Department

4-2018

Crystallographic patterns in Philippine indigenous textiles

Ma. Louise Antonette N. De Las Peñas

Agnes Garciano
Ateneo de Manila University

Debbie Marie Verzosa
University of Southern Mindanao

Eduard C. Taganap

Follow this and additional works at: https://archium.ateneo.edu/mathematics-faculty-pubs

Recommended Citation

De Las Peñas, M. L. A. N., Garciano, A., Verzosa, D. M., & Taganap, E. (2018). Crystallographic patterns in Philippine indigenous textiles. Journal of Applied Crystallography, 51(2), 456-469.

This Article is brought to you for free and open access by the Mathematics Department at Archīum Ateneo. It has been accepted for inclusion in Mathematics Faculty Publications by an authorized administrator of Archīum Ateneo. For more information, please contact oadrcw.ls@ateneo.edu.

JOURNAL OF APPLIED CRYSTALLOGRAPHY

ISSN 1600-5767

Crystallographic patterns in Philippine indigenous textiles

Ma. Louise Antonette N. De Las Peñas,[a]* Agnes Garciano,[a] Debbie Marie Verzosa[b] and Eduard Taganap[c]

[a]Department of Mathematics, Ateneo de Manila University, Loyola Heights, Quezon City, Metro Manila 1108, Philippines, [b]Department of Mathematics and Statistics, University of Southern Mindanao, Kabacan, Cotabato 9407, Philippines, and [c]Department of Mathematics and Physics, Central Luzon State University, Science City of Muñoz, Nueva Ecija 3120, Philippines. *Correspondence e-mail: mdelaspenas@ateneo.edu

Received 12 November 2017
Accepted 5 February 2018

Edited by J. M. García-Ruiz, Instituto Andaluz de Ciencias de la Tierra, Granada, Spain

Keywords: symmetry; color symmetry; Philippine indigenous textiles.

The aim of this study was to analyze a representative sample of Philippine indigenous textiles in order to capture the range of symmetries and color symmetries present. This paper examines the existence of symmetries in finite designs, and classifies the plane-group and frieze-group symmetry types of the repeated patterns in woven textiles. The tendency of a particular symmetry to be more or less common than another can indicate relationships between the symmetries and the weaving technique or the culture that produced them. This paper will also examine designs and patterns with color symmetry found in these textiles. The sample consisted of 588 repeated patterns and finite designs in textiles (389 plane, 166 frieze and 33 finite) culled from well known museums in the Philippines, personal collections of scholars, existing literature on Philippine textiles and field visits.

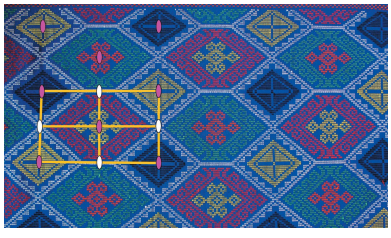
1. Introduction

Philippine woven textiles offer a glimpse into the rich cultural heritage of the Filipino people. From the northernmost parts of Luzon to the southern edges of the Sulu archipelago, the extensive range and quality of Philippine textiles are beyond question (Fig. 1). Geography and history have influenced the evolution of textile arts among the indigenous communities.

In northern Luzon, the Cordillera people, protected in part by their terrain, resisted the Spanish conquerors. Consequently, there has been more persistence of traditional arts, values and cultural identities in these areas than in the lowlands. The traditional blankets produced by the Tinguians and the Bontocs from the Cordillera region in northern Luzon and the *inabel* cotton weaves of the Ilocanos in the northern region of the Philippines offer evidence of the strong weaving tradition in these areas.

In the south, the rivalry between the Spanish and the Muslims was important in the preservation of indigenous culture. The Muslims resisted the colonization of Mindanao and were able to prevent the destruction of native arts and crafts, in contrast to what occurred in lowland Luzon and the Visayas. Their textiles include the *malong* or 'tubular garment' of the Maranaos, the *pisyabit* or 'head cover' of the Tausugs, and the *saputangan* or 'head cloth' of the Yakans. Mindanao is also home to indigenous non-Muslim communities with their own weaving traditions; their textiles include the *t'nalak* or tie-dyed fabric of the T'bolis, as well as the woven cloths of the Bagobos.

In this work, we study the existence of symmetries in finite designs and classify the plane-group and frieze-group



© 2018 International Union of Crystallography

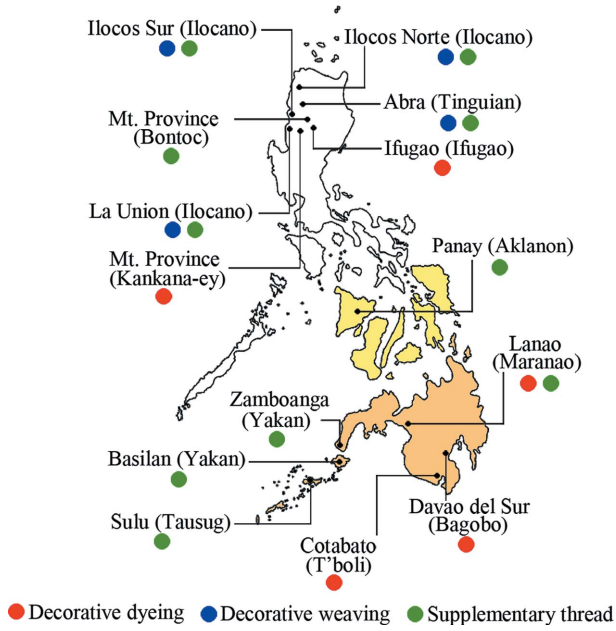


Figure 1
Weaving communities in the Philippines. The island groups of Luzon, Visayas and Mindanao are the white, light yellow and light orange colored regions, respectively. Red, blue and green dots are used to denote decorative dyeing, decorative weaving and supplementary thread weaving techniques, respectively.

symmetry types of the repeated patterns in woven textiles from indigenous communities in the Philippines. This paper will also discuss the designs and patterns with color symmetry present in the textiles. We consider the use of colors in the motifs and investigate the associated color groups of the resulting designs and patterns.

The outline of the paper is as follows: §2 presents the materials and methods used in this study, §3 details the mathematical basis for the analysis of the textiles, §4 presents the different weaving techniques employed by the indigenous communities in the Philippines, §5 discusses the symmetry group structures of the finite designs and repeated patterns present in representative textiles arising from each weaving technique, §6 explains the color symmetries present in textiles discussed previously in §5, and finally §7 provides a summary of the work.

2. Materials and methods

This study is focused on woven textiles (excluding embroidered or beaded textiles), specifically the analysis of the frieze patterns, plane patterns and finite designs of these textiles. In our study, we adapt the methodology of Washburn & Crowe (1988). A symmetrical figure within the plane is called a 'pattern' or 'design'; the terms 'pattern' or 'repeated pattern' (for emphasis) are reserved for designs with translational symmetry. A pattern is referred to as a 'frieze' if it has translational symmetries in only one direction, whereas a 'plane pattern' has symmetries that include two linearly independent translations; if a design does not have any translational symmetry, then it is referred to as a 'finite design'.

The aim of this study was to analyze a representative sample that can capture the range of symmetries and color symmetries in Philippine textiles. To achieve this, we examined samples from well known museums in the Philippines (National Museum, Yuchengco Museum, Vargas Museum, Museo Kordilyera) and private textile collections of scholars (Quintos, Salvador-Amores, Respicio), as well as the collections of the National Commission for Culture and the Arts (NCCA) and the Philippine Textile Council. We also examined the existing literature on Philippine textiles (Guatlo, 2013; Pastor-Roces, 1991; Paterno *et al.*, 2001; Respicio, 2003, 2014; Rubinstein, 1989) and expanded the sample through field visits to indigenous communities (T'boli and Yakan villages), loom weavers in the Ilocos region (Nagbacalan Loomweavers Multi-purpose Cooperative) and festivals (Dayao Festival and NCCA International Festival of Extraordinary Textiles). From these sources, a representative sample of textiles exhibiting patterns of various symmetries and color symmetries was obtained for each indigenous community.

3. Mathematical considerations

The mathematical study of a finite design or repeated pattern in a cultural ornament such as a textile entails investigation of its symmetries. An isometry of the plane (translation, rotation, reflection or glide reflection), which is a distance preserving transformation that maps a finite design or repeated pattern onto itself, is called a 'symmetry' of the respective design or pattern.

A finite design does not have translational symmetry (and consequently, has no glide reflections) and can have reflection and/or rotational symmetry. A finite design is of cyclic symmetry group type C_r if it has r-fold rotational symmetry and no reflection symmetry. On the other hand, a finite design is of dihedral symmetry group type D_r if it has reflection symmetry as well as r-fold rotational symmetry. Note that D_1 is the symmetry group of designs with bilateral symmetry only.

Frieze patterns and plane patterns are classified on the basis of the symmetries of the pattern. There are seven symmetry group types for frieze patterns, called 'frieze groups', and 17 symmetry group types for plane patterns, called 'plane groups'. Here we adopt the standard nomenclature for frieze and plane groups given by Kopský & Litvin (2002) and Hahn (2005), respectively.

In the figures that supplement our discussion of the symmetry groups of the repeated patterns, we provide the lattice units of the symmetry groups. The centers of rotation are marked by ellipses, mirror reflection axes by bold lines and glide reflection axes by dashed lines.

4. Weaving techniques

Weaving is done on a loom. The warp threads run vertically from the warp beam towards the weaver while the weft threads run perpendicular to the warp threads, that is, from left to right or *vice versa*, in relation to the weaver. Philippine

weaving techniques may be classified into three broad categories to be explained further in §4.1–4.3: (i) decorative dyeing, (ii) decorative weaving and (iii) supplementary thread techniques. Fig. 1 shows the different techniques employed in traditional weaving areas of the Philippines. Of the 588 finite designs and repeated patterns examined, 138 were woven using the decorative dyeing technique, 59 using decorative weaving techniques and 391 using the supplementary thread technique.

4.1. Decorative dyeing

Ikat is a term for describing the decorative dyeing technique. The process involves binding or knotting several threads together at certain intervals and then submerging the entire mass of threads in a dye. The knotted portions will resist the dye, from which the pattern may be seen. Some knotted threads may be unraveled and the whole mass soaked once again in a different dye to produce a more complex pattern. This process may be repeated several times. The *ikat* used in the Philippines is primarily warp *ikat*. This means that it is the warp rather than the weft threads that are soaked in dye, so that a pattern emerges when weft threads are interwoven through the dyed warp threads.

The T'bolis from Lake Sebu, Cotabato, in southern Mindanao, use wild abaca thread to produce *ikat* patterns, usually colored black, red and natural beige, the original color of the abaca leaves (Fig. 2). The warp threads are secured on a frame and portions are knotted to resist the dye. These threads are submerged in black dye and dried. Next, the areas that are to be colored red are untied and the threads boiled in red liquid. The rest of the knotted portions are then untied, and these become the areas that appear in the natural color of the abaca leaves. Weft threads are then interwoven perpendicular to the patterned warp threads to produce the cloth (Fig. 3*a*). The *ikat* cloth is referred to by the T'bolis as the *t'nalak*. The T'bolis weave the *t'nalak* relying on a mental image of the patterns, which are passed on from mother to daughter, or bestowed upon the weaver by *Fu Dalu*, the spirit of the abaca, through dreams (Paterno *et al.*, 2001). As such, the *t'nalak* is a prized possession used by the T'bolis on special occasions such as births, weddings or funerals.

The Bagobos, indigenous people from the Davao region in southern Mindanao, and the Ifugaos and Kankana-eys from

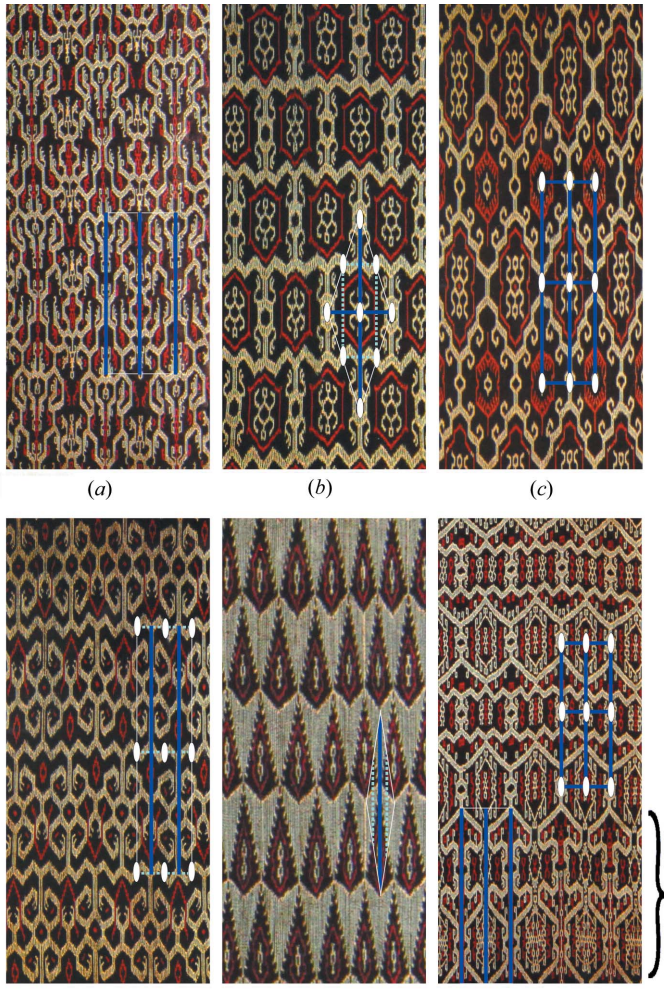


Figure 2
T'nalak with symmetry groups (*a*) *p*1*m*1, (*b*) *c*2*mm*, (*c*) *p*2*mm*, (*d*) *p*2*mg* and (*e*) *c*1*m*1. In (*f*), a *t'nalak* with primary design and symmetry group *p*2*mm*, bordered below by a frieze with symmetry group *p*1*m*1 (marked by brace). Images from Paterno *et al.* (2001) (photographed by Neil Oshima).



Figure 3
T'boli weavers. (*a*) *Agustin Sudaw* (photographed by Ma. Louise Antonette De Las Peñas, July 2014, Manila, Metro Manila); (*b*) *Charlie Dulay* (photographed by I. Castrillo, September 2017, General Santos City, South Cotabato).

the Cordillera region in northern Luzon also employ the warp *ikat*. Fig. 4 shows a blanket woven by the Bagobos, displaying crocodile motifs. The crocodile or 'big lizard' is a powerful motif used by the weaving communities among the Bagobos and Ifugaos in their tie-dyed fabrics; amphibious forms are regarded as vehicles of spirits traveling the world and beyond (Respicio, 2003).

The Maranaos from southwestern Mindanao, on the other hand, employ the weft *ikat* technique in their weaving, where the weft threads are soaked in dye. An example of the Maranao art is the *malong* shown in Fig. 5.

4.2. Decorative weaving

In decorative weaving techniques, the patterns are created not through dyeing but through the weaving process itself. The simplest among these is the plain weave, where the weft passes over and under every other warp thread (one-over-one-under). This basic weave is common among all weaving communities. Plaids or stripes are the more familiar forms of decorative weaving, and the *hablon* of the Hiligaynons, in Iloilo, western Visayas, demonstrates this kind of weave. It is usually used for the *patadyong*, the Visayan wrap-around skirt. The result of a one-over-one-under weave is a fabric whose back pattern is the 'negative' (with respect to color) of the front pattern. As such, it is sometimes called a 'face-to-face weave'.

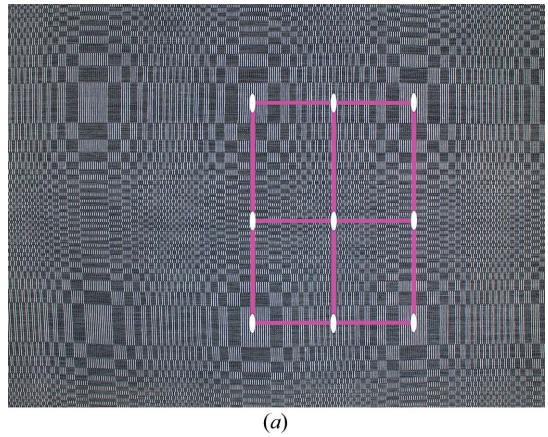
More variety can be achieved by setting differently colored warp threads on the loom in irregular intervals or by changing the color of the weft threads. One notable example is the *binakul* pattern from northern Luzon (Fig. 6); the yarns are arranged in such a way that, when the weft yarns interlock with the warp yarns, squares and rectangles of varying sizes are formed. Two hues of yarn are used (positive white and negative black, blue or deep red), and the result is an optical pattern giving an illusion of swirling circles or ripples, intended to cause dizziness to evil spirits. This kind of decorative weave technique is achieved using a pedal frame loom (Fig. 7) and is common among the weaving communities in the Ilocos region and Abra in northern Luzon.



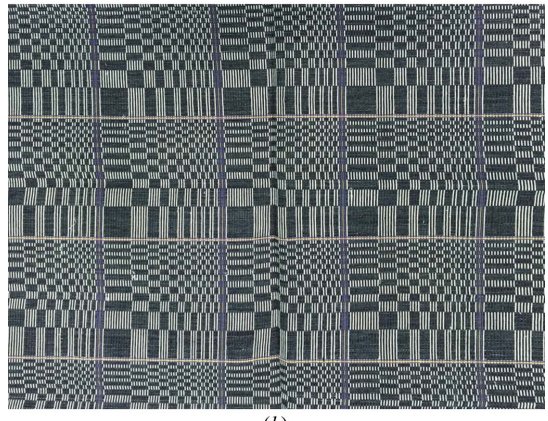
Figure 4
Warp *ikat* woven blanket from the Bagobos showing friezes with symmetry groups *p*1*m*1 and *p*2*mm*. The bottom frieze shows a crocodile design. (*Ikat from the National Museum collection*, photographed by Debbie Marie Verzosa, May 2014, Manila, Metro Manila.)



Figure 5
Malong in weft *ikat* weave from the Maranaos, showing friezes with symmetry groups *p*2*mm* and *p*1*m*1. (*Malong from the F. Quintos collection*, photographed by Ma. Louise Antonette De Las Peñas, September 2014, Quezon City, Metro Manila.)



(*a*)



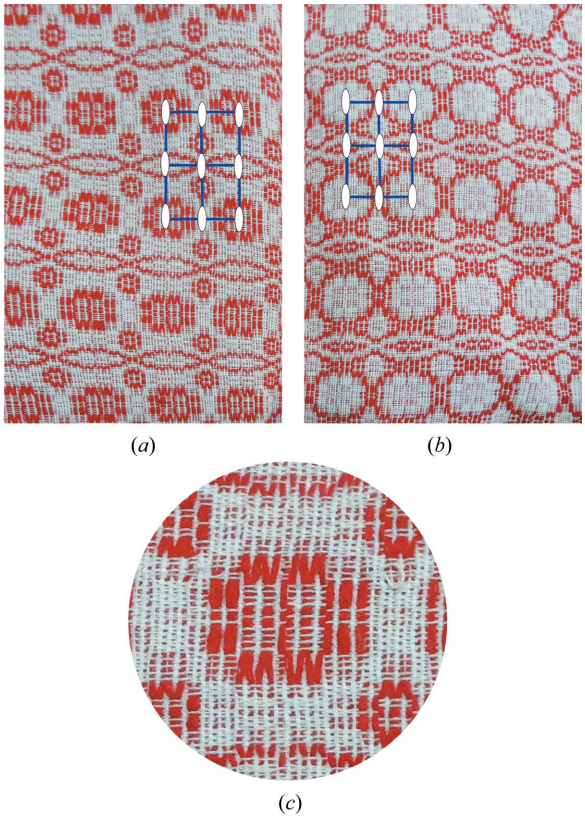
(*b*)

Figure 6
Two variants of *binakul*, the (*a*) optical and (*b*) quarter circle patterns with symmetry groups *p*2*mm* and *p*1, respectively. (*Binakul from the Phillipine Textile Council collection*, photographed by Agnes Garciano, May 2014, Manila, Metro Manila.)

Another option to achieve variety is to make the warp 'float' or skip over two or more weft threads, or to make the weft float over two or more warp threads (Fig. 8*b*). These techniques are called warp float and weft float, respectively.



Figure 7
Weaver using the pedal loom. (Photographed by Ma. Louise Antonette De Las Peñas, May 2015, Paoay, Ilocos Norte.)

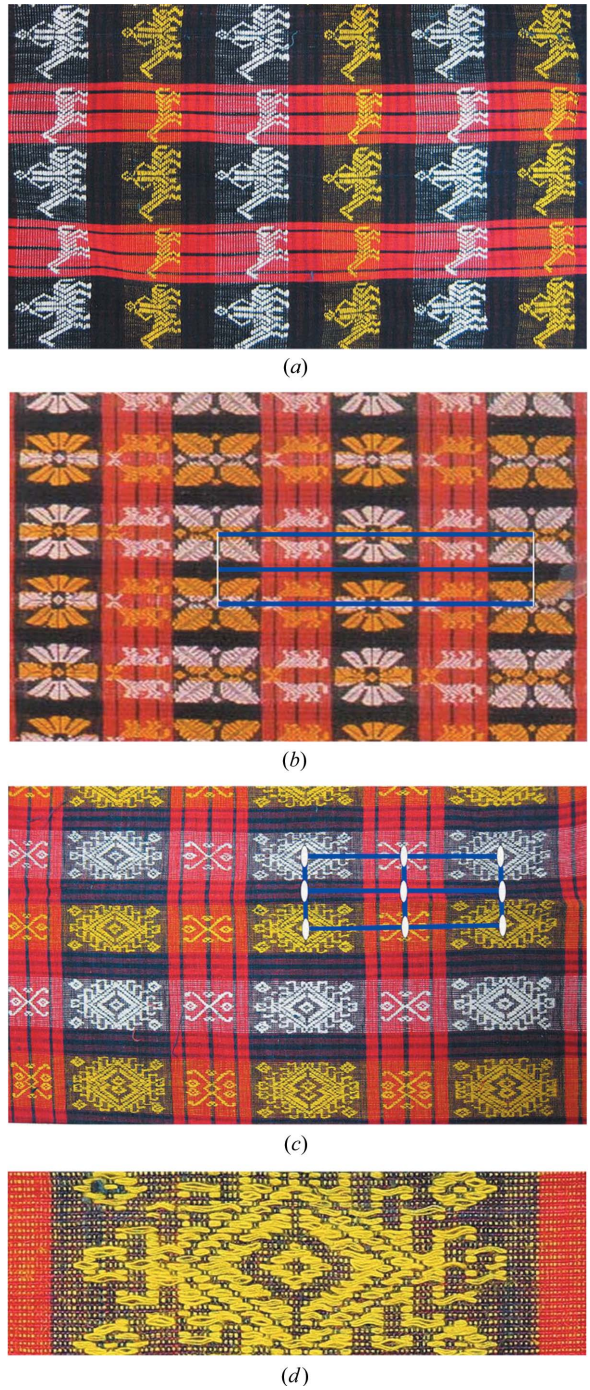


(*a*) (*b*) (*c*)

Figure 8
Textile woven using the decorative weft float technique. Patterns have symmetry group *p*2*mm*. Photographs show (*a*) the back and (*b*) the front; (*c*) detail view of the back side of the textile. (*Textile from the Nagbacalan Loomers Multi-purpose Cooperative collection*, photographed by Ma. Louise Antonette De Las Peñas, May 2015, Paoay, Ilocos Norte.)

4.3. Supplementary thread

Supplementary warp or weft techniques involve the insertion of additional extra warps or wefts to create designs. These



(*a*) (*b*) (*c*) (*d*)

Figure 9
Tinguian blankets woven in *pinilian* style with symmetry groups (*a*) *p*1, (*b*) *p*1*m*1 and (*c*) *p*2*mm*; (*d*) supplementary weft configuration employed to create a motif of the blanket in (*c*). [(*a*), (*c*) *Pinilian textiles from the F. Quintos collection*, photographed by Ma. Louise Antonette De Las Peñas, September 2014, Quezon City, Metro Manila; (*b*) Rubinstein (1989).]

threads are supplementary or extra in the sense that if they were removed the woven cloth would still be complete, though without the original pattern. In the supplementary weft technique, the additional weft threads are made to float over warp threads in a planned sequence to create the design or pattern. If the supplementary threads extend from edge to edge, they are 'continuous', but if they only go back and forth in small areas where they are needed, they are 'discontinuous'.



Figure 10
Tinguian blankets in *dinapat* style: (*a*) frieze with symmetry group *p*2*mm* and (*b*) planar pattern with symmetry group *p*2*mg*. (*c*) Detail view of (*b*). [(*a*) Rubinstein (1989); (*b*) *dinapat textile from the F. Quintos collection*, photographed by Ma. Louise Antonette De Las Peñas, September 2014, Quezon City, Metro Manila.]

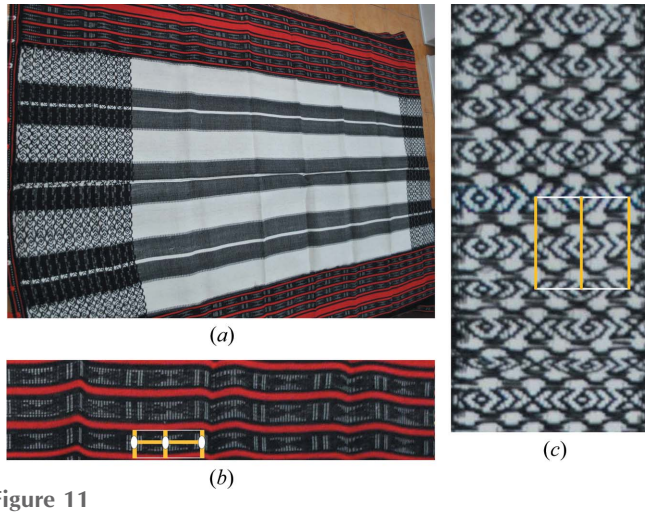


Figure 11
(*a*) *Pinagpagan*; (*b*) the friezes at the top and bottom have symmetry group *p*2*mm*; (*c*) the central panel displays a planar pattern with symmetry group *p*1*m*1 at the sides. [*Pinagpagan* (from De Las Peñas & Salvador-Amores, 2016), photographed by Dr Analyn Salvador-Amores, 2014, Baguio City, Benguet.]

4.3.1. Continuous supplementary weft. For the Tinguians and Bontocs in northern Luzon, the most common weaving technique is the continuous supplementary weft, which is evident in their blankets (Figs. 9–11). Most of the Tinguian blankets are woven using the *pinilian* (Fig. 9) or *dinapat* (Fig. 10) styles. These depict flora, leaves, human figures or animals; a common motif is the rider and horse (Fig. 9*a*). These motifs are highly valued, and the blankets are often used for funeral ceremonies. *Pinilian* means 'separated' or 'divided', referring to the checkerboard-like grid style of laying out the motifs. Note in Fig. 9(*d*) how a motif of the repeated pattern in Fig. 9(*c*) is created by the insertion of weft threads, producing 15 different horizontal weft configurations that are employed in a repeated sequence. Each weft configuration floats from left to right at repeated intervals. Meanwhile, *dinapat* denotes 'full' or 'total', referring to the repeated pattern that occupies the whole blanket. The *dinapat* blanket in Fig. 10(*b*) shows that, even without varying the breadth of the warp and weft yarns, patterns other than checkered or plaid can be produced, *i.e.* the manner in which the weft yarns are inserted produces red and white isosceles right-angled triangles.

Fig. 11(*a*) shows a *pinagpagan*, a typical funerary blanket of the Bontocs also woven using the continuous supplementary

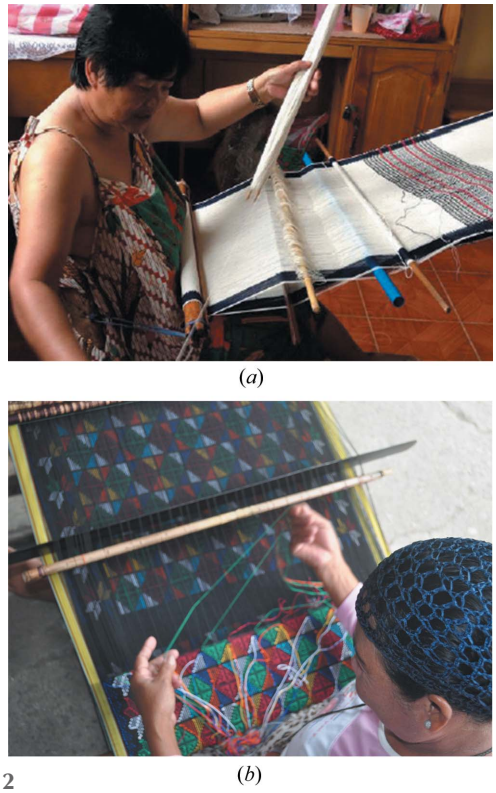


Figure 12
Weavers employing the supplementary weave technique. (*a*) Bontoc (continuous) and (*b*) Yakan (discontinuous). [(*a*) *Bontoc weaver* (from De Las Peñas & Salvador-Amores, 2016), photographed by Dr Analyn Salvador-Amores, 2014, Baguio City, Benguet; (*b*) *Yakan weaver*, photographed by Debbie Marie Verzosa, April 2015, Yakan village, Zamboanga City.]



(*a*) (*b*)

Figure 13
(*a*) Yakan textile showing the *palipattang* pattern consisting of friezes; (*b*) a representative frieze with symmetry group *p*2*mm* (lattice unit with white axes of reflection and centers of rotation) and color group *p*2*mm* (lattice unit with blue axes of reflection and pink centers of rotation). (*Textile with palipattang*, photographed by Debbie Marie Verzosa, April 2015, Yakan village, Zamboanga City.)

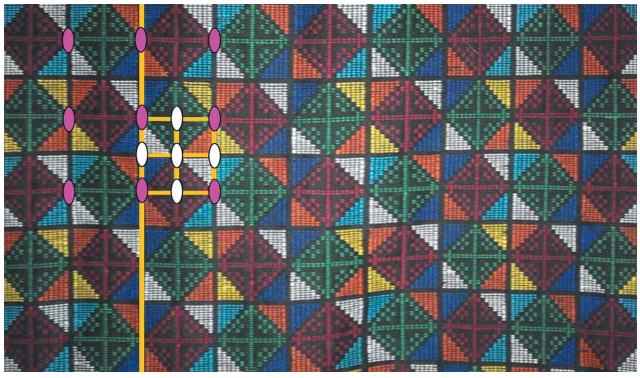


Figure 14
Yakan textile showing pattern of diamonds with symmetry group *p*2*mm* (lattice unit with yellow reflection axes; white/pink centers of rotation). Color group is *p*2 (lattice unit with pink centers of rotation). (*Textile with diamonds*, photographed by Debbie Marie Verzosa, April 2015, Yakan village, Zamboanga City.)

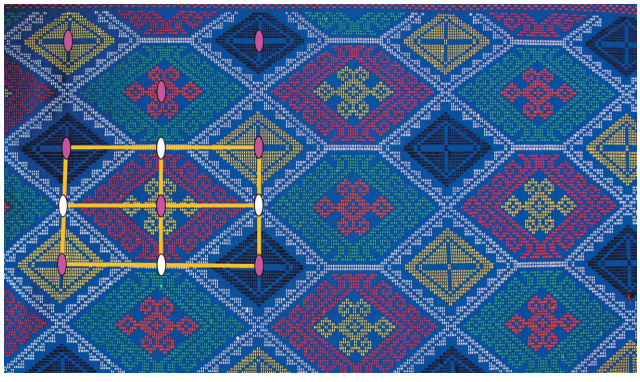


Figure 15
Yakan textile showing *bunga sama* pattern with symmetry group *p*2*mm* (lattice unit with yellow axes of reflection and pink centers of rotation). The color group is *p*2 (lattice unit with pink centers of rotation). (*Bunga sama design from the Phillipine Textile Council collection*, photographed by Agnes Garciano, May 2014, Manila, Metro Manila.)

weft technique. Three black stripes run through the central panel of the blanket, indicating its use for the deceased of high rank in society. The motif in Fig. 11(*c*), common in Bontoc funerary textiles, is referred to as the *matmata*, which means 'eyes of the ancestors', serving as a guide as one travels to the afterlife (De Las Peñas & Salvador-Amores, 2016). Fig. 12(*a*)



Figure 16
Yakan textile showing pattern with symmetry group *p*1*g*1. (*Textile with leaves of a vine*, photographed by Debbie Marie Verzosa, April 2015, Yakan village, Zamboanga City.)



(*a*) (*b*) (*c*)

Figure 17
(*a*) Yakan *saputangan* with finite designs in the center and four corners of symmetry type D_4, and friezes with symmetry group *p*2*mm*; (*b*) and (*c*) show closer views. (*Yakan suputangan from the F. Quintos collection*, photographed by Ma. Louise Antonette De Las Peñas, September 2014, Manila, Metro Manila.)

depicts a Bontoc weaver with her backstrap loom, weaving the central panel of the *pinagpagan*.

4.3.2. Discontinuous supplementary weft. A related technique is the discontinuous supplementary weft, very prominent among the Yakan weavers in southwestern Philippines. In this weave, the additional weft yarns are not woven across the entire length of the warp; they may be cut and be limited to a few warp yarns (Fig. 12*b*). Weft yarns are discontinuous, so it is possible to produce designs or patterns that change colors from left to right across the warp yarns.

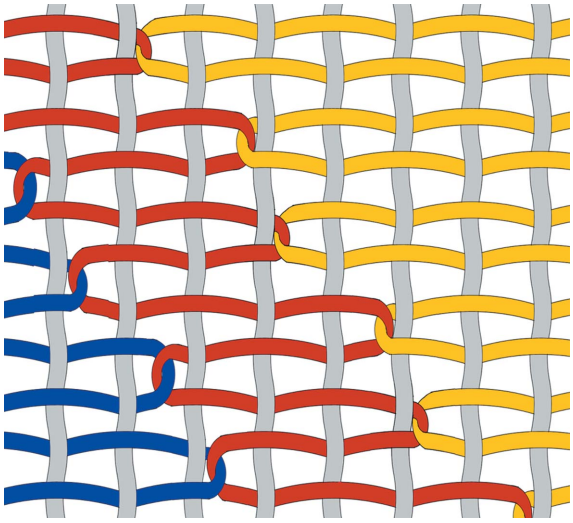
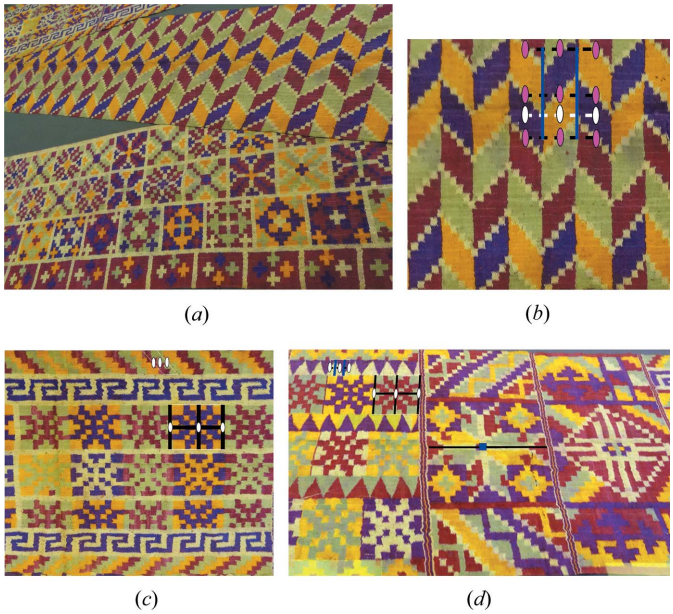


Figure 18
Tapestry weave.

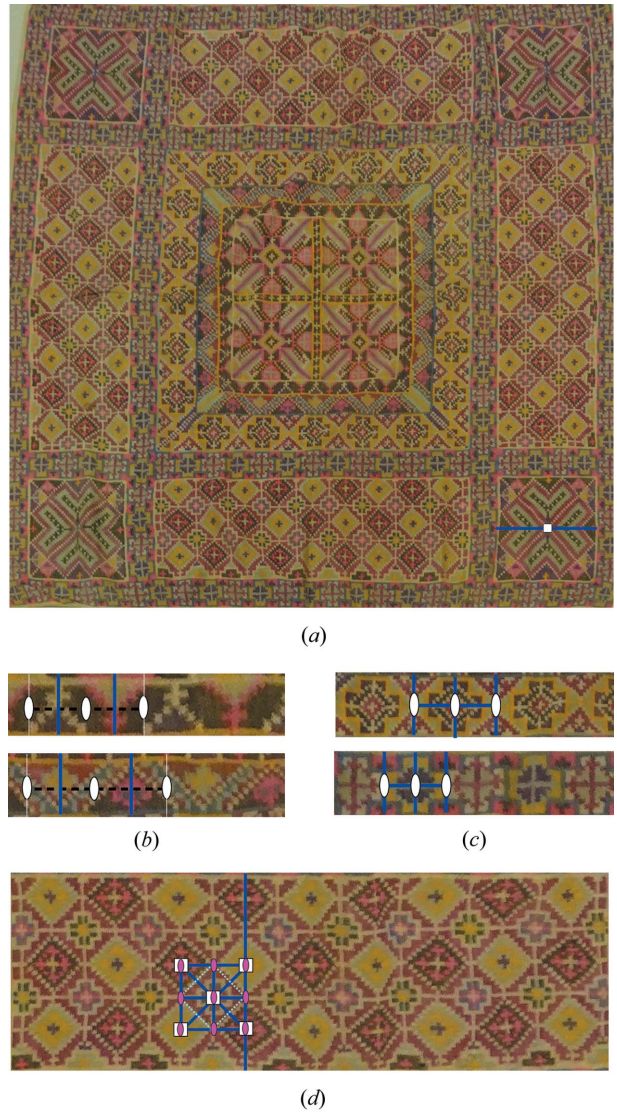


(*a*) (*b*) (*c*) (*d*)

Figure 19
(*a*) Tausug *kandit*; (*b*) plane pattern with symmetry group and color group *p2mg* (lattice unit of symmetry group is half that of color group); (*c*) frieze patterns with symmetry groups *p*211, *p*1 and *p*2*mm*; (*d*) finite design with symmetry group D_4. (*Tausug kandit from the F. Quintos collection*, photographed by Ma. Louise Antonette De Las Peñas, September 2014, Manila, Metro Manila.)

The Yakans employ this weaving technique to create colorful clothing with intricately woven designs. Their traditional clothing includes narrow cut pants resembling breeches, usually with rainbows (*palipattang*) (Fig. 13), and cloth wrapped around the waist by women as a tubular skirt or used by men as a waistcloth where a weapon can be inserted (Figs. 13–16). Traditional motifs include diamond motifs (Figs. 14 and 17), the *bunga sama* inspired by the skin patterns of a python (Fig. 15) and the *dawen-dawen*, meaning leaf of a vine (Fig. 16). The head cloth used by men is called *saputangan* and often shows geometric motifs (Fig. 17).

The Aklanon from Panay, western Visayas, also use the discontinuous supplementary weave in creating patterns in their *piña* textiles, made from the fibers of the leaves of the red Visayan pineapple.



(*a*) (*b*) (*c*) (*d*)

Figure 20
(*a*) Tausug *pisyabit*. Shown in the center and four corners are finite designs with the symmetry group D_4. Frieze patterns with symmetry groups (*b*) *p2mg* and (*c*) *p2mm*. (*d*) Plane pattern with symmetry group *p4mm*; color group *p*2 (lattice unit with pink centers of rotation). (*Tausug pisyabit from the F. Quintos collection*, photographed by Ma. Louise Antonette De Las Peñas, September 2014, Manila, Metro Manila.)

4.3.3. Tapestry. A special kind of woven textile produced using the discontinuous supplementary weft technique is the tapestry weave, where the principal weft itself is discontinuous, as opposed to the supplementary weft. In this weave, the weaver counts warp threads to be lifted so that the principal weft threads can be inserted to create the design or pattern (Gustilo, 2014). To hold the fabric together, weft yarns are dovetailed or locked with those adjacent (Fig. 18) (Rubinstein, 1989). This weaving technique is employed by the Tausugs from southwestern Mindanao; the Tausug men have their own version of the waistcloth called the *kandit* (Fig. 19*a*) and a head cloth called the *pisyabit* (Fig. 20) in tapestry weaves. The *kandit* shown is skillfully woven with a variety of geometric motifs of contrasting colors which combine friezes, plane patterns and finite designs that are characteristic of traditional Tausug textiles. Some weaves of the Bontocs and Yakans also incorporate the tapestry technique.

5. Symmetry structures by weaving techniques

5.1. Decorative dyeing

Vertical reflection symmetry was present in all the *t'nalak* textiles analyzed. The presence of vertical reflection symmetry is a consequence of the way in which the threads are knotted before the dye is applied (Fig. 3*b*). Among the symmetry group types of the plane, the patterns found were *p*1*m*1, *c*2*mm*, *p*2*mm*, *p*2*mg* and *c*1*m*1 (Figs. 2*a*–2*e*), which contain vertical reflections. In some samples, plane patterns are terminated by frieze patterns at the end of the cloth. For instance in Fig. 2(*f*), we see a plane pattern with the symmetry group *p*2*mm* and a frieze pattern with symmetry group *p*1*m*1 at the border of the cloth.

Fig 4 shows an example of a warp *ikat* weave that is one of the traditional textiles in Bagobo society, consisting of several frieze patterns with symmetry groups *p*1*m*1 and *p*2*mm*.

The weft *ikat* tubular garment from the Maranaos that is shown in Fig. 5 consists of several frieze patterns with symmetry groups *p*2*mm* and *p*1*m*1.

5.2. Decorative weaving

The *binakul* is an example of a face-to-face weave, the back portion exhibiting a plane pattern consisting of squares and rectangles. A typical example of the *binakul* with symmetry group *p*2*mm* is shown in Fig. 6(*a*). Fig. 6(*b*) shows another variant of the *binakul* with the quarter circle motif, which has the symmetry group *p*1. Interestingly, this plane pattern has a symmetry group that closely approximates *c*1*m*1, with diagonal reflection axes passing through the corners of the black squares. The repeated patterns in Figs. 8(*a*) and 8(*b*) show the back and front, respectively, of textiles also woven using the face-to-face weave. The plane patterns have symmetry group *p*2*mm*.

In our study, the symmetry analysis was focused on woven textiles showing a finite design or a repeated pattern, arising from a basic unit or motif. For this reason, although plaids and stripes fall under the decorative weaving technique, we do not consider these textiles to exhibit a motif (not a square or a rectangle) and they are not considered in the symmetry analysis.

5.3. Supplementary thread

5.3.1. Continuous supplementary weft. *Pinilian* and *dinapat* styled designs or repeated patterns in Tinguian blankets demonstrate different symmetry group types. For example, Figs. 9(*a*)–9(*c*) show plane patterns with symmetry groups *p*1, *p*1*m*1 and *p*2*mm*, respectively. The patterns in Figs. 10(*a*) and 10(*b*) have the frieze group *p*2*mm* and plane group *p*2*mg*, respectively.

The blanket in Fig. 11 is typical of a Bontoc funerary blanket consisting of three panels, the top and bottom adorned with frieze patterns with the symmetry group *p*2*mm*. Both ends of the central panel are adorned with plane patterns with the symmetry group *p*1*m*1.

5.3.2. Discontinuous supplementary weft. The *palipattang* shown in Fig. 13 consists of layers of frieze with the symmetry group *p*2*mm*. The repeated patterns of the Yakan textiles in Figs. 14 and 15 are planar patterns with the symmetry group *p*2*mm*, while that shown in Fig. 16 has the symmetry group *p*1*g*1. The *saputangan* presented in Fig. 17 highlights a traditional feature in the Yakan head cloth, consisting of diamonds in the center square, adorned with four squares at the corners with identical motifs, and frieze patterns at the sides. The center square and four corner squares adopt the inherent symmetries of the square and have symmetry group D_4. The frieze patterns have the symmetry group *p*2*mm*.

5.3.3. Tapestry. For the *kandit* in Fig. 19(*a*), one side has a plane pattern with the symmetry group *p*2*mg* (Fig. 19*b*). The other side changes pattern from four layers of frieze each with the symmetry group *p*2*mm* [see the lower portion of Fig. 19(*a*)] to seven layers of frieze, with the symmetry groups *p*211, *p*1 and *p*2*mm* (Fig. 19*c*); to eight layers of frieze with the symmetry groups *p*2*mm* and *p*2*mg* [see the left portion of Fig. 19(*d*)]; to finite designs including those with the symmetry group D_4 [see the center of Fig. 19(*d*)].

A typical *pisyabit* is shown in Fig. 20(*a*), consisting of five squares (one center, four corners) and four rectangles with friezes adorning the regions between these quadrilaterals. The squares have symmetry group D_4. The friezes have symmetry group *p*2*mg* (Fig. 20*b*) and *p*2*mm* (Fig. 20*c*). For the rectangular regions, the plane pattern has the symmetry group *p*4*mm* (Fig. 20*d*).

5.4. Discussion

The previous section presented a number of the existing symmetries found in each of the three broad categories of Philippine woven textiles. Also, as indicated in §4, there were 138, 59 and 391 repeated patterns and finite designs analyzed, respectively, for decorative dyeing, decorative weaving and supplementary thread techniques. For each weaving technique, the percentage of patterns and designs that exhibit a given symmetry group type was calculated and will be reported below.

5.4.1. Plane patterns. Given the nature of textiles which are two-dimensional, most patterns include repetition in two dimensions. Out of the 588 patterns and designs studied, 389 were plane patterns and the distribution of these patterns is presented in Fig. 21. Although these percentages are not intended to be estimates for the actual percentages of the entire population of woven textiles, they still show which symmetries are more common in a particular weaving technique.

It is clear from Fig. 21 that, in all of the samples analyzed, threefold and sixfold symmetries are absent, with the non-occurrence of the plane groups *p*3, *p*3*m*1, *p*31*m*, *p*6 and *p*6*mm*. Since the warp and weft threads in the textile are perpendicular, a three- or sixfold rotation does not map a motif onto itself (Grünbaum & Shephard, 1988).

Furthermore, the supplementary thread technique gives the most variety in terms of symmetry. It is possible that the 'freedom' offered by inserting additional threads can broaden the range of designs that may be produced using this technique. For example, the textile from the Yakans showing *dawen-dawen* (Fig. 16) has the symmetry group *p*1*g*1, which is a rare group in our sample. We have not come across a planar pattern with symmetry group *p*2*gg* from the Yakans using the supplementary thread technique, but it is not impossible to create these patterns. Instead, customary preference of patterns such as *bunga sama* or diamond *etc*. gives rise to the occurrence of other planar groups such as *p*2*mm*.

All *ikat* or decorative dye planar patterns in our sample exhibit a vertical reflection which may be a consequence of the culture that produces the patterns, because it is possible to make patterns that have no vertical reflection. However, it is remarkable that the most common symmetry group for the *ikat* is *c*2*mm* (*e.g.* Fig. 2*b*), whereas none of the decorative weave samples and only a few supplementary thread samples produced this symmetry group. Instead, the most common symmetry group for these two weaving techniques is *p*2*mm*. Both *c*2*mm* and *p*2*mm* groups have 180° rotation and reflections. However, while all centers of rotation in a *p*2*mm* pattern are on reflection axes, the same cannot be said of *c*2*mm* patterns (Fig. 22).

It is conceivable that the frequency of a *c*2*mm* symmetry group type in *ikat*, though not in the other types of weaves, is a consequence of the weaving technique. It is customary for the *ikat* patterns in our sample to have motifs that are arranged diagonally following a rhombic lattice. This is possible through the manner the threads are knotted before the dye is applied (Fig. 3*b*). However, this is not easy to achieve in decorative weaves, which demonstrate mostly patterns with rectangular lattices, such as those with *p*2*mm* as the symmetry group. (Figs. 6*a*, 8*a* and 8*b*).

The textiles in our sample that show a 90° rotation, for instance, are the tapestry weaves (see *pisyabit* in Fig. 20). As a matter of fact, it was only among the Tausug fabrics where the plane pattern symmetry group *p*4*mm* was found (Fig. 20*d*). In these examples, both warp and weft threads are of the same width, so that when they are woven perpendicular with respect to one another, a 90° rotation is certainly possible. The relative rarity of 90° rotations and lack of plane groups such as *p*4 and *p*4*gm* may be a result of cultural preference, as weavers may intentionally employ a float weave (Figs. 6 and 8) or use a more prominent thread (Fig. 14), meaning a 90° rotation is unlikely.

5.4.2. Frieze patterns. Out of the 588 patterns and designs studied, 166 were frieze patterns (43 *ikat* and 123 supplementary thread).

Fig. 23 shows that *p*2*mm* is the most common frieze group found among the supplementary thread samples and, to some extent, is quite common in the *ikat* samples. This shows that most frieze patterns have both horizontal and vertical reflection symmetry. The frieze group *p*1*m*1 is more common among the *ikat* samples than the supplementary thread samples. That is, in many *ikat* samples with this frieze group, there is a vertical reflection (which is also found among *ikat* plane

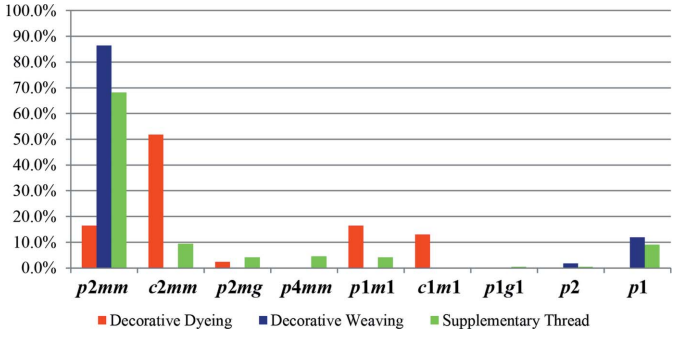


Figure 21
Percentage of each weaving technique exhibiting a particular symmetry group of the repeated pattern.

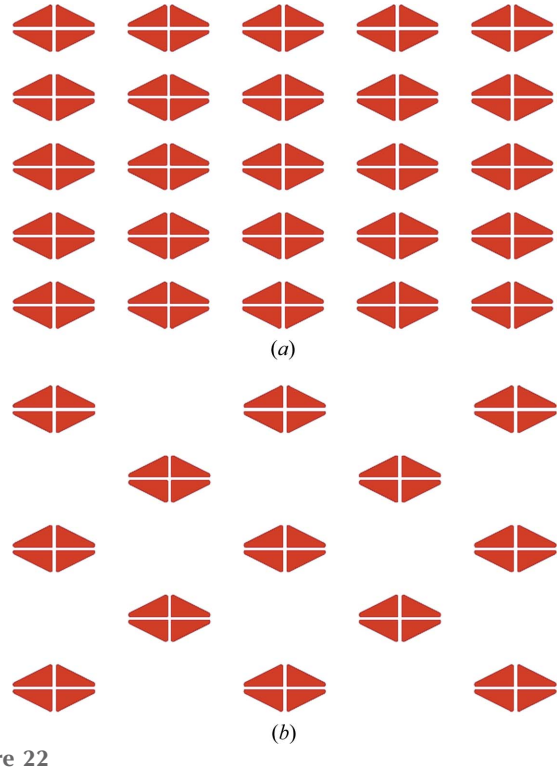


Figure 22
Plane patterns with symmetry group (*a*) *p*2*mm* and (*b*) *c*2*mm*.

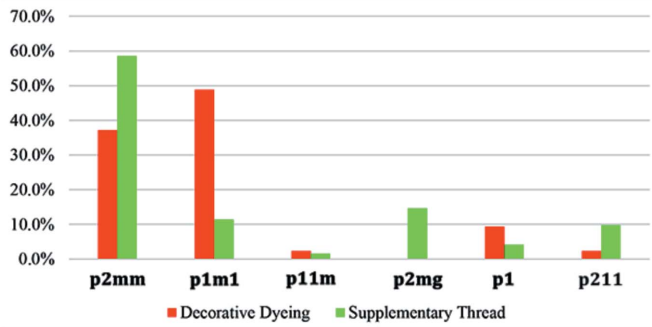


Figure 23
Percentage of each weaving technique exhibiting a frieze pattern.

patterns) but there is no glide reflection or horizontal reflection, or 180° rotational symmetry. The presence of only a vertical reflection is by no means an indication of a limitation in the weaving technique but is possibly a product of the designs employed, which are used to represent the weavers' dreams and connection with *Fu Dalu*, the spirit of the abaca (Paterno *et al.*, 2001).

Frieze patterns with the symmetry group *p2mg* occur in the supplementary thread samples, mostly among the Tausug textile. It is possible for Tausugs (or Yakans) to create friezes with glides using their weaving technique, such as those with the symmetry group *p*11*g*, which is notably not found in any of our samples. Their choice of traditional motifs that are meaningful to their culture leads to friezes with symmetry groups *p2mg* or *p2mm*.

5.4.3. Finite designs. Among the 588 patterns and designs studied, only 33 were finite designs (10 *ikat* and 23 supplementary thread). None of the *ikat* textiles showed a 90° rotation. Weaving a motif with a 90° rotational symmetry will involve a significantly different kind of knotting process than what is done within the weavers' culture. By contrast, 11 out of the 23 (47.8%) supplementary thread fabrics showed a 90° rotational symmetry. These are mostly from the Tausug tapestry weaves such as the *pisyabit* (Fig. 20), whose basic motif elements are squares (De Las Peñas *et al.*, 2014), and this symmetry is also a result of the weaver's choice of threads of the same width.

6. Color symmetry structures

In the symmetry analysis of figures in cultural artifacts, another point of consideration is studying the color symmetries of the designs and patterns. Studies pertaining to color repetition in cultural contexts have been carried out in the literature on San Ildefonso pottery designs (Crowe & Washburn, 1985), sandals of the Basketmaker and Pueblo peoples (Campbell, 1989; Teague & Washburn, 2013), Moorish ornaments (Grünbaum *et al.*, 1986), and Peruvian textiles (Washburn, 1986). Other work on color symmetry of cultural objects has been reported by Makovicky (1986, 2011, 2015, 2016).

6.1. Mathematical considerations

Following Teague & Washburn (2013), we distinguish three levels of color used in woven textiles: one-color designs/patterns, colored designs/patterns and designs/patterns with color symmetry.

'One-color' or 'monochromatic' designs/patterns occur where all motifs are executed in one color against a background of another color (Campbell, 1989), provided that the shape of the background is not the same as the shape of the motif (Teague & Washburn, 2013). In these patterns, the color of the background is not considered. One example is the pattern found in the sides of the central panel shown in the funerary textile of the Bontocs (Fig. 11*c*). The black motifs are set against a white background of different shapes. In this case, the relevant motifs for analysis will consist of the black motifs only.

Both colored designs/patterns and designs/patterns with color symmetry use more than one color. However, in a 'colored' design/pattern, no color change across motifs of the same type occurs. In this case, symmetries of the underlying design/pattern always map a motif of one color to a motif of the same color. In other words, any symmetry of the underlying design/pattern fixes each color. By contrast, designs/patterns with 'color symmetry' occur when a motif of the same shape is assigned more than one color. An example is the pattern of the Yakan fabric in Fig. 14, where triangles are assigned five colors: orange, white, yellow, dark blue and light blue.

A design/pattern with color symmetry can further be identified as a design/pattern with perfect or non-perfect color symmetry. In a design/pattern with 'perfect color symmetry', each symmetry of the uncolored design/pattern permutes the given colors. If a particular symmetry permutes the colors, it is referred to as a 'color symmetry' and it maps all parts of the design/pattern having the same color onto parts of a single color. Colored designs/patterns are generally perfectly colored because a permutation of colors is given by the identity permutation (mapping a color to itself).

Corresponding to every design/pattern with color symmetry is its associated color group. The color group consists of all the symmetries of the uncolored design/pattern that permute the given colors, that is, it is the group containing all the color symmetries. For a design/pattern with perfect color symmetry, its symmetry group is also the color group.

6.2. Color symmetries in Philippine textiles

Colors are perceived and executed in textiles differently, depending on the culture, as we have seen in the colorful designs and patterns of particular Philippine textiles. The T'bolis, for instance, use the colors black, red and natural beige, manifesting their dreams, myths and beliefs in the colored patterns of the *t'nalak*. The Iloko and Tinguians use two shades of a color to create optical repeated patterns meant to ward off bad spirits. The Yakans and Tausugs use colorful motifs to create repeated patterns in celebration of bravery in battle, joy in birth and marriage rituals.

Table 1
Summary of the color groups of textiles with non-perfectly colored designs and patterns.

The figure number of an example included in the count is indicated in parentheses.

Symmetry group	Color group						
Plane group	*c*2*mm*	*p*2	*p*1*m*1	*p*2*mm*	*p*2*mg*	*C*1	Total
*c*2*mm*				1		1	2
*p*2*mm*		2 (Figs. 14 and 15)	2	3 (Fig. 13)		2	9
*p*2*mg*					3 (Fig. 19*b*)		3
*p*4*mm*		2 (Fig. 20*d*)				1	3
							Total: 17

Symmetry group	Color group				
Frieze group	*p*2*mm*	*p*1*m*1	*p*211	C_1	Total
*p*2*mm*	1	4 (Fig. 17*a*, bottom/top frieze pattern)	1	1	7
*p*2*mg*		1			1
*p*1*m*1				1	1
					Total: 9

Symmetry group	Color group					
Finite group	D_2	D_1	C_4	C_2	C_1	Total
D_4	4 (Fig. 19*d*)		1	1	6 (Fig. 17*a*, center square)	12
D_2		1			2	3
						Total: 15

Many of the textiles we have analyzed that employ the decorative dyeing and weaving techniques are colored designs/patterns. A characteristic feature of all *ikat* repeated patterns that we have analyzed is that the motifs of the same kind have the same color, which may be attributed to the dyeing technique. In the *binakul* weave, though the warp and weft each have two hues, the overall effect is a repeated pattern consisting of squares and rectangles, where squares or rectangles of the same size appear to be of the same color. In general, decorative face-to-face repeated patterns such as the one that appears in Fig. 8(*a*) are colored patterns, where a motif is repeated throughout the fabric, without a change in color.

For textiles woven using the supplementary weave technique, there are repeated patterns with color symmetry. In Fig. 9, the patterns have perfect color symmetry since all the symmetries of the underlying symmetry structure effect a permutation of the colors, and are color symmetries. In Fig. 9(*a*)–9(*c*), there are translational symmetries that fix and other translational symmetries that interchange the colors. In Fig. 9(*b*), horizontal reflection symmetries with axes passing through the center of the motifs fix the colors, and those with axes between the motifs interchange the colors. In Fig. 9(*c*), reflection symmetries with vertical and horizontal axes passing through the center of the motifs fix the colors, and the reflection symmetries with horizontal axes passing between motifs interchange yellow and white. The Tinguian blanket in Fig. 10(*b*) displays a repeated pattern with perfect four-color symmetry. The triangle motifs are assigned four colors: red, yellow, white and light green. The reflection symmetries with horizontal axes passing through the vertices of the isosceles triangles whose bases are vertical fix the four colors. On the other hand, the 180° rotational symmetries centered at the points at which pairs of red isosceles triangles intersect fix red and green, and interchange yellow and white. Another example of a pattern with perfect color symmetry is the Yakan textile shown in Fig. 16.

The Yakan and Tausug weavers use colorful repeated patterns in their textiles, with a preference for multi-colored motifs. There are instances where certain colors are used more than others, or used by the weaver in two or more motifs. The use of colored threads in this manner gives rise to patterns with non-perfect color symmetry. For example, consider the planar pattern with seven colors shown in Fig. 14, with the symmetry group *p*2*mm*. The triangle motifs are woven with five colors (dark blue, light blue, yellow, white and orange) and the diamond motifs with two colors (red and green). Note the use of dark blue in every row of triangles. A reflection with the vertical axis passing through the point where red and green diamond motifs meet maps the dark blue to both white and orange, and does not effect a permutation of the colors. The 180° rotations with centers colored pink (shown on the lattice unit of the color group in Fig. 14) effect a permutation of the colors. Here the color group is *p*2. Another example of a planar pattern with non-perfect color symmetry is the *bunga sama* pattern shown in Fig. 15, whose symmetry group is *p*2*mm*, consisting of five colors. The color yellow appears in the centers of the diamond and hexagonal shaped motifs. The reflection with the vertical axis passing through the center of a diamond motif does not effect a permutation of the colors. It fixes the yellow color of the diamond motif, but interchanges the yellow and orange regions of two hexagons. The color group of this colored pattern is *p*2. The 180° rotations with centers colored pink, as shown in Fig. 15, effect a permutation of the colors. The frieze from the Yakan textile in Fig. 13(*b*) also displays non-perfect color symmetry. The reflection with axes marked in white maps green rectangles to both green and orange rectangles. The color group of this frieze is *p*2*mm*, an index seven subgroup in the symmetry group of the frieze; the frieze group is also *p*2*mm*. The centers and axes of symmetry in the lattice unit of the color group are colored pink and blue.

The Yakan *saputangan* in Fig. 17, the *kandit* in Fig. 19 and the Tausug *pisyabit* in Fig. 20 display designs and patterns with non-perfect color symmetry. The finite design (center square) in Fig. 17 has a horizontal reflection symmetry that maps yellow colored motifs to either a yellow or a white colored motif. For this design, the symmetry group is D_4 while the color group is C_1. The planar pattern on one side of the *kandit*

in Fig. 19(*b*) has a 180° rotational symmetry, centered at a yellow parallelogram, that maps yellow parallelograms to either yellow or red parallelograms. The color group for this planar pattern is the plane group *p*2*mg* which is of index two in its symmetry group, also of type *p*2*mg*. The centers of rotation and symmetry axes included in the lattice unit of the color group are colored pink (centers), black (axes of glide reflections) and blue (axes of mirror reflections). In the rectangular region of the *pisyabit* in Fig. 20(*d*), a reflection symmetry with axes passing through the yellow diamond motifs fixes the yellow color of the diamond motifs but maps the yellow inner region of a cross to a pink region. The color group of this colored pattern is *p*2 [centers colored pink are superimposed on the lattice unit of the symmetry group *p*4*mm* in Fig. 20(*d*)].

Table 1 presents the color groups corresponding to the non-perfectly colored designs and patterns arising from the supplementary thread technique that are included within the sample considered in this study. Note that there are patterns and designs in which the color group and the symmetry group of the underlying uncolored pattern are not of the same plane, frieze or finite group type. For example, more than 50% of the friezes with symmetry group *p*2*mm* have the color group *p*1*m*1. As previously noted, the use of the supplementary thread technique allows the weaver to create more varied repeated patterns which are reflected by the range of plane and frieze patterns. The color analysis also suggests a link between this weaving technique and the mixture of patterns with color symmetry. Unlike the decorative dyeing and decorative weaving techniques, the supplementary thread technique allows the weaver to insert (and remove) new colors in the middle of the textile, thereby achieving a wide range of perfectly and non-perfectly colored designs.

7. Conclusion

By no means is this paper exhaustive, showing only a fragment of the designs and repeated patterns found in Philippine culture. Nevertheless, our sample indicates that the types of symmetries and color symmetries produced are related to the weaving techniques and the cultures that produced them.

7.1. Symmetry groups

Our symmetry analysis in this work covers textile patterns and designs that are dictated by the warp and weft strands which are perpendicular, so it is not possible to have threefold or sixfold rotational symmetries. Thus, the groups *p*3, *p*31*m*, *p*3*m*1, *p*6 and *p*6*m* are absent in Fig. 21.

The symmetry groups of the patterns arising from the decorative dyeing technique involve reflection symmetries, namely *c*2*mm*, *c*1*m*1, *p*2*mm*, *p*2*mg* and *p*1*m*1. A large part of the sample have *c*2*mm* and *c*1*m*1 symmetry groups. The *ikat* motifs forming a rhombic lattice may be attributed to the manner in which the knots are tied before dyeing.

On the other hand, the decorative weaving techniques have motifs that form a rectangular lattice; the plane groups mostly present are *p*2*mm*, *p*2 and *p*1. The plane group *p*2*gg* is also possible using this weaving technique, by controlling the manner in which the wefts interlock with the warps at particular intervals through the pedal loom to form the planar pattern. However, these patterns are difficult to achieve in this manner and are uncommon.

The supplementary technique gives rise to a wider range of symmetry groups. These include *p*2*mm*, *c*2*mm*, *p*2*mg*, *p*1*m*1, *c*1*m*1, *p*1*g*1, *p*4*mm* and *p*1. The insertion of weft threads gives more flexibility in terms of the symmetrical patterns that arise. The choice of threads of the same width by Tausug and Yakan weavers gives rise to patterns or designs with 90° rotational symmetry.

For the frieze patterns in the sample, the symmetry groups are *p*2*mm*, *p*1*m*1, *p*11*m*, *p*2*mg*, *p*1 and *p*211. Just like for the plane patterns, most of the friezes arising from the supplementary weaving technique have symmetry group *p*2*mm*. Similar to the case of the plane patterns, the friezes with symmetry group *p*1*m*1 mostly arise from the decorative dyeing technique.

We have not seen plane groups *p*4, *p*4*gm* or *p*2*gg* or the frieze group *p*11*g* among the patterns or designs from the Tausugs or the Yakans, but it is not entirely impossible to create such patterns with these symmetry groups. Cultural preference for traditional patterns or prominent weft threads in the supplementary weave give rise to other plane groups.

7.2. Color groups

While there are colored patterns and designs arising from the decorative dyeing and decorative weaving techniques, patterns and designs with color symmetry are obtained using the supplementary technique, made possible through the insertion of colored threads. The use of multicolored threads (usually five colors and more) by Tausug or Yakan weavers to create colorful motifs as dictated by their culture gives rise to both perfectly and non-perfectly colored patterns and designs. The latter result in varying color group structures, summarized in Table 1.

7.3. Philippine National Commission for Culture and the Arts; mathematics education

The richness and variety of designs and patterns in the textiles of Philippine indigenous communities has been evident throughout this work. It is hoped that this study highlights the technical skill and ingenuity of the weavers and eventually contributes in some way to the preservation of their craft. The Philippine National Commission for Culture and the Arts (NCCA) has been working towards the preservation of weaving as a cultural tradition, and has formed Schools of Living Tradition (SLTs) in weaving communities across the country. In a weaving SLT, a master weaver teaches young weavers the techniques required to weave traditional textile designs.

Finally, our analysis of mathematical structures embedded in Philippine textiles has also formed the basis for the creation of lesson sequences in K-10 mathematics (De Las Peñas *et al.*, 2015) which we have put together with the support of the

NCCA. Specifically, exploratory activities and problem solving strategies rooted in Philippine culture were written for student use, and can be used as examples by teachers in constructing similar activities.

Acknowledgements

The authors would like to thank the following individuals who have generously shared their expertise and allowed the use of their material and images for the purposes of this research: Mr Floy Quintos, Professor Norma Respicio, Professor Donald Rubinstein, Dr Analyn Salvador-Amores, Dr Marian Pastor-Roces, Ms Rambie Lim (Philippine Textile Council), Ms Bernardette David (NCCA) and Ms Charito Cariaga (Nagbacalan Loomers Multi-purpose Cooperative). The authors are grateful to the weavers for openly sharing their time and resources during numerous sessions with them. Special thanks go to the National Commission for Culture and the Arts (NCCA) and the Ateneo de Manila University Loyola Schools.

Funding information

The following funding is acknowledged: National Commission for Culture and the Arts (NCCA); Ateneo de Manila University Loyola Schools Scholarly Work Grant.

References

Campbell, P. J. (1989). *Comput. Math. Appl.* **17**, 731–749.
Crowe, D. W. & Washburn, D. K. (1985). *J. Algebra Groups Geometr.* **3**, 263–277.
De Las Peñas, M. L. A. N., Garciano, A. D. & Verzosa, D. M. B. (2014). *Woven Universes*, pp. 5–11. Makati City: Yuchengco Museum.
De Las Peñas, M. L. A. N., Garciano, A. D., Verzosa, D. M. B. & Cheng, H. M. C. (2015). *Mathematics in Indigenous Philippine Artwork*, https://mathinphilippineart.wordpress.com/k-to-12-activities/.
De Las Peñas, M. L. A. N. & Salvador-Amores, A. V. (2016). *Philipp. J. Sci.* **145**, 89–103.
Grünbaum, B., Grünbaum, Z. & Shepard, G. C. (1986). *Comput. Math. Appl.* **12**, 641–653.
Grünbaum, B. & Shephard, G. C. (1988). *Am. Math. Mon.* **95**, 5–30.
Guatlo, R. E. (2013). Editor. *A Journey Through Philippine Handwoven Textiles.* Manila: HABI, The Philippine Textile Council.
Gustilo, M. (2014). *The Art and Order of Nature in Indigenous Philippine Textiles.* Makati City: Ayala Foundation.
Hahn, T. (2005). Editor. *International Tables for Crystallography*, Vol. A, *Space-Group Symmetry*, 5th ed. Heidelberg: Springer.
Kopský, V. & Litvin, D. B. (2002). Editors. *International Tables for Crystallography*, Vol. E, *Subperiodic Groups*, 1st ed. Dordrecht: Kluwer Academic Publishers.
Makovicky, E. (1986). *Comput. Math. Appl.* **12**, 949–980.
Makovicky, E. (2011). *Crystallographer's Alhambra: Beauty of Symmetry? Symmetry of Beauty.* Universities of Copenhagen and Granada.
Makovicky, E. (2015). *R. Fis. Acc. Lincei*, **26**, 235–250.
Makovicky, E. (2016). *Symmetry: Through the Eyes of Old Masters.* Berlin: Walter de Gruyter.
Pastor-Roces, M. (1991). *Sinaunang Habi: Philippine Ancestral Weave.* Manila: Communication Technologies.
Paterno, M., Castro, S., Javellana, R. & Alvina, C. (2001). *Dreamweavers.* Makati City: Bookmark.
Respicio, N. A. (2003). *Our Pattern of Islands: Philippine Textile Exhibition.* Melbourne: Consulate General of the Philippines.
Respicio, N. A. (2014). *Journey of a Thousand Shuttles, the Philippine Weave.* Manila: National Commission for Culture and the Arts.
Rubinstein, D. (1989). *Fabric Treasures of the Philippines.* Mangilao, Guam: Isla Center for the Arts at the University of Guam.
Teague, L. S. & Washburn, D. K. (2013). *Sandals of the Basketmaker and Pueblo Peoples: Fabric Structure and Color Symmetry.* University of New Mexico Press.
Washburn, D. K. (1986). *Comput. Math. Appl.* **12**, 767–781.
Washburn, D. K. & Crowe, D. W. (1988). *Symmetries of Culture: Theory and Practice of Plane Pattern Analysis.* University of Washington Press.

Deployment of Mathematical Resources to a Philippine High School through a Community LTE Network

Ma. Louise Antonette N. De Las Peñas
mdelaspenas@ateneo.edu
Department of Mathematics
Ateneo de Manila University, Philippines

Maria Alva Q. Aberin
maberin@ateneo.edu
Department of Mathematics
Ateneo de Manila University, Philippines

Agnes D. Garciano
agarciano@ateneo.edu
Department of Mathematics
Ateneo de Manila University, Philippines

Juan Carlo F. Mallari
jmallari@ateneo.edu
Department of Mathematics
Ateneo de Manila University, Philippines

Jumela F. Sarmiento
jsarmiento@ateneo.edu
Department of Mathematics
Ateneo de Manila University, Philippines

Mark Anthony C. Tolentino
mtolentino@ateneo.edu
Department of Mathematics
Ateneo de Manila University, Philippines

Debbie Marie B. Verzosa
dmbverzosa@usm.edu.ph
Department of Mathematics and Statistics
University of Southern Mindanao, Philippines

Abstract: *In the Philippines, one challenge that continues to be faced by the Department of Education in bringing educational content in a blended learning modality is the lack of internet access of the learners. This paper discusses the distribution, through a community LTE network, of mathematical resources for Grades 7 to 10 to teachers and students of a particular high school in the Philippines. It also gives details on particular technological tools (mathematical applications) that were created to help the mathematical learning of students in a remote setting.*

1. Introduction

In the past two years of the Covid 19 pandemic, schools worldwide have adapted various modalities in teaching and learning. For the school years 2020-2021 and 2021-2022 in the Philippines, the Department of Education (DepEd) has called for a blended learning modality for schools in lieu of face-to-face instruction [1]. The problem lies in providing access to quality education to children who belong to households with limited or no access to the internet. In fact, based on the Philippine National ICT survey [2], only 17.7% of households have access to the internet. As such, the DepEd's blended learning approach focused largely on non-internet sources such as printed modules, radio, and television. Clearly, there is a need to explore alternative avenues for the delivery of information. Under a government-funded project *Mathematical Resources for Distance Learning Using Community LTE Networks* that started in April 2021, the authors collaborated with local DepEd school divisions to deliver mathematical content that included mathematical applications (apps), teaching guides/instructional videos, and performance tasks or activity sheets through community LTE networks.

Based on the literature, community LTE networks are small-scale local networks that are usually set up to address the connectivity needs in communities that are not reached by traditional telecommunication companies [3]. Community LTE networks have been successfully deployed in Bokondini, a mountainous region located in the Papuan highlands in Indonesia [3], and Oaxaca, an indigenous state in the southeastern part of Mexico [4]. In the Philippines, the possible use of community LTE networks has been considered and studied in [5]. More recently, community LTE network base stations have already been deployed in Philippine municipalities in Cavite, Rizal and Zambales [6].

There has always been a need to provide support for the mathematics education of Filipino learners. Even before the COVID-19 pandemic, the challenges in achieving the twin goals of mathematics education [7] are well-documented [8, 9]. To help ensure that distance learning takes place in the pandemic, the project carried out by the authors focused on designing mathematical apps, for dissemination, that showcased content on the following foundational concepts: place value, fraction number sense, and number sense strategies for Grades 1-6; integers, functions, geometry, trigonometry for Grades 7-10; and Statistics for Grades 1-11. The concepts are fully aligned with the official DepEd's Most Essential Learning Competencies (MELCs) [1] and with the existing literature on mathematical learning.

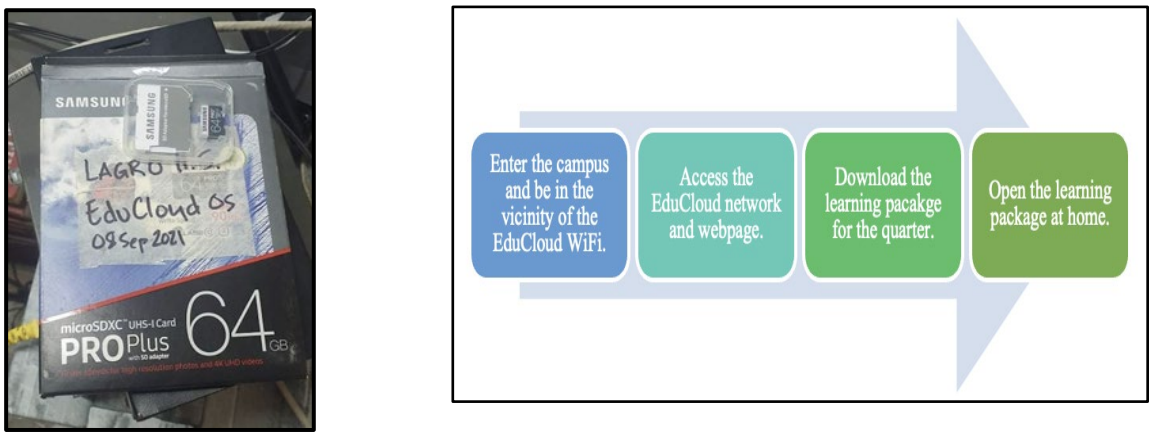
This paper documents the deployment of the mathematical resources of Grades 7 to 10 in school year 2021-2022 to a high school in Quezon City in the Philippines using a community LTE network.

2. A Local Network for Distribution of Educational Resources

During discussions by the authors with one school division in the country- DepEd Quezon City, it was agreed that for the deployment of the mathematical resources through a community LTE network, several factors will be considered in choosing a public school to set up the network. These factors were: i) there should be a substantial percentage of the student population that did not have ready access to the internet; and ii) faculty members and students would be ready for the adoption of the use of technological tools in teaching and learning mathematics, respectively. A choice was Lagro High School in Quezon City, within the country's capital region. In general, teachers in Lagro High School had experience in teaching with technology, such as google classroom (which was carried out in a small number of classes that had internet connectivity) and had exposure to some mathematical applications. Moreover, at the start of the pandemic, the Quezon City local government provided

every public-school student of Quezon City with tablets for use to help them access educational materials. Given the circumstances and the set-up of distance learning in Lagro High School for the school year 2021-22, it was deemed beneficial to set up a local community LTE network at the Lagro High School campus.

The mobile cloud network architecture used is called *EduCloud* that has been developed by the Ateneo Innovation Center and that allows for a resilient distribution of educational content for remote asynchronous learning [10]. Initially deployed at Kaong National High School in Silang, Cavite in the Philippines, EduCloud "offers a low-cost, low technology requirement, easily deployable and wide range of compatibility systems for the distribution of open educational resources" [10, p. 402] and "initial performance tests show that the network architecture is efficient in data transfer and accommodating synchronized usage." [10, p. 402]. Collaboration of the authors with the EduCloud team, DepEd Quezon City, and Lagro High School led to an agreement of an installation process of one to two months before the start of the school year, in September 2021 (Figure 2.1(a)). As parents/students would go to school to get hard copies of modules and other learning materials at the start of every quarter of the school year, it was agreed that the mathematical resources would be organized by academic quarter and uploaded into EduCloud in preparation for the school visits. The general process followed by parents/students in accessing resources via EduCloud is shown in Figure 2.1(b).



(a) (b)

Figure 2.1 (a) The EduCloud network setup in an office strategically located in Lagro High School for optimal use; and (b) the general process followed by parents/students when accessing EduCloud.

To guide the parents/students, signages were placed at appropriate locations (i.e., within the vicinity of the EduCloud WiFi) in the Lagro High School Campus. The signages contained instructions, in both English and Filipino, for accessing EduCloud, which involved two steps: (1) connecting to the EduCloud WiFi network using the provided password; and (2) opening the EduCloud webpage by typing in the provided address in any browser in the user's device. Once in the EduCloud webpage, a parent/student simply had to select the learning package corresponding to the correct grade level and quarter. The selected package would then be downloaded as a single compressed file, which reduced the time spent by the user compared to downloading files one at a time. In this way, personal interactions and possible COVID-19 transmissions between individuals who visit the campus could also be reduced.

Before the start of the school year, in August 2021, the authors conducted *Project A.L.A.M. (Ateneo-Lagro HS using Apps from Mathplus Resources)*, which was a capacity-building webinar for

the mathematics teachers on the use of the technological tools. The webinar had parallel sessions per grade level where the mathematical resources were introduced. In addition to demonstrating how to play the apps and games presented in each grade level package, the authors offered suggestions on how to incorporate them in teaching and assessment. During the webinar, the EduCloud team also gave a talk on the EduCloud set-up in the school and how to download the resources in the math package. Shortly after, a separate webinar was also given by the teachers and school administrators for parents/students. When classes started, authors were assigned to different grade levels which ensured constant communication with the teachers to address their questions and other needs. Follow-up training and meetings were held as necessary which were also occasions for getting feedback from the teachers on the use of the mathematical resources. For newly developed apps, comments were solicited from teachers and students for the improvement of app functionalities.

3. Mathematical Resources

The mathematical resources deployed through the EduCloud system were selected and developed following the competencies identified by the Philippines' Department of Education [1]. Teachers from Lagro High School also specified least mastered skills for their students in the previous school years, e.g., 2020-2021. These are learning outcomes that proved difficult to achieve and provided valuable insight as to what topics required mathematical resources that could promote mastery and student engagement especially in remote learning.

The mathematical apps consisted of Android apps as well as Geogebra applets. These were organized into packages (i.e., a single compressed file) based on grade level and quarter (Figure 3.1). There were two versions of the packages, one for teachers and another for parents/students. In addition to the mathematical resources, guides (with English and Filipino versions) for installing apps and for running Geogebra applets were also made available (Figure 3.2). Moreover, some common apps (e.g., a scientific calculator) were included in each package. It was important that teachers were guided as to how to approach the subject matter with the use of the apps, so teaching guides and instructional videos accompanied the mathematical apps for the teachers.

An interested reader is invited to visit https://mathplusresources.wordpress.com/, the official website of the Ateneo Mathplus Resources Laboratory, where the mathematical resources can be downloaded through the internet. Some apps contained in the learning packages are described in this section to provide the pedagogical bases behind their design, and to highlight the important features available within the apps.

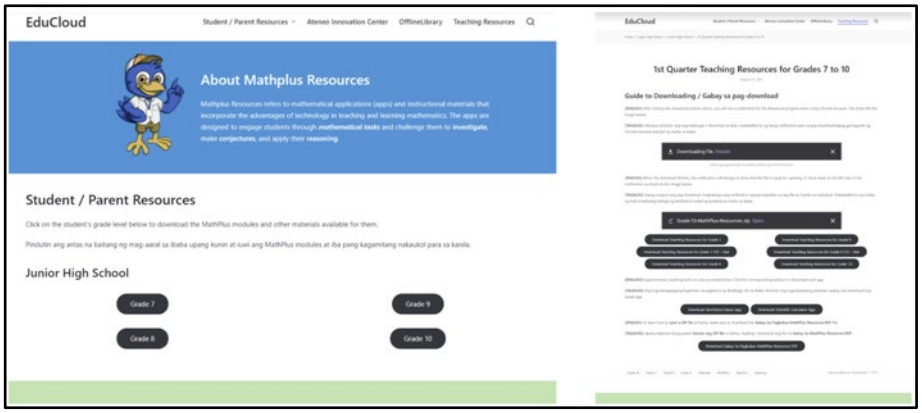


Figure 3.1 The EduCloud webpage showing organization of the mathematical resources

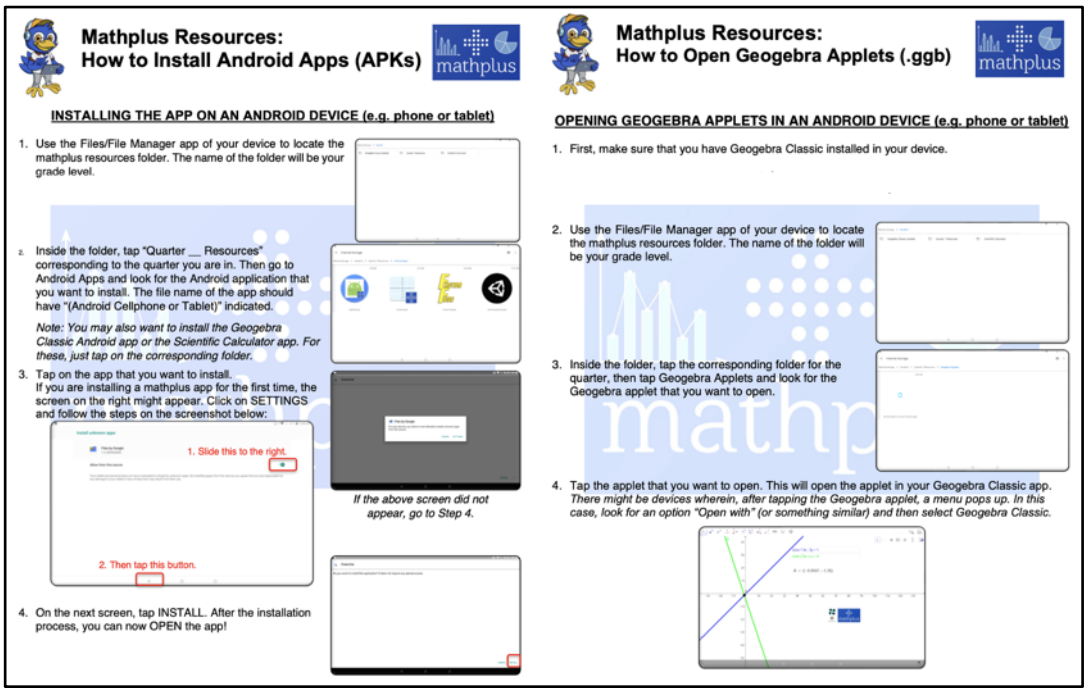


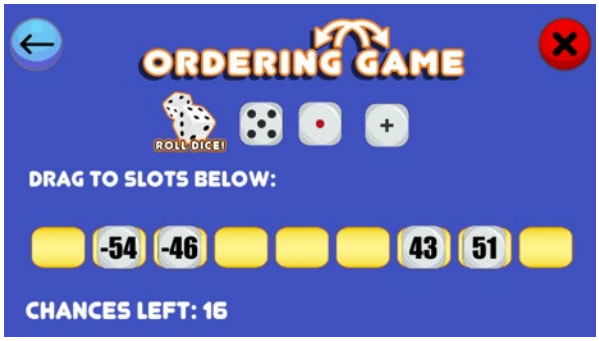
Figure 3.2 Sample instructional guides

One essential element was providing apps on number. A sense of number magnitude is a foundation for knowledge of the real number system [11]. The number line is a natural representation of each number as well as the sequential relationship between numbers [12]. Arranging real numbers on a number line is one of the basic learning competencies expected of high school students [1, 13]. To provide students with a variety of activities on the number line, two apps (*Catch the Carrot* and *Ordering Game*) were included in the package for Grade 7 (Figures 3.3(a)-(c)). In *Catch the Carrot*, a number is flashed on the screen and the player must estimate the location of this number on the number line (Figure 3.3(a)). There are options to work with whole numbers, fractions, decimals, integers, or irrationals. In *Ordering Game*, numbers appear one at a time on the screen. The player must drag each number to the line and the goal is to complete a row of ascending numbers (Figure 3.3(b)). Once a number is placed on the row, it cannot be moved anymore. A player may fail to complete the row if the next numbers would not fit in the vacant slots within the given number of moves shown at the bottom of the screen. In both games, the player must apply estimation skills and strategic thinking to complete the task within the time or number of chances given. Activities in the printed modules given to students by their teachers are supplemented by exercises using the two apps. For example, an activity in the Grade 7 printed module [13] asks students to estimate the location of the square root of a number on the number line. This has a parallel activity in the *Catch the Carrot*. Thus, students can keep on practicing using the game (Figure 3.3(a)).

The ability of high school students to factor polynomials is a "fundamental skill in solving problems in mathematics that scaffolds up leading to achievement in college level mathematics" [14, p. 116]. Many other math concepts such as simplifying or performing operations on rational expressions, and finding roots of polynomials or solving inequalities, rests on the ability of students to factor polynomials correctly. The use of technological tools to assist in developing the skill of factoring polynomials in an interesting and engaging way becomes even more critical in a remote-learning environment.



(a)



(b)

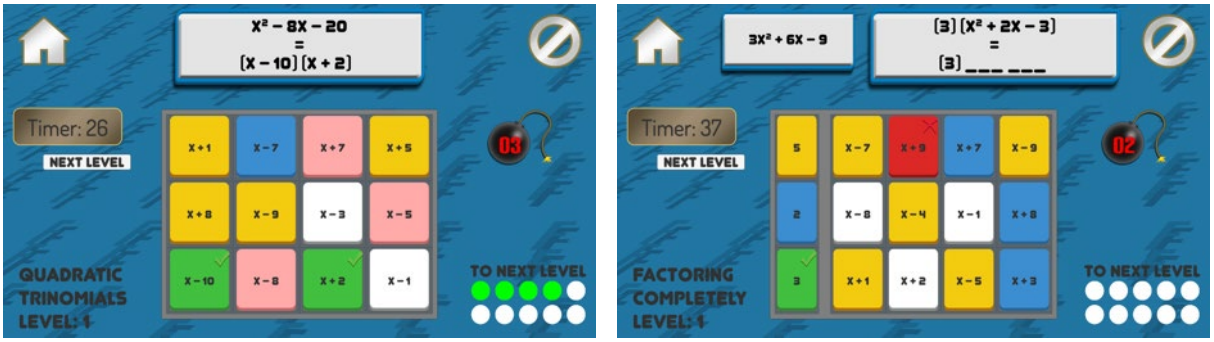


(c)

Figure 3.3 (a) Activity on number; screenshots of (b) *Catch the Carrot*, (c)-(d) *Ordering Game*

In view of the following learning competency "factors completely different types of polynomials (polynomials with common monomial factor, difference of two squares, sum and difference of two cubes, perfect square trinomials, and general trinomials)" [1, p. 230] for Grade 8 students, an app named *Factor Fuse* was developed. Students are given different types of polynomials to factor. The app contains tasks for factoring the highest common monomial factor, factoring a quadratic or cubic polynomial, or factoring a polynomial completely. Upon being shown a polynomial at the top part of the screen, a student chooses a factor of that polynomial. Sample screenshots of what appears when a student factors a quadratic trinomial or factors a polynomial completely are shown in Figures 3.4(a) and (b), respectively. A bell or buzzer sound rings depending on whether the student chose the correct factor or not. This feedback is important to a student who is still gaining the skill of factoring and needs some manner of prompting or direction on how to proceed. Upon hearing the bell sound, the student then chooses another factor from a table of polynomials. The student continues to do this until the polynomial is completely factored.

Factor Fuse is intended to be played by a student with minimal or no supervision by a teacher. Playing the app for practice even in a remote learning environment helps to strengthen a student's factoring skills.



(a) (b)

Figure 3.4 Screenshots on (a) factoring quadratic trinomial; and (b) factoring completely

In the needs assessment phase with teachers, it was determined that one of the challenging topics to teach was proving geometric theorems. It was very difficult for students to formulate parts of a two-column proof. Moreover, it would help to arrive at a technological tool that would allow the students to visualize the concepts behind every step of the proof and serve to assess learning remotely. For this reason, the authors designed apps for proving, such as *Two Column Proof-Parallelograms* (Figure 3.5(a)). This app addresses the following competencies for Grade 9: i) "understanding key concepts of parallelograms" [1, p. 236] and; ii) "determine the conditions that make a quadrilateral a parallelogram" [1, p. 236]. To navigate a proof, the student clicks the "Next" button (Figure 3.5(b)). A line of the proof appears, and the response is entered in the blank provided. To visualize a statement/phrase, the right arrow button is clicked; and an animation occurs. For example, for a proof that requires a reasoning involving the Side-Angle-Side (SAS) congruence, the side, included angle, side and the corresponding triangle are animated, as a guide to the students. The teacher may opt to discuss the proof of a theorem in synchronous class after the students have tried their hand on the proof, or perhaps use a theorem as an example, instead of giving it as an exercise.

In the *Two Column Proof* app, the student may save the answers to the proof. Once he/she has completed answering all the items, and saved the work, the answer key appears (Figure 3.5(c)-(d)). The student may opt to answer the activity again, even after he/she has viewed the answer key. Upon saving, there will now be another file saved. See Figure 3.5(e) where three files have been saved for one activity. The teacher can track (by asking the student to give a screenshot of this page of the app) if the student has practiced several times. When asking one proving question in an exam, the teacher can check from this screen if the student has more than one attempt in doing the exercise.

As the app on two column proof on parallelograms was quite helpful for the teachers and students, other similar apps were developed to address Grade 9 learning competencies on trapezoids and kites [1]. Figure 3.5(f) shows a sample proving activity on kites.

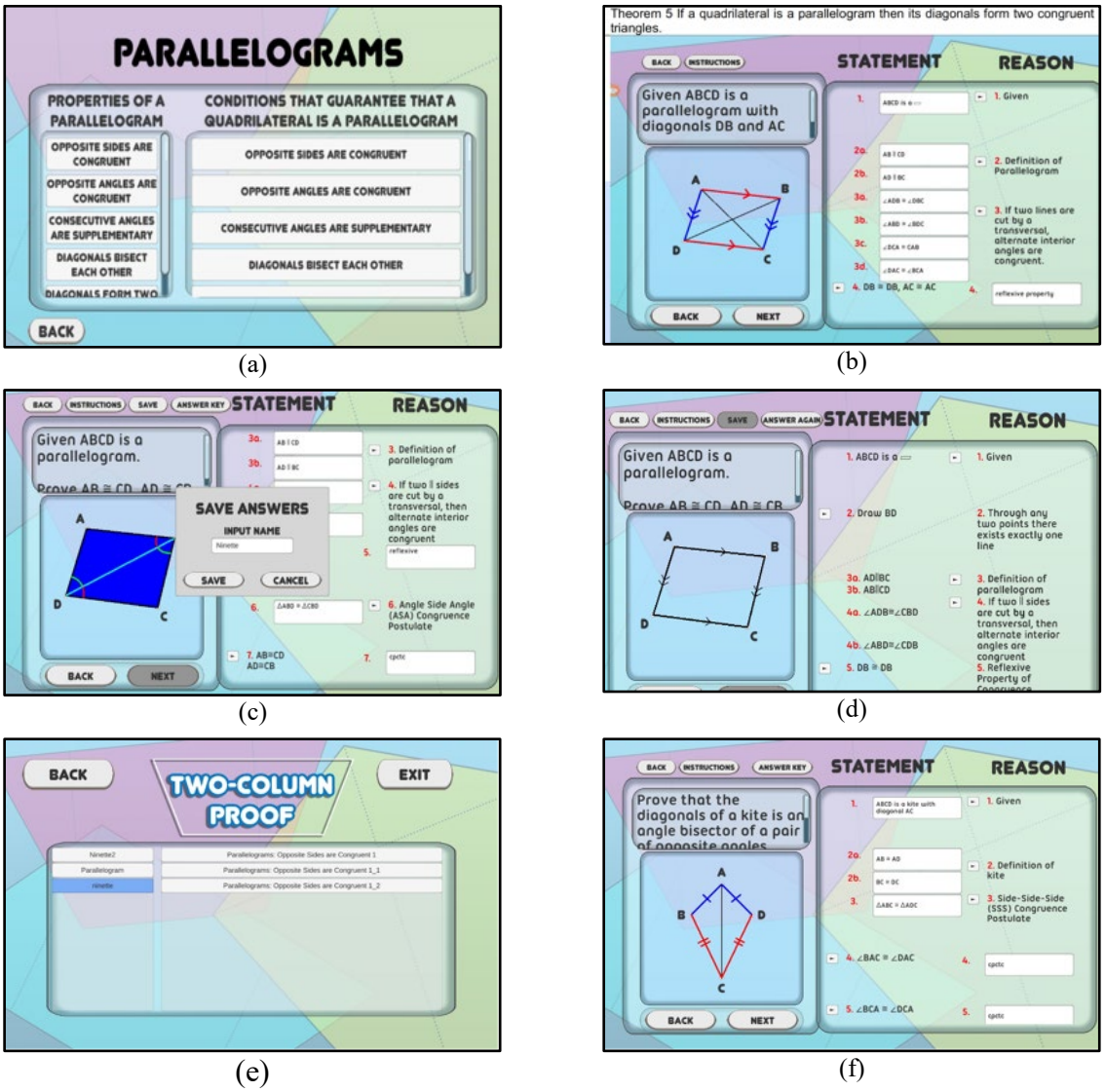
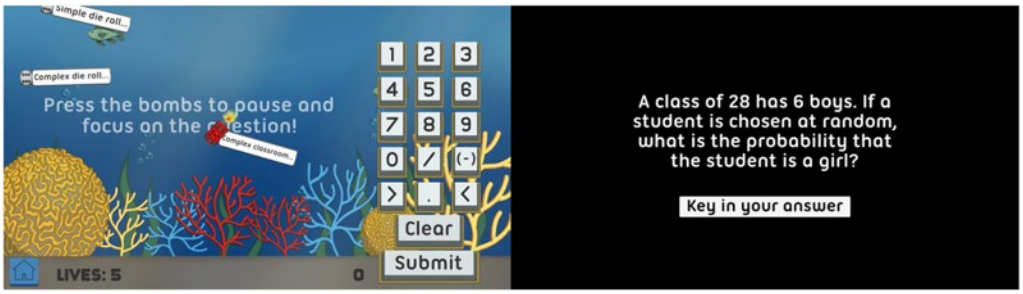


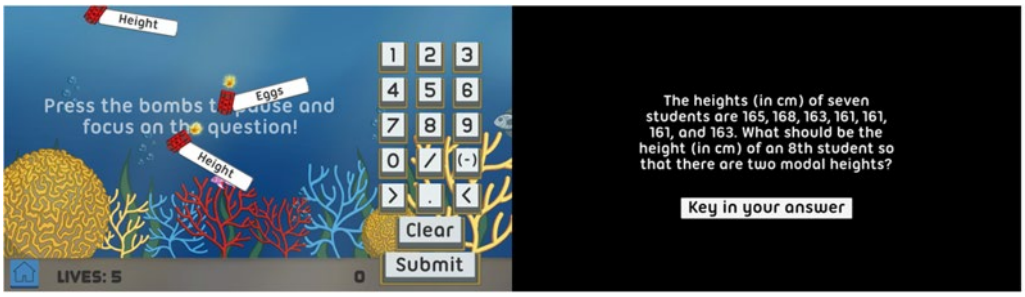
Figure 3.5 Screenshots of (a) menu on parallelograms; (b) sample proving exercise on a property of a parallelogram, namely, "the diagonals of parallelograms form two congruent triangles;" (c) when saving the proof; (d) sample answer key; (e) three different files showing proofs of the same theorem; and (f) sample proving exercise on kites

For Statistics, the least mastered skills included the computation of descriptive statistics, including measures of variability and central tendency, for grouped and ungrouped data. The authors decided to create two apps: *Just Keep Solving* for the mastery of concepts in probability and statistics related to the aforementioned topics, and *Statistics* for the computation of descriptive statistics for and the visualization of datasets. In *Just Keep Solving*, problems in probability and statistics are shown as banners falling towards the ocean floor. Tapping on a question banner pauses the movement of all banners on screen, allowing students time to carry out their computations. Banners will fall as students key in their answers, adding tension to the game and encouraging students to think quickly and recognize question patterns. The game interface and sample questions are shown in Figure 3.6. The app features two levels for probability and three levels for statistics, covering topics ranging from random variables and probability distributions to measures of central tendency and variability. The levels on probability are focused on the following learning competencies for Grades 8 and 11: i)

"finds the probability of a simple event," [1, p. 234] ii) solves problems involving probabilities of simple events," [1, p. 234] iii) "illustrates a probability distribution for a discrete random variable and its properties," [1, p. 540] iv) "computes probabilities corresponding to a given random variable," [1, p. 540] and v) "solves problems involving mean and variance of probability distributions" [1, p. 540]. Meanwhile, the statistics levels cover the following competencies for Grade 7: i) "calculates the measures of central tendency of ungrouped… data," [1, p. 230] ii) "uses appropriate statistical measures in analyzing and interpreting statistical data," [1, p. 230] and iii) "calculates the measures of variability of… ungrouped data" [1, p. 230].



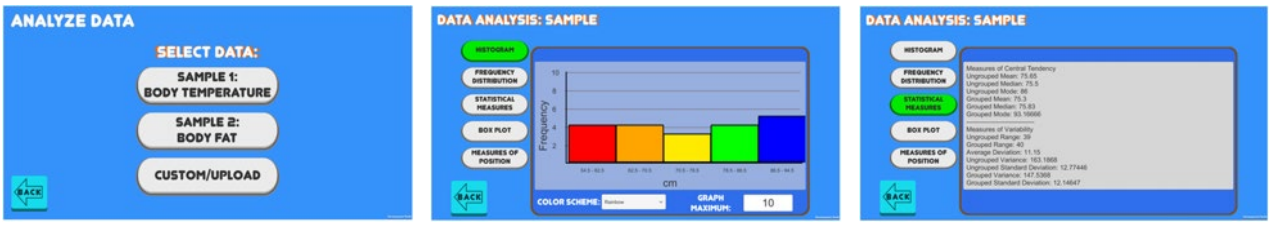
(a)



(b)

Figure 3.6 Screenshots of the *Just Keep Solving* interface and sample questions for (a) probability and (b) statistics

The *Statistics* app, meanwhile, provides tools for computing measures of central tendency and variability, as well as visualizing data. As shown in Figure 3.7, students can use either the app's built-in datasets or their own. In addition to computed statistical measures for grouped and ungrouped data, analysis includes a frequency distribution table, a histogram, and a box plot.



(a) (b) (c)

Figure 3.7 Screenshots from the *Statistics* app showing (a) the data selection screen; (b) a sample histogram; and (c) computed statistical measures

The dynamic geometry feature afforded by the Geogebra software provides opportunities for students to explore, discover, prove conjectures and apply mathematical concepts. Learner's visual and conceptual abilities in mathematics are improved through explorations via Geogebra applets. Moreover, the color, graphics, and interactivity of GeoGebra applets can catch and hold the attention of the students and motivate them to learn and make meaning of their learning [15]. Because of these affordances, Geogebra applets were identified and provided for teaching particular topics. The Trigonometry applets shown here are examples. The applets provide dynamic worksheets and opportunities for exploration which support development of procedural and conceptual knowledge of trigonometric ratios.

One applet (Figure 3.8) allows students to determine the sine, cosine, and tangent of acute angles in the context of a right triangle. Given the sides of a right triangle, students input the values of the desired trigonometric ratio. Immediate feedback is given whenever the correct answer is given. Pressing the NEW PROBLEM button creates another problem to solve. Thus, the applet provides numerous practice exercises which facilitate mastery of the subject matter. Given this nature, it replaces the list of exercises that is normally given to students. It can conveniently be used as a formative assessment in class.

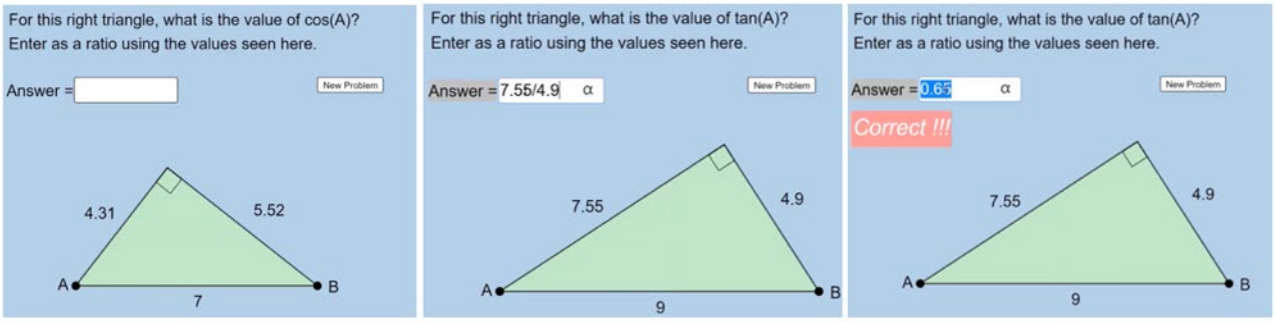


Figure 3.8 Screenshots pertaining to exercises on determining trigonometric ratios

The second applet (Figure 3.9) illustrates the conceptual meaning of the sine, cosine and tangent ratios when defined in the context of a right triangle. If the sine (or cosine) ratio button is pressed, an animation is activated showing the opposite side (or adjacent side) of the angle moving towards the hypotenuse. When it coincides with the hypotenuse, an explanation of trigonometric value is displayed at the upper portion of the screen. If the tangent ratio button is chosen, the animation shows the opposite side moving towards the adjacent leg. A similar explanation is presented at the end of the animation.

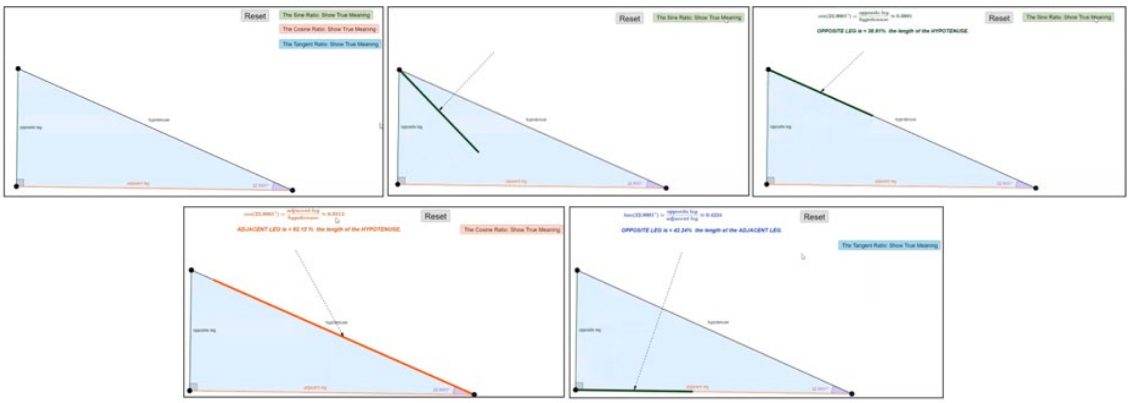


Figure 3.9 Screenshots: an animation of the conceptual meaning of the trigonometric ratios

Finally, the third applet (Figure 3.10) provides an exploratory activity for students to understand the definition and properties of the sine, cosine, and tangent ratios. It allows the student to create triangles of different sizes by dragging the vertices of the triangle. Moving point *A* (Figure 3.10(a)-(c)) adjusts the size of the triangle. In doing so, students can observe that the values of the trigonometric ratios do not change even if the lengths of the sides are different. This can lead them to understand that the values of the trigonometric ratios do not depend on the lengths of the sides if the angles of the triangle remain constant. However, moving point *C* (Figure 3.10(d)-(e)) transforms the triangle with different angles. The students can observe that the values of the trigonometric ratios are no longer constant. The affordance of dragging the vertex and creating dynamic transformations of the triangle provides opportunities for exploratory activities leading to visual discovery of the properties of the trigonometric ratios. It transforms figures which is not possible in the traditional paper-and-pencil approach [16, 17] (as cited in [15]). Such engagement with the applet can provide meaningful learning and deeper understanding.

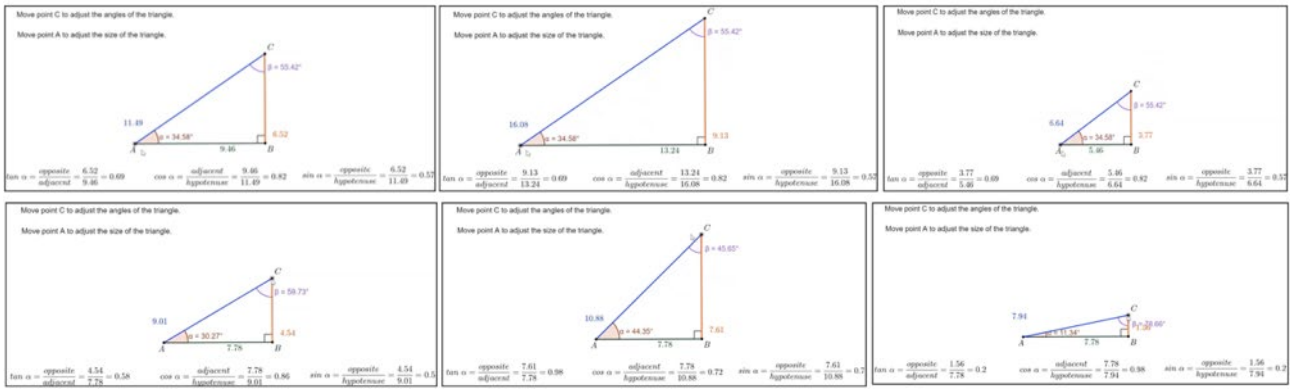


Figure 3.10 Screenshots: (a)-(c) Transformed triangles by dragging vertex *A* with constant values of the trigonometric ratios; (d)-(e)Transformed triangles by dragging vertex *C* resulting to different trigonometric ratios

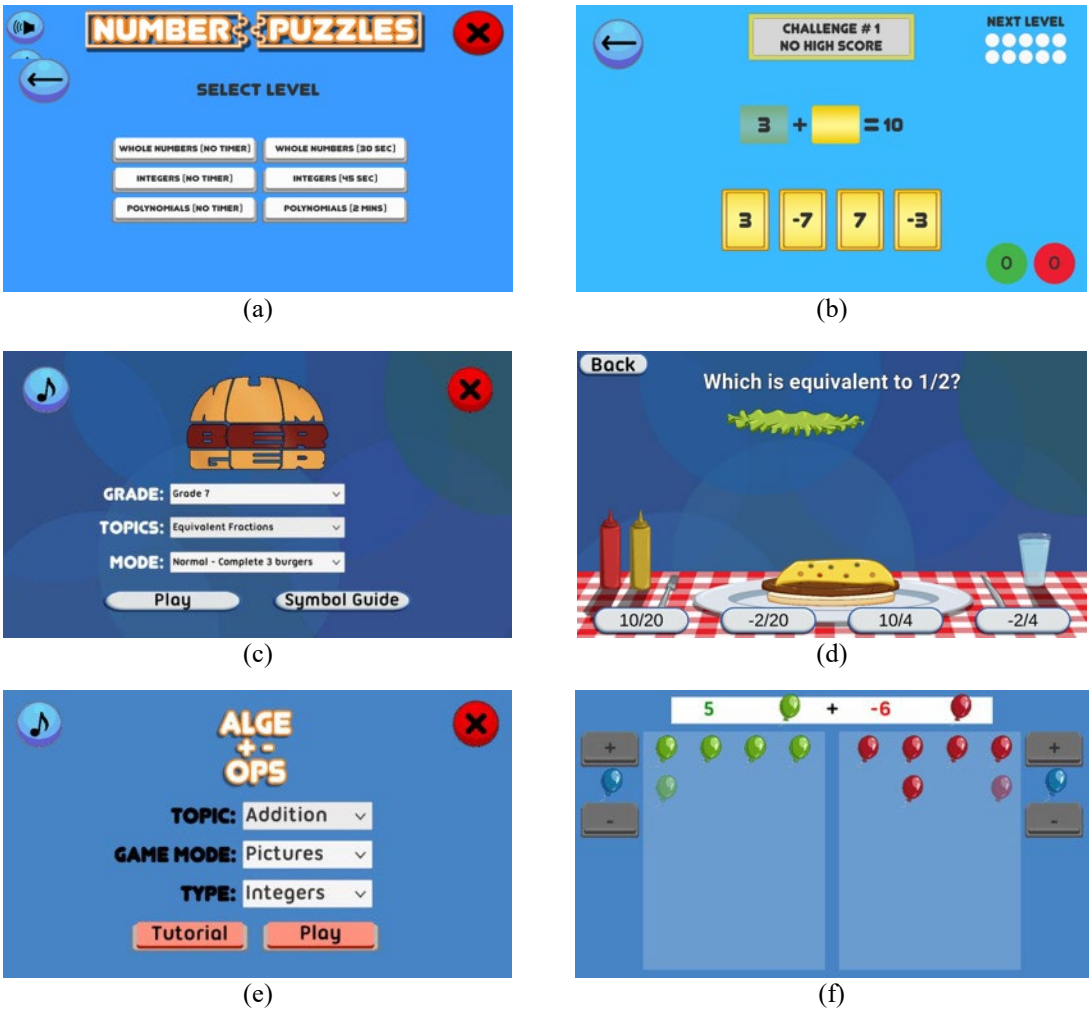
4. Remediation Program

To help students who have difficulty in mastering mathematical concepts and skills, a remediation program named Academic Remediation on Operations on Numbers (ARON) was implemented in Lagro High School. Specifically, this intervention program was designed for Grade 7 students who have yet to gain mastery of computational skills involving fractions, decimals, and integers. These students were selected based on the results of a diagnostic test administered at the beginning of the school year. Through the community LTE network, three mathematical apps were deployed to the group of students and teachers involved in ARON: *Number Puzzles*, *Numberger* and *AlgeOps*.

Number Puzzles (Figure 4.1(a)-(b)) contains a series of problems involving the four fundamental operations. These problems involve numerical expressions with some unknowns. The expressions may involve whole numbers, integers and even polynomials. *Numberger* (Figure 4.1(c)-(d)) includes an option involving equivalent fractions. Set in a game-like environment, students identify which fractions are equivalent to a given fraction. These apps were chosen by the authors in coordination with Grade 7 teachers handling the remediation program. The teachers asked for apps that will provide more opportunities for students to practice and master the foundational mathematics skills. *AlgeOps* (Figure 4.1(e)-(f)) provides students with visualization of integer operations using two

models: colored counters and number line. Positive and negative integers are represented by green and red counters, respectively. Students can relate the operations of addition and subtraction of integers by inputting an appropriate number of counters and seeing how the number of counters change because of the operation. The student then indicates the answer by moving a marker to an appropriate position on a number line.

A survey via Google Form that aimed to evaluate the apps in three areas was given to students and teachers: (a) ease of use; (b) gameful experience and (c) alignment with learning objectives. Results indicated a positive response from students and teachers. Majority of the students had no difficulty using the apps. Students found the app useful in understanding the lesson and solutions. The apps also helped them find the answers easily and quickly. They also noted that the apps provided a lot of opportunities for practice. Moreover, the teachers found the apps useful in helping the students stay motivated and engaged throughout the lesson. They also commented that the numerous exercises provided by the apps helped the students master the foundational skills in mathematics.



(a) (b)

(c) (d)

(e) (f)

Figure 4.1 Screenshots from (a)-(b) *Number Puzzles*; (c)-(d) *Numberger*; and (e)-(f) *AlgeOps*

5. Challenges

The use of the community LTE network EduCloud was intended to offset some of the challenges associated with distance learning in the Philippines, where internet access is not guaranteed [2]. However, the implementation of EduCloud is not free from challenges.

The main challenge pertained to the process of downloading the files from the community LTE network. The IT team of DepEd Quezon City had earlier installed safety measures on the tablets, preventing students from downloading content from unlisted sources. The students' tablets had to be re-configured to allow access to the EduCloud network. A second reason was that the files in the learning packages were unfamiliar to parents and students. Parents who were more familiar with technology had more success and were better able to find solutions while others needed more guidance. While parents could successfully download the files from the EduCloud framework in the school premises, they could not install the files on their own. Many parents also had to be informed about the nature of files–GeoGebra vs Android apps, for example. Further, since some of the apps are available in PC or Android versions, there was a need to inform parents to install the correct files in their device. Teachers had to create detailed instructions and infographics to guide parents and students, especially on downloading and installing the Geogebra apps.

A second challenge was due to physical access to EduCloud in the network site amidst protocols related to the pandemic during some of its peaks, such as in January 2022. During such times, parents could not access the materials and it was also difficult for IT school personnel to assist, given strict health safety protocols. Fortunately, when COVID-19 cases leveled off or decreased, safety measures were eased, and parents could travel to the network site and download the packages. Further, since the packages were organized according to academic quarters, then parents needed to access the materials only four times in a year.

A third challenge related to the apps themselves. Since this is an ongoing project, new apps are undergoing preliminary testing, and feedback from users (teachers, parents, and students) continue to be gathered. For example, feedback on the Proving apps in geometry showed that the apps provided helpful visualizations of the proving process, but additional markings in the diagrams could clarify some of the given statements in the proving tasks.

6. Conclusion and Future Direction

Achieving the goals of mathematics education is a perennial challenge [9], and this has been made even more difficult due to the sudden shift in learning modalities due to the pandemic. Inconsistent access to stable internet forced teachers and students to utilize non-Internet sources such as printed modules, radio, and television. However, mathematical technological tools cannot be deployed through such media. This project demonstrated that through a community LTE network, mathematical apps can be shared even without the internet. To maximize the learning potential of these apps, they were organized according to academic quarters to help the users select the important apps they needed to learn the expected competencies. Further, the apps came with teaching guides and clear instructions for downloading so that teachers and parents could rightfully guide the learners even in a remote setting.

Proper deployment of the material was achieved through a combination of research-based content and the commitment of DepEd school divisions, IT personnel, and teachers. The dedication of parents in making sure the material got to their children was also a big factor in success of deployment. These were needed to address challenges that would normally arise in such projects. The authors would

continue collaborating closely with schools to generate more feedback and gather empirical data on the extent to which the apps affect student learning.

There are more challenges to face in the coming school year. One is on further remediation, as DepEd has identified more students who had not met the learning competencies required of their grade level in the past two years of the pandemic. Moreover, even as more schools expect to offer onsite classes, there are students who would continue remote learning modality due to health issues. Discussions on how the mathematical apps as well as the community LTE network will be used to help address these challenges are ongoing with the authors and particular DepEd school divisions.

Acknowledgements This paper is one of the outputs of the Ateneo Mathplus Resources Laboratory housed at the Department of Mathematics, School of Science and Engineering, Ateneo de Manila University. The authors would like to thank the Department of Science and Technology-Philippine Council for Industry, Energy, and Emerging Technology Research and Development (DOST-PCIEERD) and the University Research Council (URC), Ateneo de Manila University for the support of the development of the interactive resources discussed in this paper. Further, we thank the Ateneo Innovation Center's EduCloud team headed by Dr. Nathaniel Libatique. We also thank our i) app developers Mr. Victor Antonio M. Ortega, Mr. Nigel Benedict Cargo, Mr. Jose Teodoro Lacson; and ii) art designer Mr. Amiel Damian F. Justiniani. We also thank Dr. Joel P. Feliciano of DepEd Quezon City and Ms. Nora Alcantara Villalobos and the teachers of Lagro High School for their cooperation and support during this project.

References

[1] Department of Education. (2020). K to 12 Most Essential Learning Competencies with Corresponding CG Codes. Retrieved 7/20/2022 from https://commons.deped.gov.ph/K-to-12-MELCS-with-CG-Codes.pdf

[2] Department of Information and Communications Technology. (2019). 2019 National ICT Household Survey Results. Retrieved 7/20/2022 from https://dict.gov.ph/ictstatistics/2019-national-ict-household-survey-results/

[3] Sevilla, S., Johnson, M., Kosakanchit, P., Liang, J. and Heimerl, K. (2019). Experiences: Design, Implementation, and Deployment of CoLTE, a Community LTE Solution. In *The 25th Annual International Conference on Mobile Computing and Networking (MobiCom '19)*. New York, USA: Association for Computing Machinery.

[4] Kosakanchit, P. (2020). Setting Up a Community LTE Network in Oaxaca, Mexico. Retrieved 7/20/2022 from https://blog.colte.network/2020/03/02/setting-up-a-community-lte-network-in-oaxaca-mexico/

[5] Hilario, C.A.G., Barela, M.C., De Guzman, M.F.D., Loquias, R.T., Raro, R.V.C.B., Quitayen, J.J.J., and Marciano, J.J.S. (2020). LokaLTE: 600 MHz Community LTE Networks for Rural Areas in the Philippines. In *2020 IEEE Global Humanitarian Technology Conference (GHTC)* (pp.1-8). IEEE.

[6] Ocampo, Y. (2022). Philippines Deploys Community LTE Networks in Rural Areas. Retrieved 7/20/2022 from https://opengovasia.com/philippines-deploys-community-lte-networks-in-rural-areas/

[7] Department of Education (2016). K to 12 Curriculum Guide: Mathematics. Retrieved 7/20/2022 from https://www.deped.gov.ph/wp-content/uploads/2019/01/Math-CG_with-tagged-math-equipment.pdf
[8] Bernardo, A.B.I. and Limjap, A.A. (2012). Investigating the Influence of Teachers' Pedagogical Beliefs and Reported Practices on Student Achievement in Basic Mathematics. In *12th International Congress on Mathematical Education*. Seoul, Korea. Retrieved 7/20/2022 from https://www.mathunion.org/fileadmin/ICMI/Conferences/ICME/ICME12/www.icme12.org/upload/submission/1975_F.pdf
[9] Verzosa, D.M.B. and Vistro-Yu, C.P. (2019). Prospects and Challenges in Implementing a New Mathematics Curriculum in the Philippines. In C.P. Vistro-Yu & T.L. Toh (Eds.), *School Mathematics Curricula* (pp. 207-226). Singapore: Springer.
[10] Villanueva, J.C.P.M., Melendres, M.A.V., Lagunzad, C.G.B., and Libatique, N.J.C. (2021). Design and Deployment of a Mobile Learning Cloud Network to Facilitate Open Educational Resources for Asynchronous Learning. In Rodrigo, M. M. T. et al. (Eds.), *Proceedings of the 29th International Conference on Computers in Education*. Asia-Pacific Society for Computers in Education.
[11] Siegler, R.S., Thompson, C.A., and Schneider, M. (2011). An Integrated Theory of Whole Number and Fractions Development. *Cognitive Psychology,* 62, 273–296.
[12] Woods, D.M., Geller, L.K., and Basaraba, D. (2018). Number Sense on the Number Line. *Intervention in School and Clinic*, 53(4), 229-236.
[13] Department of Education Schools Division Office of Pasig City (2020). Mathematics Grade 7 Quarter 1– Self-Learning Module 15: Square Root of Rational and Irrational Numbers, 7-9.
[14] Kilgore, K.E. and Capraro, M.M. (2010). A Technological Approach to Teaching Factorization. *Journal of Mathematics Education*, 3(2), 115-125.
[15] Mudaly, V. and Uddin, R.S. (2016). Technology in Mathematics: Use of Geogebra Applets. *PONTE International Scientific Researches Journal*. 72(9).
[16] Laborde, C. (2001). Integration of Technology in the Design of Geometry Tasks with Cabri-Geometry. *International Journal of Computers for Mathematical Learning*, 6, 283- 317.
[17] Ruthven, K. (2005). Expanding Current Practice in Using Dynamic Geometry to Teach about Angle Properties. *Micromath*, 21(2), 26-30.

Shih, JL. et al. (Eds.) (2023). Proceedings of the 31st International Conference on Computers in Education. Asia-Pacific Society for Computers in Education

Design of a Mobile App to Promote Understanding and Fluency in Finding the Equation of a Line

Agnes D. GARCIANO[a,*], Maria Alva Q. ABERIN[a], Ma. Louise Antonette N. DE LAS PEÑAS[a], Juan Carlo F. MALLARI[a], Jumela F. SARMIENTO[a], Mark Anthony C. TOLENTINO[a], & Debbie Marie B. VERZOSA[b]
[a]*Department of Mathematics, Ateneo de Manila University, Philippines*
[b]*Department of Mathematics and Statistics, University of Southern Mindanao, Philippines*
*agarciano@ateneo.edu

Abstract: This paper focuses on the design of a mobile app called *Pick or Fish* that fosters comprehension and mastery of the concepts of slopes, *y*-intercepts and equations of lines. The app's pedagogical value lies in its potential to help students understand and become proficient in these concepts. The app is suitable for use on low-cost mobile devices. It functions within an engaging game-like setting featuring visual elements that enable students to see the effect of parameter changes on the direction of a line. The beginner and advanced levels of the app have scaffolding features that gradually introduce the students to the key aspects of linear functions. The mechanics of the app, its pedagogical basis and how integration in the classroom may be achieved as teachers plan the lesson, facilitate open-ended discussion and encourage independent use of the app are also discussed.

Keywords: Lines, slopes, *y*-intercept, equations of lines, mobile app

1. Introduction

There is a view that from both a mathematical and pedagogical perspective, the core and fundamental concept underlying algebra, trigonometry, probability, statistics and calculus is that of *function* (Schwartz & Yerushalmy,1992). In the Philippine high school mathematics curriculum, linear function is one of the first types of functions that students learn. In Grade 8, the topics of slope, *y*-intercept, and equation of a line are discussed. An in-depth understanding of these concepts is necessary in problem solving and mathematical modeling involving linear functions. Furthermore, this is also necessary for doing more advanced work in linear regression, interpolation, and rates of change in calculus. The skill of writing equations of lines enables students to make correct mathematical models of real-world data exhibiting linear trends and make forecasts. Thus, the acquisition of conceptual knowledge and procedural fluency on calculations involving linear functions are important for high school students to have.

In this paper, we present the design of a mobile app called *Pick or Fish* that is intended to help students gain a better understanding of slope and y-intercept and acquire fluency in forming the equation of a line. The app is one of the technological tools developed and designed by the authors made available for use in Philippine schools through the internet, community LTE networks (De Las Peñas et al., 2022) or local networks powered by datacasting technology (De Las Peñas et al., 2023). These tools support the twin goals of mathematics education in the Philippines, which is to develop critical thinking and problem solving (DepEd, 2016). In emerging economies such as the Philippines, studies have shown that mathematics is only superficially learned (Verzosa & Vistro-Yu, 2019), and that the focus of mathematics classrooms is on rules and procedures (Nag et al., 2014; Verzosa, 2020).

Pick or Fish was designed as a mathematical mobile application (app) that runs in an interactive, game-like environment. It is intended to be played by students to gain proficiency

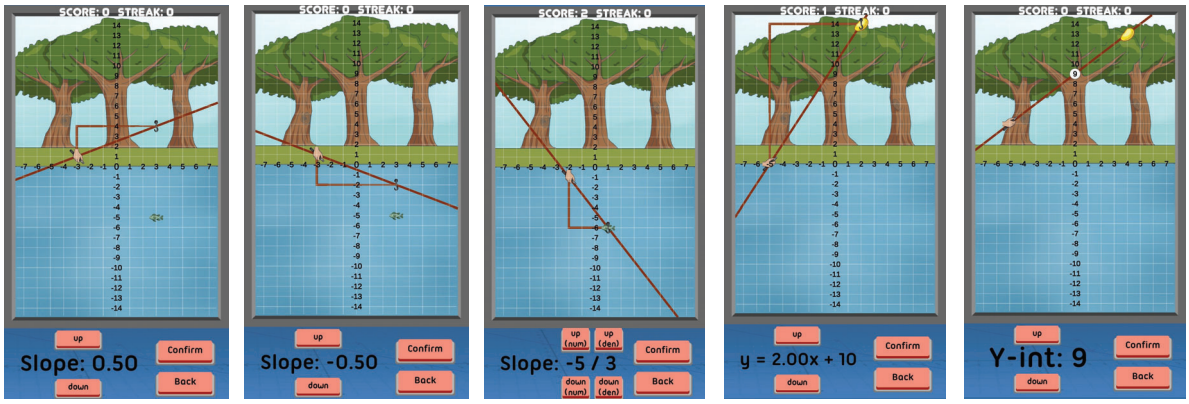
in finding slopes and forming equations of lines. Interactive digital tools such as mobile apps designed in a game -like environment have the potential to enhance the interest of students (Chao et al, 2018). Interest to carry out the tasks is important for students to gain mastery and fluency in the topic. While there are other existing apps and software that allow the exploration of slopes and equations of lines, one of the important features of *Pick or Fish* is that it is built in a game like environment, and there is the inclusion of scaffolding mechanisms that allow a student to work at a particular level of difficulty based on their learning progression on the topic. The latter feature makes the app apt for student centered learning. As an emerging economy, the Philippines has also comparatively limited educational resources. Further, many Filipino students come from low-income families who could not afford graphics calculators, laptops/computers or medium- to high-range mobile phones. Thus, the *Pick or Fish* was designed to be freely available and able to run on earlier Android versions of mobile phones.

2. Description of the *Pick or Fish* App

The app is designed to help students develop their understanding of the concept of slope and *y*-intercept of a line and consequently help build their skill in forming the equation of a line. There are two levels: *Beginner* and *Advanced* and there are several topics per level. In each level, the task is to pick a fruit or fish by either varying the value of the slope, or the *y*-intercept, or both.

2.1 Beginner Level

In all topics in the *Beginner* level, the screen shows a hand with a rod and a target object (either a mango fruit or a fish). The student needs to press the “up” and “down” buttons found at the bottom of the screen to move the rod so that it would touch the fruit or the fish. Doing this, the student can see how adjusting the slope corresponds to changing the steepness of the line as represented by the rod. By playing the game repeatedly, the student can connect positive and negative slopes to the orientation of the line (Figures 1(a) and 1(b), respectively).



(a) (b) (c) (d) (e)

Figure 1. Screenshots of the *Beginner* level of *Pick or Fish*

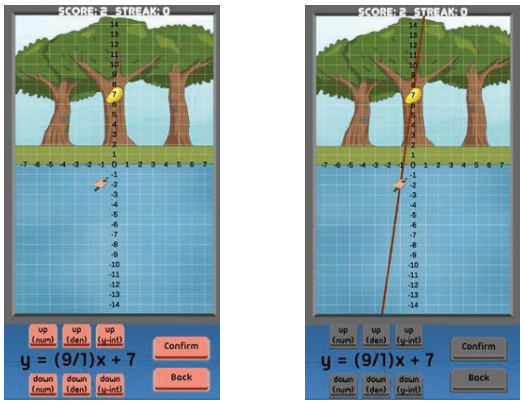
The app scaffolds instruction by first focusing on slopes having integer values, then fractional values. The student can see how the slope also corresponds to the ratio of the vertical and horizontal changes in the line. For example, in Figure 1(c), the slope is -5/3 which corresponds to the ratio in a triangle with vertical dimension 5 and horizontal dimension 3.

In the succeeding topic, *Slopes—Equation of a Line,* an equation of a line in the form $y = mx + b$ is given, but only the value of m needs to be changed (Figure 1(d)). This is the first introduction of the student to the slope-intercept form of the equation of a line. Familiarity with this form is important as it shows that a line is determined by two parameters m and b. The next topic is the *y-intercept.* where the student presses the up and down buttons to get the correct value of the *y*-intercept (Figure 1(e)). Here, the student sees how changing the *y*-

intercept will not change the steepness but would rather shift the line up or down. The last topic under the Beginner Level is the *Slope-Intercept Form of a Line.* Here, the student provides the values of the slope *m* and *y*-intercept b in the equation $y = mx + b$.

2.2 Advanced Level

In the *Advanced* level, the student can utilize their developing notions of slope and *y*-intercept to make the line reach the targeted mango or fish. The topics in this level are like those in the *Beginner* level, except that the rod is not visible at the start–only the hand and target are shown (Figure 2(a)). Thus, the student must apply what they learned in the *Beginner* level and press the necessary buttons (without visual scaffolding) to produce the correct equation that would make the rod reach the target (Figure 2(b)).



(a) (b)

Figure 2. Screenshots of the *Advanced* level, Topic: *Slope-intercept Form of a Line* rod is not shown at the start; (b) rod is shown after pressing “Confirm” button.

A student who makes a mistake has the chance to correct himself. For instance, when working on a sample question related to Slope-*Intercept Form of a Line*, both slope and *y*-intercept values are needed. The student may correctly input the value of the *y*-intercept, which is visible on the screen. However, determining the value of the slope requires more thought. Without visual help he must calculate the ratio of the vertical and horizontal distances between the hand and the object. If the student enters an incorrect value, he receives feedback that the rod misses passing through the object. This feedback serves as a clue, helping the student identify whether the slope must be increased or decreased and gives him an opportunity to input the correct answer.

2.3 Topic/Level Progression

The scaffolded questions built into the app help students to work at the appropriate level of difficulty. Students can go back to any of the levels and topics at any time if they need more practice, or they can do so as guided by their teacher. In each level, once the student presses “Confirm,” the app provides visual feedback and a bell or buzzer sound for the student to know if the answer entered is right. In addition, a score found at the top part of the screen enables students to monitor their progress and explore other topics in the app. These features help learners gauge their level of mastery and guide them as they progress through the topics.

3. Pedagogical Basis

The *Pick or Fish* app was designed to elucidate how variables express a relationship in a linear equation. The concept of *variable* is not easily learned because students may not have limited conceptions of how variables are used in mathematics. For Küchemann (1981, p. 104), a

variable represents “a range of unspecified values, and a systematic relationship is seen to exist between two such sets of values.” Ely and Adams (2012) identified two critical properties from Küchemann’s definition: First, a variable is indeterminate, and can stand for a range of values such as in “$y = x + 7$,” and second, it is part of a systematic relationship such that when one variable changes, another quantity may vary with it. Drawing on the history of mathematics, they further argue that the development of the notion of a variable represented a major breakthrough from using letters merely to represent an unknown or determinate quantity, such as in the equation “$2 + x = 5$.” This development took centuries, making it understandable why students struggle with the transition from unknown to variable–they think that a letter must represent a specific value (Ely & Adams, 2012). This is possibly why students find it difficult to solve tasks involving representational transitions from a graph to an equation or vice versa (Ceuppens et al., 2018).

The visual scaffolds in the *Pick or Fish* facilitate the cognitive shift required to help students see relationships between the equation of a line to its graph and gain a better understanding of the systematic relationship between the variables in the equation. This is done through a game environment, particularly in the *Beginner* level, where students can see how adjusting coefficients in a linear equation leads to changes in the corresponding graph. These scaffolding elements which are introduced gradually through the progression of topics, along with visual aids and feedback mechanisms, help students cultivate a strong conceptual understanding of slope and *y*-intercept concepts, as well as to improve their fluency and proficiency in forming the equations of lines.

4. Game Design Features

Shi and Shih’s (2015) Game-Based Learning (GBL) Design Model was used as the framework for the development of *Pick or Fish*. This model highlights 11 game design factors (*game goal, game mechanism, interaction, freedom, challenge, game fantasy, narrative, sociality, sensation, mystery, and game value*) for the design, analysis, and evaluation of game-based learning applications. Ten of these factors were incorporated into *Pick or Fish* and are discussed in detail below.

The *game goal* for *Pick or Fish* is to deepen one’s understanding of slopes and the formulation of the equations of lines. This is achieved via its drill-like *game mechanism* where the player must provide the correct slope, *y*-intercept or equation of a line that passes through two given points (the hand and the fruit or fish). This drill-like mechanism complements the game goal as it allows players to repeatedly practice with different questions or tasks provided in the app to develop their fluency in the topic. Contributing to the *game mechanism* are the factors *interaction, freedom, and challenge.* Players *interact* with the game by tapping buttons to increase or decrease the slope and/or *y*-intercept of a line. They are given the *freedom* to select from multiple topics and levels that, as previously mentioned, allow for scaffolded instruction. These levels *challenge* the player to correctly answer five questions in succession, after which they will receive a prompt to move on to the next topic. These elements are important for developing learners’ fluency in the topic as they can use the app to set their own learning pace (i.e., staying with a current topic or level until they have developed sufficient mastery) or to address specific learning gaps (i.e., selecting the topic or level that corresponds to a competency that they need to improve).

As *Pick or Fish* can be considered a single-player arcade game, it places no emphasis on *sociality* and features a simple *game fantasy* and *narrative* in which players are tasked to pick fruits off trees or catch fish by correctly identifying the equations of lines. The *game fantasy* is then further enhanced by the game’s *sensation* and *mystery* aspects. For *sensation*, the game uses a simple but vibrant background with an overlain Cartesian plane, as well as sound prompts to let the player know when they input a correct or incorrect answer. It must be noted that on this Cartesian plane, for the slope topics under the game’s *Beginner* level, adjusting the slope not only changes how the line is tilted but also changes the line’s “rise” and “run” as represented by the appropriate triangle. This topic or level progression adds *mystery* to the game, with each new topic or level providing a new experience to the player.

Finally, the *game value* arises from the combination of all these factors, as *Pick or Fish* is aimed at providing players an engaging environment in which they can develop their understanding of equations of lines.

5. Integration and Use of the *Pick or Fish* App

This section describes how *Pick or Fish* can be utilized as a pedagogical tool. The steps below were adapted from the work of Woods et al (2018).

Step 1. Determine learning goals. The app is appropriate and useful in attaining the learning competencies involving slope, *y*-intercept and equation of a line. The design makes it easier for students to make the shift needed to see connections between a line's equation and graph and to comprehend the systematic link between the variables in an equation.

Step 2. Plan ahead. The teacher should plan for the logistical requirements of using the free to download app. Students should be taught how to properly download the software on a desktop or an Android device by their teachers. An Android emulator is required to run the program if it was downloaded to a PC.

Step 3. Present and practice. This step entails the presentation of the lesson and a demonstration on how to play the app. The teacher can ask, "Should the rod be moved up or down, left or right?" or "What happens to the direction of the rod when the up button is clicked?" These instruction questions will prevent students from using guesswork as a strategy to find the slope, *y*-intercept and equation of the line. Students have the chance to practice and become comfortable with the app during this phase.

Step 4. Process and verbalize. Finally, open-ended questions will allow students to explain their thinking process and reasoning. This can be done by asking reflection questions like "What helps you determine which button to click?", "How do you know that slope should be positive? or negative?", "What is the meaning of *y*-intercept?", or "How can one form the equation of a line?"

For practice and mastery, the teacher may advise the pupils to utilize the app asynchronously or outside of class. Students can share screenshots of their outputs over time in order to monitor their development. A follow-up discussion can be done to address difficulties and misconceptions of the students.

6. Conclusion and Future Direction

A deep conceptual understanding and fluency in creating equations of lines is a basic learning competency expected of high school students. However, many students still find the ideas of slope and *y*-intercept abstract and lack the proficiency in using these elements to formulate the equation of a line. This problem is carried on to more advanced work in mathematics where the concept of linear functions and equations are necessary.

In this paper, the *Pick* or *Fish* app is introduced as an interactive tool that can help students gain more insight and fluency in finding the slope and *y*-intercept of a line and write its equation. The app is aligned with the Philippine Department of Education's Most Essential Learning Competencies in Grade 8 Mathematics (DepEd, 2020). The app's game-like features help motivate learning and entice students to play to improve their understanding of abstract concepts. The popularity of mobile phones and tablets in the Philippines makes the app easily accessible as it can be readily downloaded and require low storage.

The next step in this research is to conduct studies on determining the effectiveness of the app in student learning. The app may still be improved by the inclusion of undefined slopes of vertical lines. An aspect that may be pursued in the future is the incorporation of more scaffolding mechanisms that can offer help to students who may struggle with the transition from beginner to advanced levels. Gathering more feedback from students and teachers on the use of the app will also help direct future research in this area.

Acknowledgements

This paper is a product of the research work of the Ateneo Mathplus Resources Laboratory of the Department of Mathematics, School of Science and Engineering, Ateneo de Manila University (ADMU). The authors express their gratitude to the Department of Science and Technology-Philippine Council for Industry, Energy, and Emerging Technology Research and Development and the University Research Council, ADMU for their valuable support in the development of the mathematical resources made for Grades 1 to 11 Mathematics. We also thank our app developers Victor Antonio M. Ortega, Mr. Nigel Benedict Cargo, Mr. Jose Teodoro Lacson, and Mr. Amiel Damian F. Justiniani for the apps and video art assets.

References

Ceuppens, S., Deprez, J., Dehaene, W., & Cock, M. D. (2018). Design and validation of a test for representational fluency of 9th grade students in physics and mathematics: The case of linear functions. *Physical Review Physics Education Research, 14,* 02015.

Chao, W.-H., Yang, C.-Y., Hsien, S.-M., & Chang, R.-C. (2018). Using mobile apps to support effective game-based learning in the mathematics classroom. *International Journal of Information and Education Technology, 8*(5), 354-357.

De Las Peñas, M. L. A. N., Aberin, M. A. Q., Garciano, A. D., Mallari, J. C. F., Sarmiento, J. F., Tolentino, M. A. C., & Verzosa, D. M. B. (2022). Deployment of mathematical resources to a Philippine high school through a community LTE network. In W. C. Yang, M. Majewski, D. Meade, & W. K. Ho (Eds.), *Proceedings of the 27th Asian technology conference in mathematics* (pp. 126-140). Mathematics and Technology, LLC.

De Las Peñas, M. L. A. N., Kwong, J. C. T., Banzon, P. A. B., Martinez, P. A., Lapada, W. I. G., Eballa, J. N., Sebastian, J. E. C., Asido, M. D., San Juan, J. D. M., Versoza, D. M. B., Sarmiento, J. F., Garciano, A. D., Tolentino, M. A. C., Aberin, M. A. Q., & Mallari, J. C. F. (2023). Mathematical mobile apps via Rural Casting. In P. Kommers, I. A. Sánchez, & P. Isaías (Eds.), *Proceedings of the international conferences on e-Society 2023 and mobile learning 2023* (pp. 343-350). International Association for Development of the Information Society.

Department of Education. (2016). *K to 12 Curriculum Guide: Mathematics (Grade 1 to Grade 10)*. https://www.deped.gov.ph/wp-content/uploads/2019/01/Math-CG_with-tagged-math-equipment.pdf

Department of Education. (2020). *K to 12 most essential learning competencies with corresponding CG codes*.

Ely, R., & Adams, A. E. (2012). Unknown, placeholder, or variable: what is x? *Mathematics Education Research Journal*, *24*(1), 19–38.

Küchemann, D. E. (1981). Algebra. In K. M. Hart (Ed.), *Children's understanding of mathematics 11–16* (pp. 102-119). London: John Murray.

Nag S., Chiat S., Torgerson C., & Snowling, M. J. (2014). Literacy, foundation learning and assessment in developing countries: final report. *Education Rigorous Literature Review*. Department for International Development, United Kingdom.

Schwartz, J., & Yerushalmy, M. (1992). Getting students to function on and with algebra. In E. Dubinsky & G. Harel (Eds.), *The concept of function: aspects of epistemology and pedagogy* (pp. 261–289). Mathematical Association of America.

Shi, Y.-R., & Shih, J.-L. (2015). Game factors and game-based learning design model. *International Journal of Computer Games Technology, 2015*, 1-11.

Verzosa, D.M.B. (2020). Teachers' approaches and drawn representations in explaining solutions to fraction tasks. Manuscript in preparation.

Verzosa, D.M.B., & Vistro-Yu, C.P. (2019). Prospects and challenges in implementing a new mathematics curriculum in the Philippines. In C.P. Vistro-Yu & T.L. Toh (Eds.), *School mathematics curricula* (pp. 207-226). Singapore: Springer.

Woods, D.M., Geller, L.K., & Basaraba, D. (2018). Number sense on the number line. *Intervention in School and Clinic, 53*(4), 229-236.

Ateneo de Manila University

Archīum Ateneo

Mathematics Faculty Publications | Mathematics Department

2020

Designing Mobile Apps to Promote Numeracy and Statistical Reasoning

Ma. Louise Antonette N. De Las Peñas
Ateneo de Manila University, mdelaspenas@ateneo.edu

Mark L. Loyola
Ateneo de Manila University, mloyola@ateneo.edu

Jumela F. Sarmiento
Ateneo de Manila University

Mark Anthony C. Tolentino
Ateneo de Manila University, mtolentino@ateneo.edu

Debbie Marie Versoza
Ateneo de Manila University

Follow this and additional works at: https://archium.ateneo.edu/mathematics-faculty-pubs

 Part of the Mathematics Commons, and the Science and Mathematics Education Commons

Custom Citation

Ma. Louise Antonette N. De Las Peñas, Mark L. Loyola, Jumela F. Sarmiento, Mark Anthony C. Tolentino, Debbie Marie B. Verzosa (University of Southern Mindanao), (2020), Designing Mobile Apps to Promote Numeracy and Statistical Reasoning. Electronic Proceedings of the 25th Asian Technology Conference in Mathematics (eds Yang, W.C. & Meade, D.), null, 167-182.

This Article is brought to you for free and open access by the Mathematics Department at Archīum Ateneo. It has been accepted for inclusion in Mathematics Faculty Publications by an authorized administrator of Archīum Ateneo. For more information, please contact oadrcw.ls@ateneo.edu.

Designing Mobile Apps to Promote Numeracy and Statistical Reasoning

Ma. Louise Antonette N. De Las Peñas
mdelaspenas@ateneo.edu
Department of Mathematics
Ateneo de Manila University, Philippines

Debbie Marie B. Verzosa
dmbverzosa@usm.edu.ph
Department of Mathematics and Statistics
University of Southern Mindanao, Philippines

Jumela F. Sarmiento
jsarmiento@ateneo.edu
Department of Mathematics
Ateneo de Manila University, Philippines

Mark Anthony C. Tolentino
mtolentino@ateneo.edu
Department of Mathematics
Ateneo de Manila University, Philippines

Mark L Loyola
mloyola@ateneo.edu
Department of Mathematics
Ateneo de Manila University, Philippines

Abstract: *Even with major advancements in the field of mathematics education, many students still do not attain the learning competencies prescribed by official curricula. This may be partially explained by students' immersion in classrooms characterized by superficial strategies or rote-learning methods. This paper reports on the design of mobile applications (apps) developed by the authors as part of an ongoing project funded by a national government agency and intended to promote structural thinking and statistical reasoning. It describes the general features of the apps, as well as the pedagogical principles upon which the apps' designs were anchored on. These principles are grounded on research and established practices on number sense and statistical learning. Collaborations with the Philippine Department of Education for widespread implementation and sustainability are also discussed.*

1. Introduction

Multinational benchmarks of mathematics learning indicate that many students still had not attained the learning competencies prescribed by official curricula [1]. Possibly this is because mathematics classrooms in several countries promote rules and procedures with little attention to underlying mathematical concepts [2, 3, 4]. Without effective intervention, children can remain weak in basic numeracy or number recognition [5]. For example, in the Philippines, elementary school children's place value knowledge is limited to knowledge of labels (i.e., hundreds, tens, ones) rather

than on quantity [6]. A similar problem occurs in the learning of statistics, where the focus is on computation rather than on statistical literacy or reasoning skills [7].

A potential solution is offered by mobile technology [8], which can provide a sustainable educational alternative for mathematical learning [9]. However, a large number of available apps are basically glamorized worksheets which are mostly of the "drill and practice" type [10]. These apps may just perpetuate the situation where rote learning is the norm. Thus, there is a need to design apps that are informed by theories emerging from the wealth of knowledge gained from research on numeracy and statistical learning. This paper reports on five purposely designed numeracy apps for Grades 1 to 6; and one web app, a statistical database for Grades 1 to 11, to develop these two broad areas of mathematics.

The apps are products of an ongoing supported project "Technology Innovations for Mathematical Reasoning, Statistical Thinking and Assessment" by the Department of Science and Technology-Philippine Council for Industry, Energy and Emerging Technology Research and Development (DOST-PCIEERD). The apps acquired additional relevance due to the urgent educational difficulties driven by the Covid-19 pandemic and the projected adaptations that teachers or learners may face as they transition into blended learning. Although the apps were designed for Filipino learners, they address learning outcomes that are present in many other national curricula as well.

This paper is divided into several parts. The next section describes some general considerations that guided the design of the apps. The third section presents the pedagogical basis for the numeracy apps and the web statistical learning app. The last section presents the collaboration with the Philippine Department of Education (DepEd).

2. Number Sense and Statistical Learning

This section discusses the rationale for designating number sense and statistical reasoning as target educational outcomes of the apps. These two broad areas of mathematics can be considered "big ideas" because they (1) consist of foundational concepts that are necessary for more advanced learning, (2) encompass a network of concepts and strategies, (3) provide an organizational structure for further learning, and (4) are apparent in learners' activity [11]. Thus, support for these "big ideas" or fundamental ideas is called for. Further, many of the existing mathematical apps overlook much of the current research about mathematical learning [10].

There are several countries where reports of poor number sense and statistical reasoning exist (see [12, 13, 14, 15, 16] and the references therein). However, since the apps were designed through a funded project by the Philippine government to develop innovative tools in supporting the implementation of the Department of Education's K to 12 curriculum, it was a priority to design the apps based on published research on the mathematical achievement of Filipino learners. Nevertheless, these apps would also be of interest to mathematics educators and researchers from many other countries with similar educational contexts.

Number sense is often neglected in Philippine classrooms. For example, conceptual place value considers two distinct ideas: *quantity value* (knowing that 45 is 40 and 5) and *column value* (knowing that the 4 in 45 represents 4 tens) [17, 18]. However, Filipino students do not seem to fully acquire these two fundamental concepts because their knowledge is often limited to labels (e.g., in a 3-digit number, the middle digit is called the "tens place") [6]. This outcome suggests that the treatment of place value in Philippine classrooms is on memorizing the labels "hundreds, tens, and ones" rather than on connecting these labels to number quantity (quantity value) or the quantity represented by each digit (column value). Possibly, this poor understanding of place value is the reason why Filipino students cannot apply mental strategies to solve tasks such as 208–10, where regrouping is involved

[6]. In view of this situation, the apps designed in this project focus on conceptual place value. These apps are different from previously designed apps for Filipino students, where traditional place value instruction and the learning of standard algorithms are emphasized.

Statistical learning is another major concern in the Philippines. The Philippine K to 12 program specifies a very rigorous statistics curriculum. The K to 12 program was implemented in 2012, with the spiral approach as one of its new elements. Whereas previously, statistics was taught in only a few grade levels, in the new program, all teachers from Grades 1 to 11 are expected to teach statistics. The problem is, statistics remains one of the most difficult topics to teach, and even local tertiary level textbooks are replete with statistical misconceptions [19]. The database for statistics learning seeks to address this problem by providing teaching guides that can support teachers who themselves have limited background in statistics. Further, in the spirit of Census at School programs in other countries, the database aims to collect authentic data so that Filipino students would find additional relevance in what they are learning and a better grasp of understanding statistical concepts. An additional obstacle is that in the Philippines, statistics is the last topic in the curriculum of each grade level from Grades 1-11. Any delay in curriculum implementation, by calamities such as floods or earthquakes, will mean that statistics is cut short or not taught at all. With the database for statistical learning, students and teachers may continue the learning of statistics through synchronous or asynchronous activities through the worksheets and teaching guides provided within the database.

3. General Design Considerations

The numeracy apps were designed following these guiding principles. The first principle was inclusivity. The apps were intended to be available to as many learners and teachers as possible. Thus, the project team decided that the apps should remain free, requiring only one-time access to the Internet (during download). Even when there is no Internet access, the app must be transferable wirelessly (Bluetooth) or through an external device (such as USB or memory card).

Second, the apps were designed to be ready-to-use or ready-to-play. Students should be able to figure out how to use the apps even with minimal instruction and limited supervision. Further, students can play or use the apps at their own time. Thus, teachers may assign the apps as supplementary material for mathematical learning.

Third, the apps were designed to align with the DepEd curriculum, which encompasses the five strands of mathematical learning, namely, numbers and number sense, measurement, patterns and algebra, geometry, probability and statistics [20]. The project team consulted teachers, math heads, and principals to ensure that the apps are suitable for Filipino learners.

Fourth, the apps were designed to be interactive and dynamic. These were unlike static worksheets where students primarily perform drill and practice. The apps were designed in a game-like environment. The idea is that, by doing better in the game, learners develop a stronger understanding of mathematical concepts. In all grade levels, the apps facilitate exploration, visualization, and justification.

The design of the web statistical app was guided by similar principles. First, the web app was designed to be mobile-ready so that this may be conveniently accessed and used not just on computers but also on mobile devices such as tablets and smartphones. Second, the interface of the app is simple, easy-to-use, and appropriately designed for young children. Moreover, since the web app requires access to the Internet, measures were put in place to address possible interruptions due to intermittent or limited Internet connectivity. Finally, the student worksheets and teaching guides, aligned with the Statistics and Probability competencies for Grades 1-11 in the DepEd curriculum, were prepared to complement the use of the web application.

4. Pedagogical Design

This section discusses the pedagogical basis of the design of the apps. The five numeracy apps are downloadable from https://mathplusresources.wordpress.com/ and the web app is accessible from mathplusresources.com.

4.1 Quick Images

The *Quick Images* app presents some dot patterns and the user has to figure out how many dots are shown (Figure 4.1). It is meant to address the issue that some learners still rely on counting by ones even well into their upper elementary school years. These learners think of numbers primarily as a count sequence starting from one, rather than as a more sophisticated part-whole relationship [21]. For example, they think of "four" as the end of the count sequence 1, 2, 3, 4, without realizing that "four" is the same as three and one or two and two. In [22], a recommended intervention is the use of subitizing (or "instantly seeing how many"). It was noted that certain arrangements of dots can be quickly recognized as a certain number, such as recognizing that one four-dot and one three-dot dice pattern give the number seven. This recognition involves **conceptual subitizing**, or a knowledge of numbers as a composite of parts. Familiar spatial arrangements and five or ten frames were used as organizing features because these can facilitate arithmetic and place value knowledge [22, 23]. It was further recommended that the dot patterns be shown very quickly, so that students are prompted to draw on the part-whole composition of number rather than on the unitary count sequence. *Quick Images* allows the user to choose between three speed levels: slow (2.5 s), medium (1.5 s) and fast (0.5 s).

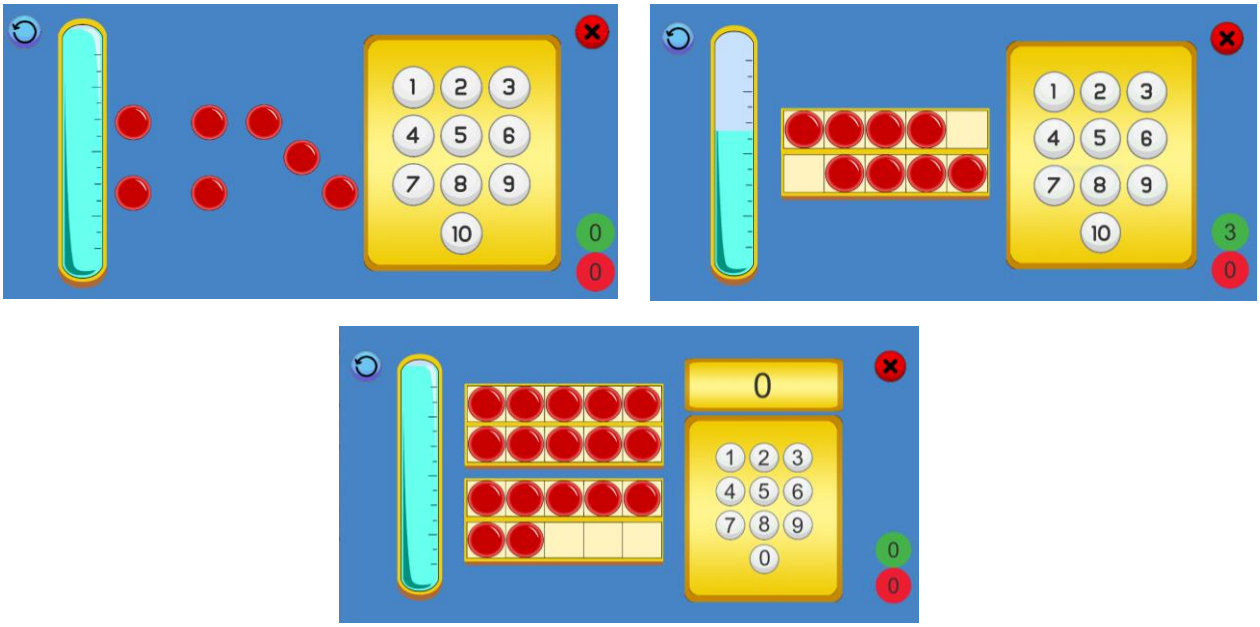


Figure 4.1 Screenshots from *Quick Images*

4.2 Frame Game

The *Frame Game* app (Figure 4.2) was designed to facilitate learners' skill in incrementing or decrementing by 10s, 100s, and so on, which is a foundational piece of knowledge for developing ten-structured thinking [21] and conceptual place value [18]. In *Frame Game*, a target number is given, and users need to press the buttons (±1, ±10) so that the target number is reached. Visual support is provided by a pictorial representation of the current number. As with *Quick Images*, the objects shown in *Frame Game* are arranged in tens to provide an organizational structure in the child's mind.

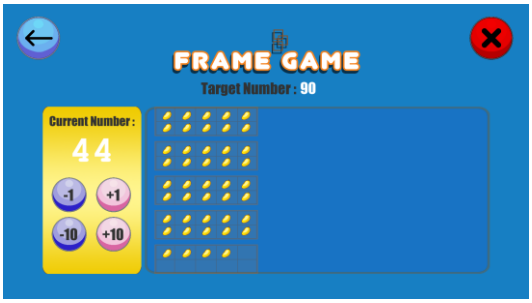
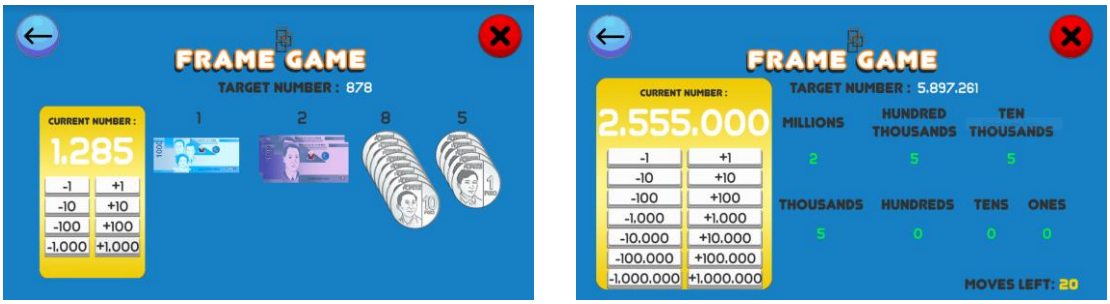


Figure 4.2 Screenshot from *Frame Game*

Frame Game also aligns with several grade levels in the official elementary school curriculum. In this app, the user has an option to choose the number range, from 1 to 100 for Grade 1 through 1 to 10,000,000 for Grade 5. It also addresses various levels of abstraction. Objects are initially shown using pictorial representations of each object (Figure 4.2), then using money (Figure 4.3a), and finally using purely symbolic or digit representations (Figure 4.3b).



(a) (b)

Figure 4.3 Screenshots of higher levels of abstraction from *Frame Game*

4.3 Grid Game

The *Grid Game* app is similar to *Frame Game* in that the user must reach a target number by pressing appropriate buttons. The main visual support is the hundred grid (Figure 4.4). There are also higher levels of abstraction in *Grid Game*. Initially, all numbers can be seen in the grid (Figure 4.4). At a higher level of abstraction, a blank grid is shown (Figure 4.5a). At the highest level of abstraction,

no grid is shown (Figure 4.5b). To address competencies in several grade levels, *Grid Game* offers the user an option to choose the number type: whole numbers (for Grades 1-2), decimals (for Grades 4-5), and integers (for Grades 6-7).

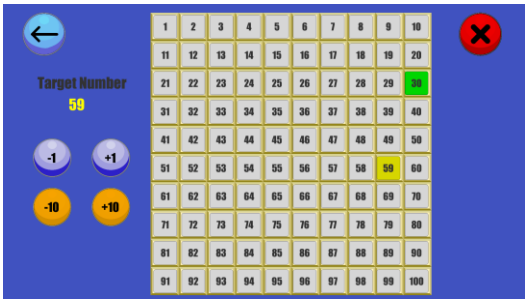
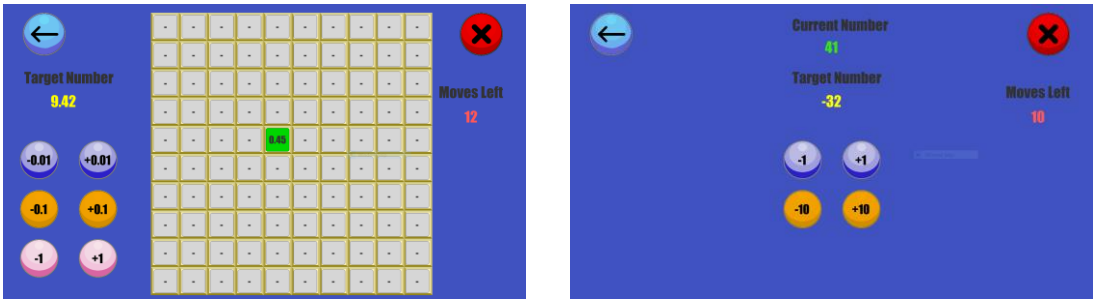


Figure 4.4 Screenshot from *Grid Game*

To ensure that users go beyond unitary counting (e.g., pressing +1 multiple times to go from 24 to 52), both *Frame Game* and *Grid Game* provide a higher level where users are required to apply the most efficient strategy to reach the target number. This is done by indicating the number of allowable moves ("Moves Left" in Figure 4.3b and Figure 4.5). When users run out of moves, they have the option to repeat the same problem and improve their solution.



(a) (b)

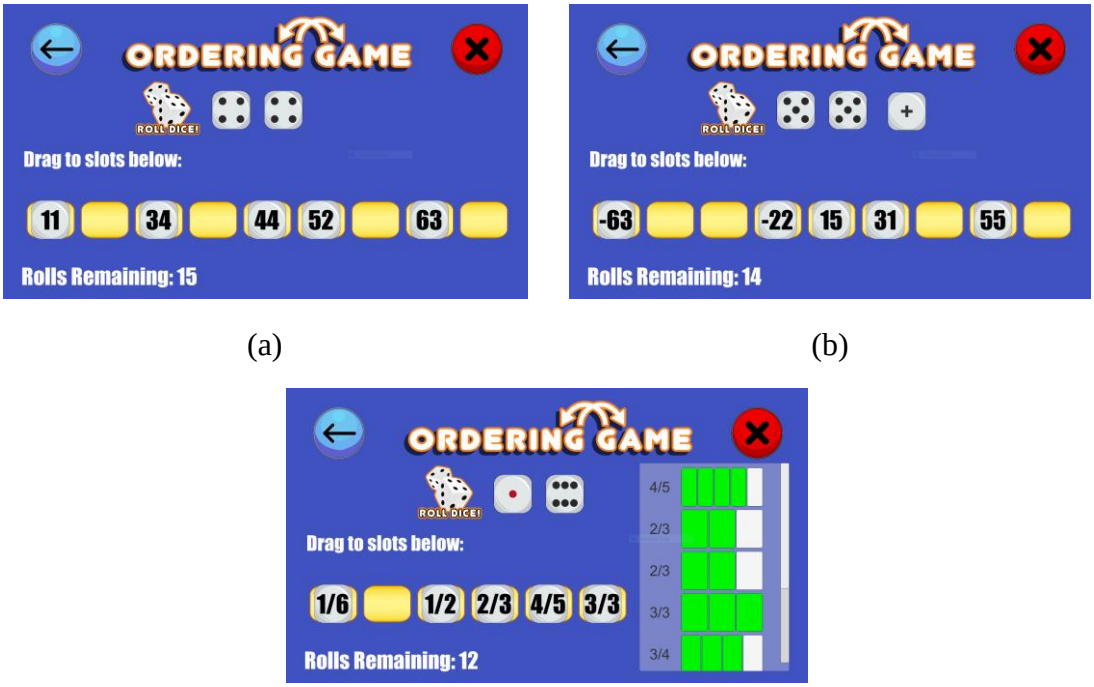
Figure 4.5 Screenshots of higher levels of abstraction from *Grid Game*

4.4 Ordering Game

The design of the *Ordering Game* app (Figure 4.6) was based on the premise that numerical development is hinged on an understanding of number magnitudes; and extending this understanding to increasingly large whole numbers and to a broader set of numbers including integers and rational numbers [24]. A simple proposed intervention is to incorporate activities that utilize the mental number line. It was found that learners from low-income families gained a more precise understanding of numerical magnitude after playing a numerical board game for just four 15-minute sessions [25]. Some resources [26] are already available online, so we developed another game, which was inspired by an activity by Marilyn Burns (cited in [27]).

In *Ordering Game*, users generate numbers by rolling the virtual dice. They place the generated numbers along a row, with the condition that the numbers are increasing from left to right (Figure

4.6). The goal is to fill up the entire row within the allowed number of rolls. The allowed number of rolls can be altered by choosing a level—a higher level allows fewer numbers of rolls before which the user must complete the task. The game becomes more difficult as it progresses because it becomes more likely that a generated number cannot be correctly placed in any of the remaining slots. As with the previous apps, *Ordering Game* also aligns with several grade levels because there are options for number range (e.g., from two- to five-digit whole numbers) and for number type (whole numbers, integers, fractions). In the case of fractions, a visual representation of each fraction is also shown (see Figure 4.6c). The goal is to strengthen the users' images of fraction magnitude. *Ordering Game* also gives feedback to indicate when a user places a number on an incorrect spot along the row.



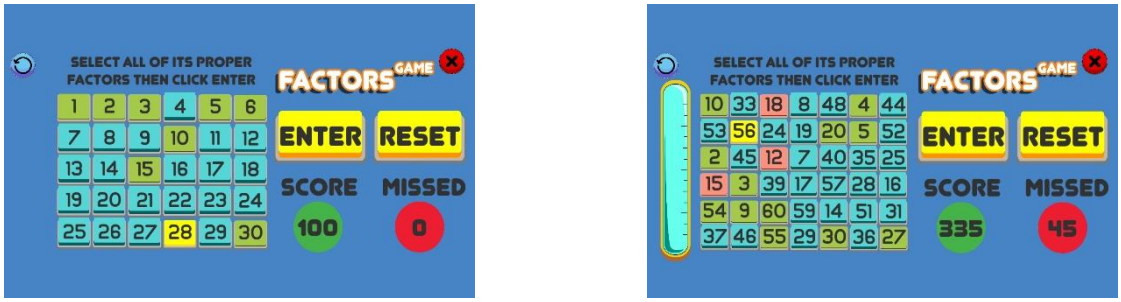
(a) (b)

(c)

Figure 4.6 Screenshots from the *Ordering Game* for (a) whole numbers, (b) integers, and (c) fractions

4.5 Factors Game

The *Factors Game* app was designed so that learners demonstrate understanding of prime and composite numbers in a game setting. There are four levels: 1-30, 1-40, 1-50 or 1-60 and three modes: unlimited time, two minutes or one minute. The player selects a number and then selects all its proper factors. The total score gained is the sum of the number chosen and the correct factors selected. The total missed score is the sum of missed factors and wrong factors selected. The game ends when all the numbers on the grid are selected or missed or the time is up. A summary appears after the game which gives the scored and missed points and correct, wrong or missed factors. The highest score attained by the player is also included in the summary. In the easiest level (Figure 4.7a), numbers 1 to 30 are placed on a 5 x 6 grid arranged in order while in the 1-60 level (Figure 4.7b), randomly selected numbers from 1 to 60, not in order, are placed on the 6 x 7 grid.



(a) (b)

Figure 4.7 Screenshots from the *Factors Game* (a) Level: 1-30 and (b) Level: 1-60

The *Factors Game* is aligned with the official elementary school curriculum. It addresses the need for the learners, from various grade levels, to identify factors and multiples of a given number. Moreover, it also trains the learners to differentiate prime from composite numbers. Without a time-limit, the goal is for the player to choose the factors of the number selected, correctly and avoid having missed points. The time limit will require players to strategize in selecting the numbers. Players will find out that they can score more points by selecting certain numbers.

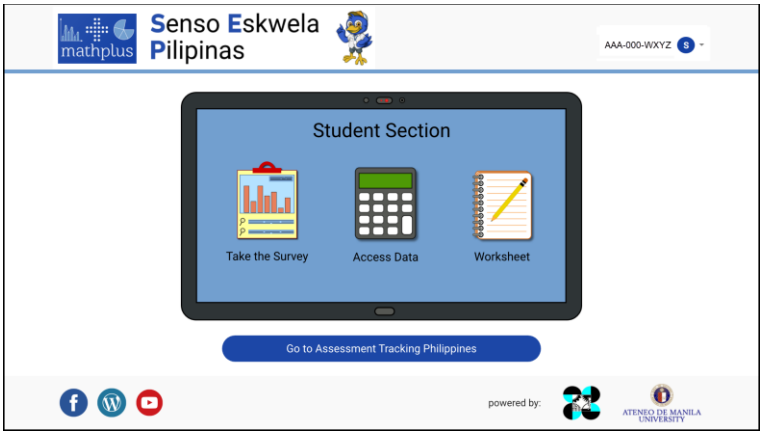
4.6 Senso Eskwela Pilipinas

A web app that has potential in improving statistics instruction in the early school years is the *Senso Eskwela Pilipinas* (*SEP*). The *SEP* is patterned after the international project called *CensusAtSchool*. First established in the UK in 2000, *CensusAtSchool* has the "dual thrust to enliven data handling activities within the classroom while also educating children about the principles and processes involved in conducting a census" [28, p.1]. Since then, *CensusAtSchool* has also been established in other countries like Australia, Canada, USA, New Zealand, and Japan. For instance, the Japanese *CensusAtSchool* project focused to address the observed needs of Japanese statistics teachers and students [29].

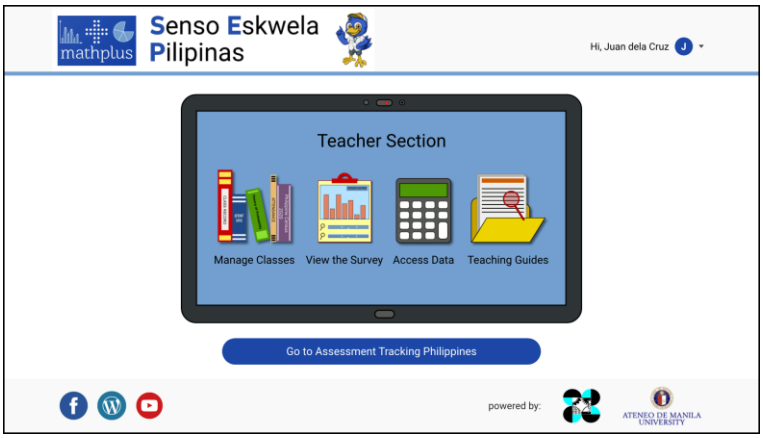
SEP serves as an online platform for gathering, storing, and accessing readily available and relatable data for learning and teaching statistics. In the login page, the user is asked if he or she is a student or a teacher. Grade 1-12 students, who are given access to *SEP* via a code given by their teacher, can answer a 28-question survey about themselves, such as their age, height and weight, favorite food and drinks, hobbies and interests, and others. Some questions involve puzzles and games that can captivate their interest. The students' responses are then stored in the *SEP* database and become part of the data that can be accessed by teachers and students nationwide. Although the same set of questions is used for all grade levels, a broad range of statistics topics can be studied depending on the kind of data the students will retrieve. Student worksheets are also provided to complement the teaching guides for teachers and also to allow students do activities independently (Figure 4.8a). At a basic level, the data can be used to teach students about creating pictographs and other charts or graphs. At a more advanced level, the same data can be used to teach the Central Limit Theorem or hypothesis testing. Moreover, since there are responses from various grade levels and locations, it is possible to do comparative analyses using different statistics topics.

Meanwhile, teachers get access to *SEP* by creating an account for free. The features designed for teachers allow them to manage classes, retrieve their students' survey results, access the data, and view teaching guides (Figure 4.8b). These teaching guides, prepared by the project team, are aligned

with the official DepEd curriculum. In general, teachers, students, and even unregistered guests may download a random sample from the database to be used for activities or tasks in their teaching or learning of statistics. Moreover, teachers and students can retrieve the data of their own class(es) (Figure 4.8c).



(a)



(b)



(c)

Figure 4.8 (a) Student page, (b) Teacher page, and (c) Students' Access Data page of *SEP*

Similar to the *CensusAtSchool* framework, *SEP* aims to enrich the experience of teachers and students in teaching and learning statistics. In particular, "[t]he sense of belonging that participating pupils feel, purely because they know their own responses will become part of a database of responses from their peers, is fascinating and motivating for them." [30, p. 175] Moreover, the questions in the *SEP* survey have been carefully drafted to appeal not only to the interests of grade school and high school students but also to emphasize experiences that are uniquely Filipino. For example, one question in the survey about games that the respondent likes to play is intentionally focused on Pinoy (Filipino) games (Figure 4.9). Through questions like this, the team aims to maximize the relatability of the *SEP* survey so that students can naturally develop a sense of curiosity about the responses of their classmates or other students elsewhere. In turn, this will hopefully help them formulate statistics questions more naturally and increase their engagement in answering these questions through their statistics lessons.

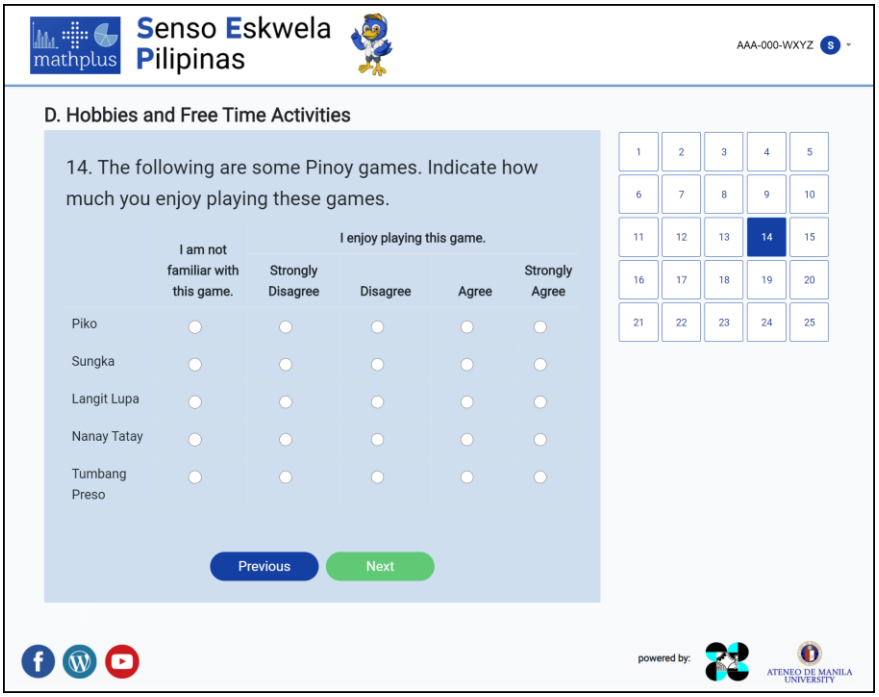


Figure 4.9 An example survey question of *SEP* on Pinoy (Filipino) games

5. Collaboration with the Department of Education

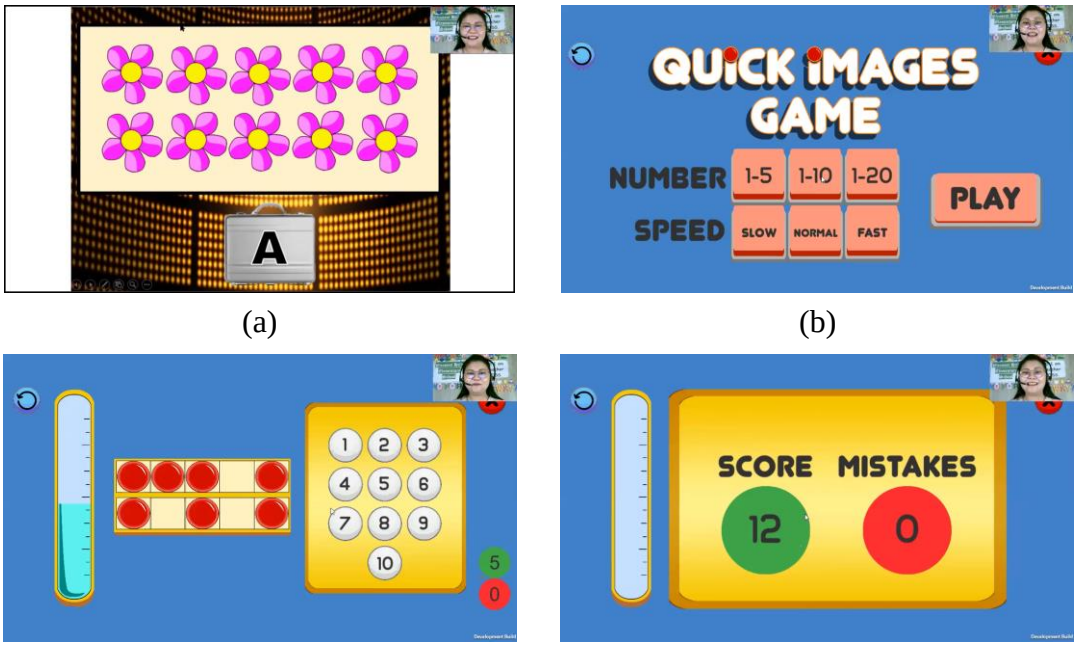
In 2020, the events arising from the Covid-19 pandemic propelled changes in modalities of teaching mathematics in schools in many countries around the world. In the Philippines the Department of Education (DepEd) has announced the shift to blended learning in schools from K to 12 in the school year 2020-2021. Radio, television, online and modular learning are being prepared and updated for use this school year. At the same time, teachers are being trained on utilizing newer platforms and innovative tools to prepare for this new mode of instruction. One of the methods of teaching that authors proposed to the Curriculum and Instruction Unit of the DepEd Central Office was the use of the mathematical apps as additional resources for teachers in blended learning to carry out instruction. One of the biggest opportunities for learning is that the apps can be integrated in the lesson wherein the students may explore and visualize concepts, use real data for exploration, practice drills and exercises remotely, even without much supervision and guidance of teachers or parents.

This approach will also provide students and teachers the opportunity to use their smartphone, tablet or laptop as a tool for mathematical learning. Since the numeracy apps are free and the web app requires no subscription; and the apps run on moderate system requirements, these make the resources very accessible to a wider range of teachers and students. The numeracy apps may be used offline; and will benefit students who do not have the resources and opportunities to engage in online learning due to Internet issues.

As the DepEd Central Office is working towards making the apps available in the DepEd learning resources and portal for the information and use of teachers nationwide, the authors already started partnerships with local DepEd divisions in Metro Manila so that teachers can start their training and exposure on the use of the apps. These partnerships produced an ongoing series of webinars to introduce the mathematical apps to teachers and familiarize them with how the apps are aligned with the most essential learning competencies prescribed by the curriculum. While the webinars were carried out in collaboration with local DepEd divisions, these were made available to teachers nationwide. The webinars were streamed live and were also made available shortly by the divisions in their social media channels so teachers could revisit the webinars at their own pace and time. Videos were also created to inform teachers about the apps to supplement the webinars. The apps were discussed in radio programs to reach the teachers in remote areas.

One collaborative effort worth mentioning was the project with Grades 1 to 6 teachers of one local DepEd division where the teachers created their own video lessons with the aid of the numeracy apps. In the video, teachers taught a lesson and introduced the game-based math apps as a means to strengthen the math concepts discussed. This endeavor was very fruitful as it provided other teachers the much-needed material to use for their lessons. Other teachers may use the videos as starting points to create their own lessons.

One of these videos is a 5-min video demonstration [31] of a Grade 1 teacher employing *Quick Images* as a complementary activity to a lesson on number identification. Screenshots of the lecture are shown in Figure 5.1. The video was posted in a city schools division portal and is available for use by teachers who may want to integrate the app in their own blended learning classroom.



(a) (b)

(c) (d)

Figure 5.1 Screenshots of a video lesson integrating *Quick Images*

In the first half of the video, the teacher reviewed the counting numbers from 1 to 10 with the aid of static images of various objects found in school. She guided the students in identifying the number of objects shown in each image (Figure 5.1a). The second half of the video shows the teacher using *Quick Images* to practice or enrich the students' quick number identification skills. She first introduced *Quick Images* and explained its interface and important features (Figure 5.1b) before playing the *Quick Images* app herself (Figure 5.1c). For students who may have missed identifying the number of objects before they disappeared, she occasionally pressed the "show again" button that allows the app to redisplay the objects. Towards the end of the video, the teacher encouraged the students to independently play with the app and send her a screenshot of the page which displays their score (Figure 5.1d).

Teachers who created the videos and prepared lessons using the apps answered feedback/ evaluation forms. They found the apps interactive, easy to use, and can cater to beginning, average and advanced students. Moreover, they believe students will be able to explore and practice mathematical skills through the applications. Some commented that the apps are helpful and adaptable into a blended learning setting because it offered a strategy to make mathematics interesting and accessible when the teacher is not always present as in a physical classroom setting. One teacher commented "The advantage is really to the young learners, as it is appropriate not only in learning math concepts but improves their mood/ attitude in learning."

The apps are currently undergoing field-testing; observations of the apps' contribution to conceptual development would be presented during the conference.

6. Conclusion and Future Direction

In this paper the design of five numeracy apps for various learning competencies in Grades 1 to 6, as well as the framework for a statistical web app for Grades 1 to 11 is presented which is part of a continuing project funded by a national government agency, to create a digital mathematics learning environment for implementation in schools. Studies on Grades 7 to 10 apps also created under the project have shown that the apps offer additional opportunities to learn mathematical content even with minimal supervision [32]. Example, a statistical analysis of the students' scores after using the proving app *Prove It* even for a short period of time showed evidence of a significant improvement in proof-writing [33].

As of this writing, more apps from Grades 1 to 10 intended for other strands in the mathematical curriculum, such as Geometry and Algebra are being created and field tested for refinements.

Initial feedback from teachers and students for the numeracy apps for Grades 1 to 6 indicates their huge potential for mathematical learning. Moreover, the apps have been showcased and discussed by the team in some webinars [34, 35, 36, 37, 38] for grade school and high school educators. The responses of the teachers in these webinars imply their appreciation towards the availability of the apps as well as their interest and intent to use these in their classes. One teacher echoed the value of technology in teaching amidst the pandemic: "With pandemic or not, technology is always there to help us deliver the knowledge [t]hat our learners deserve to receive." Another teacher commented "Instead of playing the usual games offered online, these mathematical apps can be a very good substitute beneficial to young children… with proper guidance and supported with other resource…" while another teacher affirmed that "gamification of the lessons is the new trend in education." In particular, within a week after the webinar *Senso Eskwela at iba pa* [38] where *SEP* was formally launched, more than 50 teachers registered a combined total of more than 350 classes for participation to *SEP*. Moreover, the participants were asked to evaluate the apps *SEP, Quick Images, Grid Game, Ordering Game,* and *Frame Game*. Majority of the teachers have indicated that

these apps are easy to use, adaptable to a blended classroom, and can be catered to beginning, average, and advanced students.

The next step will be to examine more closely the educational effectiveness of these apps in facilitating student performance. Studies will also be carried out on how *SEP* impacts statistical learning. Among the planned activities in collaboration with DepEd schools in the next months will be for these purposes.

There is also the assessment database that is currently under construction. The database will store assessment data gathered within (or outside) this project for evaluation and analysis. It is anticipated that the database can be useful to teachers and researchers who seek to improve the mathematical learning of students. Research-based assessments used to track the mathematical performance of thousands of pupils in Australia [39] will inform the implementation of the assessments in the Philippine setting. It is anticipated that the research-based assessments will provide a richer description of the strategies used by Filipino learners than can be gleaned from the more traditional multiple-choice assessments such as the Philippine National Achievement Test (NAT). In particular, these assessments can determine whether Filipino students are merely imitating procedures demonstrated by their teachers or using their critical thinking to solve mathematical tasks.

The authors invite the readers to visit the project website at https://mathplusresources.wordpress.com/ where future updates and refinements to the developed resources will be made available.

Acknowledgements The authors would like to thank the Department of Science and Technology-Philippine Council for Industry, Energy, and Emerging Technology Research and Development (DOST-PCIEERD) and the University Research Council (URC), Ateneo de Manila University for the support of the development of the interactive resources discussed in this paper. Acknowledgement also goes to Dr. Ma. Mercedes T. Rodrigo and the Ateneo Laboratory for the Learning Sciences (ALLS) for providing technical assistance. Deep gratitude is extended to our app developers: Mr. Victor Antonio M. Ortega and Mr. Nigel Cargo (numeracy apps).; Mr. Marc Pison and the Logik team (*SEP*); and app designer Mr. Amiel Damian F. Justiniani. We also would like to acknowledge Ms. Mary Rose E. Villanda, Pleasant Hills Elementary School - School District of Mandaluyong for the video lesson featuring *Quick Images.*

References

[1] OECD (2019). PISA 2018 Results (Volume I): What Students Know and Can Do. Paris: PISA, OECD Publishing. Retrieved 10/4/2020 from https://doi.org/10.1787/5f07c754-en.

[2] Bergqvist, T. & Lithner, J. (2012). Mathematical Reasoning in Teachers' Presentations. *The Journal of Mathematical Behavior*, 31, 252-269.

[3] Cox, R. D. (2015). "You've Got to Learn the Rules": A Classroom-level Look at Low Pass Rates in Developmental Math. *Community College Review*, 43(3), 264-286.

[4] Nag, S., Chiat, S., Torgerson, C., and Snowling, M. J. (2014) Literacy, Foundation Learning and Assessment in Developing Countries: Final Report. *Education Rigorous Literature Review*. United Kingdom: Department for International Development.

[5] Wright, R. J., Martland, J., and Stafford, A. K. (2006). *Early Numeracy: Assessment for Teaching and Intervention. Second Edition*. Thousand Oaks, CA: Paul Chapman Publishing.
[6] Verzosa, D. (2015). Developing the Number Knowledge of Low-attaining First-to-fourth-graders. In C. Vistro-Yu. (Ed.), *Proceedings of the 7th ICMI-East Asia Regional Conference on Mathematics Education* (pp. 217-224). Quezon City: Philippine Council of Mathematics Teacher Educators (MATHTED), Inc.
[7] Reston, E. and Bersales, L. G. (2008). Reform Efforts in Training Statistics Teachers in the Philippines: Challenges and Prospects. In C. Batanero, G. Burrill, C. Reading, and A. Rossman (Eds.), *Joint ICMI/IASE Study: Teaching statistics in school mathematics. Challenges for teacher education. Proceedings of the ICMI study 18 and 2008 IASE round table conference.* Monterrey, Mexico: ITESM.
[8] Goodwin, K. and Gould, P. (2014). Race in the Outback: Investigating Technology Designed to Support Number Development in a Preschool Serving an Under-Resourced Community. In J. Anderson, M. Cavanagh & A. Prescott (Eds.), *Curriculum in focus: Research guided practice (Proceedings of the 37th annual conference of the Mathematics Education Research Group of Australasia)* (pp. 247–254). Sydney, Australia: MERGA.
[9] Kim, P., Buckner, E., Kim, H., Makany, T., Taleja, N., and Parikh, V. (2012). A Comparative Analysis of a Game-based Mobile Learning Model in Low-socioeconomic Communities of India. *International Journal of Educational Development*, 32, 329–340.
[10] Highfield, K. and Goodwin, K. (2013). Apps for Mathematics Learning: A Review of 'Educational' Apps from the iTunes App Store. In V. Steinle, L. Ball & C. Bardini (Eds.), *Mathematics education: Yesterday, today and tomorrow (Proceedings of the 36th annual conference of the Mathematics Education Research Group of Australasia).* (pp. 378-385). Melbourne, Australia: MERGA.
[11] Siemon, D., Bleckly, J., and Neal, D. (2012). Working with the Big Ideas in Number and the Australian Curriculum: Mathematics. In B. Atweh, M. Goos, R. Jorgensen & D. Siemon, (Eds.), *Engaging the Australian National Curriculum: Mathematics – Perspectives from the Field* (pp. 19-45). Online Publication: Mathematics Education Research Group of Australasia.
[12] Courtney-Clarke, M. & Wessels, H. (2014). Number Sense of Final Year Pre-service Primary School Teachers. *Pythagoras,* 35(1)
[13] Sengul, S. (2013). Identification of Number Sense Strategies Used by Pre-Service Elementary Teachers. *Kuram Ve Uygulamada Egitim Bilimleri,* 13, 1965-1974.
[14] Wang, X., & Chen, G. (2010). Survey and Analysis of the Statistical Reasoning among High School Students in China and Dutch Wenfang. *Journal of Mathematics Education,* 2(1), 15-26
[15] Mohamed, M. & Johnny, J. (2010). Investigating Number Sense Among Students. *Procedia - Social and Behavioral Sciences,* 8, 317-324
[16] Yang, D., Li, M., & Lin, Chih-I. (2008). A Study of the Performance of 5th Graders in Number Sense and its Relationship to Achievement in Mathematics. *International Journal of Science and Mathematics Education,* 6, 789-807
[17] Thompson,I. and Bramald, R. (2002) An Investigation of the Relationship between Young Children's Understanding of the Concept of Place Value and their Competence at Mental Addition. Retrieved 9/5/2020 from https://www.atm.org.uk/write/MediaUploads/Journals/MT184/Non-Member/ATM-MT184-14-15-Extra.pdf

[18] Ellemor-Collins, D. and Wright, R. (2011) Developing Conceptual Place Value: Instructional Design for Intensive Intervention. *Australian Journal of Learning Difficulties*, 16.1, 41-63.
[19] David, I. P. and Maligalig, D. S. (2006). Are we Teaching Statistics Correctly to our Youth? *The Philippine Statistician,* 55(3-4), 1-28.
[20] Department of Education (2016). K to 12 Curriculum Guide: Mathematics. Retrieved 8/26/2020 from https://www.deped.gov.ph/wp-content/uploads/2019/01/Math-CG_with-tagged-math-equipment.pdf
[21] Fuson, K. C., Smith, S. T., and Lo Cicero, A. M. (1997). Supporting Latino First Graders' Ten-structured Thinking in Urban Classrooms. *Journal for Research in Mathematics Education,* 28, 738-766.
[22] Clements, D.H. (1999). Subitizing: What Is it? Why Teach it? *Teaching Children Mathematics*, 5(7), 400-405.
[23] Özdem, Ş. and Olkun, S. (2019). Improving Mathematics Achievement via Conceptual Subitizing Skill Training. *International Journal of Mathematical Education in Science and Technology*, 1-15.
[24] Siegler, R. S. and Lortie-Forgues, H. (2014). An Integrative Theory of Numerical Development. *Child Development Perspectives*, 8(3), 144-150.
[25] Siegler, R. S. and Ramani, G. (2008). Playing Linear Numerical Board Games Promotes Low-income Children's Numerical Development. *Developmental Science,* 11(5), 655-661.
[26] Ulbright, J. (n.d.). Arranged Ten Frames. Retrieved 8/29/2020 from https://www.geogebra.org/m/cb4CSBMj
[27] Silva, M. Place Value Activity Package. Retrieved 9/5/2020 from http://www.mathematicshed.com/uploads/1/2/5/7/12572836/place-value-activity-pack.pdf
[28] Connor, D. (2002). CensusAtSchool 2000: Creation to Collation to Classroom. In *Proceedings of the Sixth International Conference on Teaching of Statistics, Cape Town.* Voorburg, The Netherlands: International Statistical Institute.
[29] Aoyama, K., Watanabe, M., and Tamura, Y. (2013) Statistics Learning Environment for Students through Japanese CensusAtSchool Project. *IASE/IAOS Satellite* Retrieved 9/5/2020 from https://iase-web.org/documents/papers/sat2013/IASE_IAOS_2013_Paper_2.1.2_Aoyama.pdf?1402525000
[30] Davies, N., Richards, K., Aliaga, M., and Nichols, R. (2010) CensusAtSchool. *Significance* 7(4), 175. Retrieved 9/5/2020 from http://onlinelibrary.wiley.com/doi/10.1111/j.1740-9713.2010.00458.x/pdf
[31] Mathematics Teacher Association of Mandaluyong. (2020). *Pagbibilang mula 1 hanggang 10* [Video file]. Retrieved 9/5/2020 from https://www.facebook.com/watch/?v=354568545569833&extid=LBsxn6IpsNATl4zx
[32] De Las Peñas, M. L. A. N., Aberin, M. A. Q., Bautista, E. P., Francisco, F. F., Tabares, W. C., and Verzosa, D. M. B. (2018). *Development of Interactive Software and Teaching Guides for Grades 7-10 Mathematics*. Manila: DOST-PCIEERD.
[33] Verzosa, D. M. B., De Las Peñas, M. L. A. N., Aberin, M. A., and Garces, L. P. D. M. (2019). App-based Scaffolds for Writing Two-column Proofs. *International Journal of Mathematics Education in Science and Technology*, 50, 766-778.
[34] DepEd Tayo – Mandaluyong City. (2020). Webinar on the Use of Mathematical Apps for Blended Learning [Video file]. Retrieved 10/5/2020 from https://www.facebook.com/depedmandaluyongcity/videos/295266868265384/

[35] Mathplus Resources. (2020). Math Plus Apps for Blended Learning (Grade School) [Video file]. Retrieved 10/5/2020 from https://youtu.be/7-BIrCT5a3k
[36] GEG Quezon City. (2020). Virtual Orientation on the Use of Math Apps for Blended Learning [Video file]. Retrieved 10/5/2020 from https://www.facebook.com/GEGQuezonCity/videos/571343836879548/
[37] Mathplus Resources. (2020). Math Plus Apps for Blended Learning (High School) [Video file]. Retrieved 10/5/2020 from https://youtu.be/PGo2kq4JQYA
[38] ADMU School of Science and Engineering. (2020). Senso Eskwela Pilipinas at iba pa (Day 2) [Video file]. Retrieved 10/5/2020 from https://www.facebook.com/admusose/videos/767231277154400/
[39] Bobis, J., Clarke, B., Clarke, D., Thomas, G., Wright, R., Young-Loveridge, J., and Gould, P. (2005). Supporting Teachers in the Development of Young Children's Mathematical Thinking: Three Large Scale Cases. *Mathematics Education Research Journal,* 16, 27-57.

Ateneo de Manila University

Archīum Ateneo

Mathematics Faculty Publications | Mathematics Department

2021

Designing Performance Tasks in Mathematics Using Technological Tools

Ma. Louise Antonette N. De Las Peñas
Ateneo de Manila University, mdelaspenas@ateneo.edu

Debbie Marie Versoza
Ateneo de Manila University

Maria Alva Q. Aberin
Ateneo de Manila University, maberin@ateneo.edu

Agnes Garciano
Ateneo de Manila University, agarciano@ateneo.edu

Jumela F. Sarmiento
Ateneo de Manila University, jsarmiento@ateneo.edu

See next page for additional authors

Follow this and additional works at: https://archium.ateneo.edu/mathematics-faculty-pubs

 Part of the Mathematics Commons, and the Science and Mathematics Education Commons

Custom Citation

De Las Penas, M. L.A.N., Verzosa, D.M.B., Aberin, M.A.Q., Garciano, A.D., Sarmiento, J.F., Tolentino, M.A.C. (2021). Designing Performance Tasks in Mathematics Using Technological Tools. In W.C. Yang, D. Meade, M. Majewski (Eds.), Proceedings of the 26th Asian Technology Conference in Mathematics. Mathematics and Technology, LLC. https://atcm.mathandtech.org/EP2021/invited/21912.pdf

This Conference Proceeding is brought to you for free and open access by the Mathematics Department at Archīum Ateneo. It has been accepted for inclusion in Mathematics Faculty Publications by an authorized administrator of Archīum Ateneo. For more information, please contact oadrcw.ls@ateneo.edu.

Authors

Ma. Louise Antonette N. De Las Peñas, Debbie Marie Versoza, Maria Alva Q. Aberin, Agnes Garciano, Jumela F. Sarmiento, and Mark Anthony C. Tolentino

This conference proceeding is available at Archīum Ateneo: https://archium.ateneo.edu/mathematics-faculty-pubs/208

Designing Performance Tasks in Mathematics Using Technological Tools

Ma. Louise Antonette N. De Las Peñas
mdelaspenas@ateneo.edu
Department of Mathematics
Ateneo de Manila University, Philippines

Debbie Marie B. Verzosa
dmbverzosa@usm.edu.ph
Department of Mathematics and Statistics
University of Southern Mindanao, Philippines

Maria Alva Q. Aberin
maberin@ateneo.edu
Department of Mathematics
Ateneo de Manila University, Philippines

Agnes D. Garciano
agarciano@ateneo.edu
Department of Mathematics
Ateneo de Manila University, Philippines

Jumela F. Sarmiento
jsarmiento@ateneo.edu
Department of Mathematics
Ateneo de Manila University, Philippines

Mark Anthony C. Tolentino
mtolentino@ateneo.edu
Department of Mathematics
Ateneo de Manila University, Philippines

Abstract: *In the Philippines, the performance task is one of the major summative assessments in the K to 12 curriculum. This paper discusses how performance tasks may utilize mathematical apps within the context of blended learning. Guidelines on designing performance tasks, as well as the GRASPS framework, are discussed. Performance tasks that cut across various grade levels and strands of mathematics are presented. These involve divisibility (Numbers), integer and polynomial operations (Algebra), triangle centers (Geometry), and statistics (Statistics). The performance tasks described in this paper can provide an initial idea for the design of other summative assessments and contribute to the literature on the use of technological tools in assessment and evaluation.*

1. Introduction

The Covid19-pandemic has driven a shift of modalities in teaching and learning in schools worldwide. In the Philippines, the Department of Education has called for a blended learning modality for schools in lieu of face-to-face instruction in 2020. These modalities include printed modules,

radio, television, and the Internet. New challenges emerged from the sudden shift in educational practice, and some of these challenges relate to assessment. Traditional paper-and pencil assessment methods are no longer completely applicable. Schools are expected to adopt "assessment practices that can meaningfully support student development and respond to different contexts at this time" [1].

There is already a wealth of strategies for using technology in formative assessments even prior to the pandemic [2-7]. However, new schemes for summative assessments remain to be a key challenge for teachers, especially if these are to be done online [8]. This paper reports on the creation and design of performance tasks utilizing innovative technological tools such as mathematical applications as a potential strategy for summative assessment. Technology can provide new and more dynamic ways to face some mathematical tasks, with respect to paper and pencil methods, such as modeling real-life phenomena, making conjectures and proving properties based on experimentation, predicting, pattern-seeking [7, 9]. As Spector et al. [6, p. 60] said, "In addition to providing the means to support personalized learning and a smart education system responsive to the needs of learners and teachers and their learning environments, new technologies can support key 21st century skills … – notably critical thinking and problem solving."

In the Philippines, one mandated summative assessment is in the form of a performance task, which comprises 40% of students' final grade [10]. This paper reports on the design and construction of sample performance tasks on numbers, algebra, geometry, and statistics using mathematical apps that a mathematics teacher can utilize at appropriate junctures in the curriculum. The use of mathematical apps in the design of each task is deliberate and aims to aid the students to carry out calculations that will be used in the analysis and solutions of the problems indicated in the tasks.

2. Performance Tasks

Philippine policy guidelines [10] define performance tasks as tasks that allow students to demonstrate their knowledge in various ways. These are typically done over a period of time and the purpose is to provide students the opportunity to integrate their understanding of the main topics covered during the academic quarter (about ten weeks). There is a wide range of possible formats, such as skills demonstration, group work, multimedia presentations, research or investigative projects, or written output.

The assessment guidelines were refined in response to the pandemic [1]. Assessment exemplars and rubrics were provided, together with suggestions for students with low, medium, or high access to technology. For mathematics, suggested performance tasks include the following: constructing graphs from conducted surveys, multimedia presentations, outdoor math, probability experiments, problem posing, reasoning and proof, using manipulatives to show math concepts and solve problems using measurement tools and devices. Performance tasks may also integrate multiple competencies not only in mathematics but also in the other subject areas.

Wiggins and McTighe's GRASPS model [11] is a recommended guide in carrying out performance tasks. This model is applicable when the performance task involves a real-life situation. GRASPS stands for: Goal (a task must state the problem or challenge to be resolved), Role (a task must explain who students are in the scenario and what they are being asked to do), Audience (a task must specify who the students are solving the problem for, who they need to convince of the validity and success of their solution for the problem), Situation (a task must provide the context of the situation and any additional factors that could impede the resolution of the problem), Product, Performance, and Purpose (a task must explain the product or performance that needs to be created and its larger purpose, and Standards and Criteria for Success (a task must dictate the standards that must be met and how the work will be judged by the assumed audience).

3. App-based Performance Tasks

The performance tasks in this section utilize apps that are part of Mathplus resources (mathplusresources.wordpress.com), which are products of an ongoing government supported project "Technology Innovations for Mathematical Reasoning, Statistical Thinking and Assessment" by the Department of Science and Technology- Philippine Council for Industry, Energy and Emerging Technology Research and Development (DOST-PCIEERD). All apps can be accessed through the website https://mathplusresources.wordpress.com/.

3.1 Grades 1 to 6

A divisibility rule is a shorthand way of discovering whether a given number is divisible by a fixed divisor without performing the division, usually, just by examining its digits. Knowledge of divisibility and factoring rules can facilitate computations needed for more advanced topics such as fraction operations. This topic is listed in the most essential learning competencies [12] for students in Grades 4 and 5. Students are expected to solve both routine and non-routine problems using rules on factors, multiples, and divisibility of numbers.

Integrating technologies into learners' activities has the potential to keep them more engaged in studying divisibility rules and factoring. Hence, the assessment activities and performance tasks on divisibility and factoring enumerated in this section begin with the use of technology.

One app designed to facilitate learning number sense is the *Divisibility Game*. Students may choose which divisibility rules they want to practice on and then either play to gain mastery, increase speed in answering questions, or simply engage in drill work. In the Divisibility Game, students can choose the divisor, for example 3, 6, 9 and the number of items (Figure 3.1 (a)). Then students select numbers that are divisible by the divisor.



Figure 3.1 Screenshots of the *Divisibility Game*

For example, in Figure 3.1 (b), among the numbers, which are divisible by 3? Students may also choose to play with 12 four-digit items such as in Figure 3.1 (c) or with 16 four-digit items such as in Figure 3.1 (d).

Another app is the *Factor Game*. Through the game, students demonstrate understanding of prime and composite numbers. The game addresses the need for the students, from various grade levels, to identify factors and multiples of a given number. Students select a number and all its factors. In the easiest level (Figure 3.2 (a)), numbers 1 to 30 are placed on a 5×6 grid arranged in order while in the 1-60 level (Figure 3.2 (b)), randomly selected numbers from 1 to 60, not in order, are placed on the 6×7 grid. Students may also select a time limit. The time limit can push students to strategize in selecting the numbers to get a higher score within the available time.



(a)



(b)

Figure 3.2 Screenshots of the *Factors Game*

In preparation for the performance task, a series of informal activities and formative assessments may be performed. Students may initially explore each game option in the app in a form of free play. At this point, the idea is to engage the students' interest and motivate them to reach as many levels as possible. Teachers may monitor their students by asking them to send screenshots or a written record of their progress. The students' outputs may enable teachers to see some learning patterns and provide timely feedback and interventions when necessary.

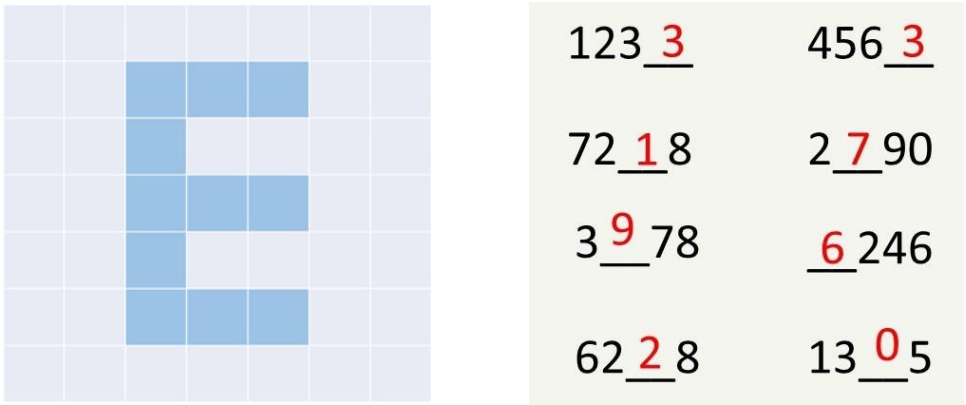
Students can also perform more structured activities. As a first activity, students can perform a "Shading Activity" using the divisibility rule. Given a grid of numbers, students are asked to shade all numbers divisible by 3 as in Figure 3.3 (a). In the end, the shaded numbers will form a familiar letter of the alphabet, object or an animal, as in Figure 3.3 (b).



(a) (b)

Figure 3.3 Shading activity

A sample performance task related to this, is one where the teacher asks the students to design their own puzzles. A student creates a puzzle by filling-up the grid (Figure 3.4 (a)) with two-digit numbers so that the numbers in the shaded squares are divisible by a number of their choice and the numbers in all other squares are not.



(a) (b)

Figure 3.4 (a) A sample grid (b). Fill in the blanks activity

A variation to this task requires students to create a puzzle by designing their own image, for example, an arrow, animal, or any object so that when students shade the correct squares with numbers divisible by their chosen number, the students will end up with the image. Furthermore, students may also use grids of different sizes, with a mixture of two-digit and three-digit numbers. In addition, in this task, teachers may require students to come up with three or more grids where the images would follow a story. At the end of the task, students write their reflection. Sample guide questions are i) How did you choose the numbers to be placed on the grid? ii) Any difficulties encountered? Explain these challenges. iii) What did you learn from this task?

Another activity is the "Fill in the Blanks Activity" where students fill in the blanks with numbers that will satisfy the given conditions on divisibility. For example, students fill in the blanks so that the 4-digit number is divisible by 9 as in Figure 3.4 (b). At the end of the activity, teachers may ask if there are other solutions and ask students to explain. A sample performance task related to this is requiring students to answer more difficult non-routine problems such as those appearing in national

competitions. Sample problems are: i) If the nine-digit number A1234567B is divisible by 45, determine all possible values of A and B (2019 Metrobank-MTAP-DepEd Math Challenge - Grade 6 National Finals), ii) How many ways are there to arrange four 3's and two 5's into a six-digit number divisible by 11? (19th Philippine Mathematical Olympiad - National Oral Stage). At the end of the task, students explain the steps they followed to solve the problem. Although this performance task does not involve a real-life situation that is central to the GRASPS model, the task is aligned with other alternatives indicated by the official guidelines [10] because it provides students an opportunity to integrate their knowledge of divisibility in an environment of inquiry and problem solving.

3.2 Algebra

Addition and subtraction of integers and polynomial expressions are fundamental algebraic skills that students need to master. Different models have been utilized to represent the concepts underlying integer operations [13, 14]. However, there seems to be no app that covers polynomial expressions. *AlgeOps* is an interactive visual mathematical app [15] which makes use of pictorial and symbolic representations of integers and polynomials (see Figure 3.5 for screenshots from the app). The app incorporates both the neutralization and number line models [16] which offer a more holistic representation of integers and eventually, strengthens instruction on integer and polynomial operations.

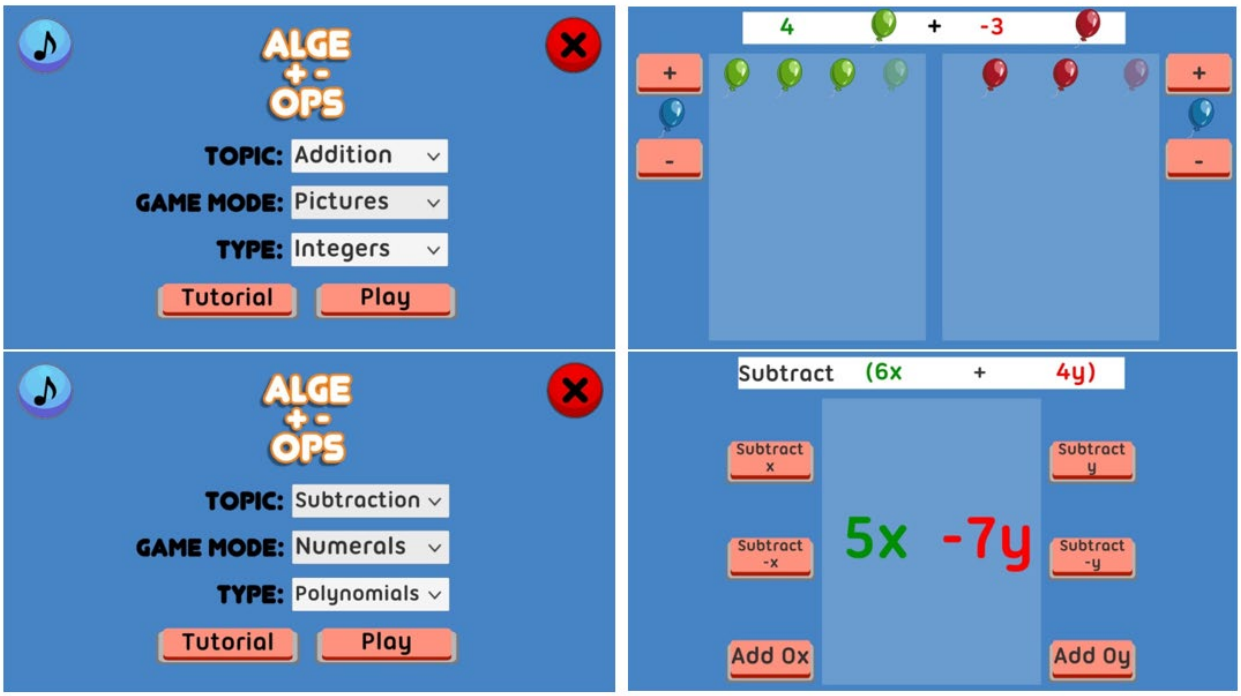
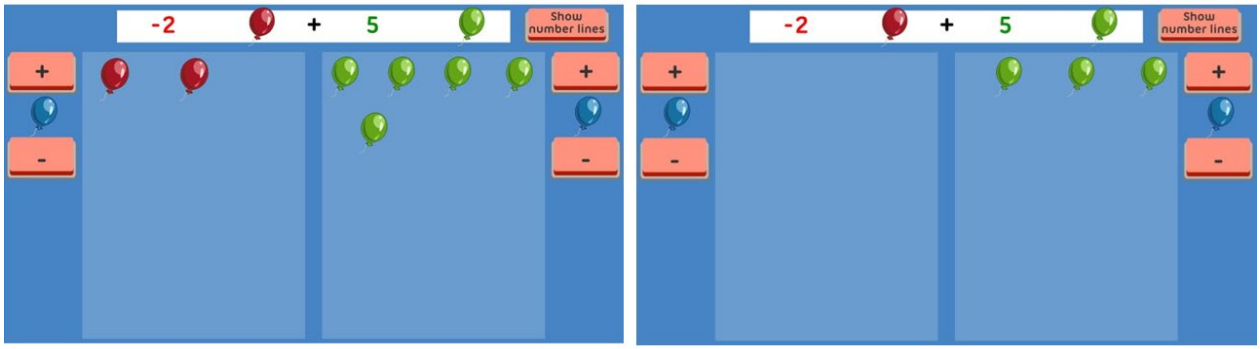


Figure 3.5 Screenshots of the different modes of an updated version of *AlgeOps*

To facilitate the operations, students use the different buttons found on the left and right panels of the screen to construct the expressions. For example, Figure 3.6 (a) illustrates the addition problem of (-2) + 5. It is represented by two red balloons and five green balloons. These visuals are displayed when the (-) left button is clicked twice and the (+) right button is clicked five times. Moreover, the

three green balloons representing the correct answer (Figure 3.6 (b)) remain after the animation of the neutralization method.



(a) (b)

Figure 3.6 A representation of the addition problem: (-2) + 5 = 3

Performance tasks are usually open-ended and often yield more than one correct answer. This characteristic is manifested in the following task:

Students are to construct different situations where the sum of the balloons is the same as the given diagram. The diagram shows two green balloons plus two green balloons (or $2x + 2x$) and three green apples plus two red apples (or $3x - 2y$). The diagram (Figure 3.7) can be unique to each student since it is generated by the device used by the student.

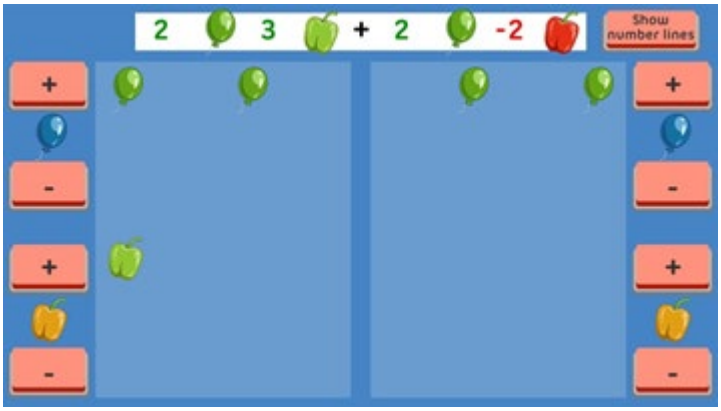


Figure 3.7 Addition of polynomials

Indeed, the task involves providing different values of *a, b, c*, and *d* that satisfy the equation $ax + by + cx + dy = 4x + y$. As such there will be numerous responses that students can provide satisfying the equation. Additional constraints on the values of *a, b, c*, and *d* can be added to make it more challenging. For example, the values must range from -20 to 20 and the values need not be integers. In consideration of guidelines for assessment, this performance task can involve competencies in another subject area. For example, the student's output requiring creative storylines and visual representations of the polynomials can be used competencies in writing and the arts.

The use of *AlgeOps* in designing the performance task provides a digital medium that replaces the usual pen and paper format of assessment. The generation of unique exercises to students illustrates

its efficiency and the animation of the neutralization method provides a dynamic representation of the addition and subtraction of integers and polynomials.

3.3 Geometry

One of the advantages afforded by technology in performance tasks in geometry is the dynamic geometry environment [9]. As will be discussed here, a digital geometry environment in assessment can develop mathematical and technological competencies and connect mathematical ideas to everyday experiences and to other disciplines [17]. It can also promote mathematical and technological competencies, such as developing and applying new mathematical knowledge through problem solving; developing mathematical reasoning, developing visualization skills to assist in processing information, making connections and solving problems, and generalizing and exploring properties of objects [7]. In addition, the feedback provided by a dynamic geometry environment supports interaction during assessment [7, 18].

One example of a performance task for Geometry in Grade 7 employing the GRASPS model is a real-life application in which students gain experience and discover the significance of triangle centers such as the centroid, incenter, circumcenter and orthocenter. This can be facilitated with a *Geogebra* applet "Bisectors of Triangles" that allows the student to explore the various centers (Figure 3.8).

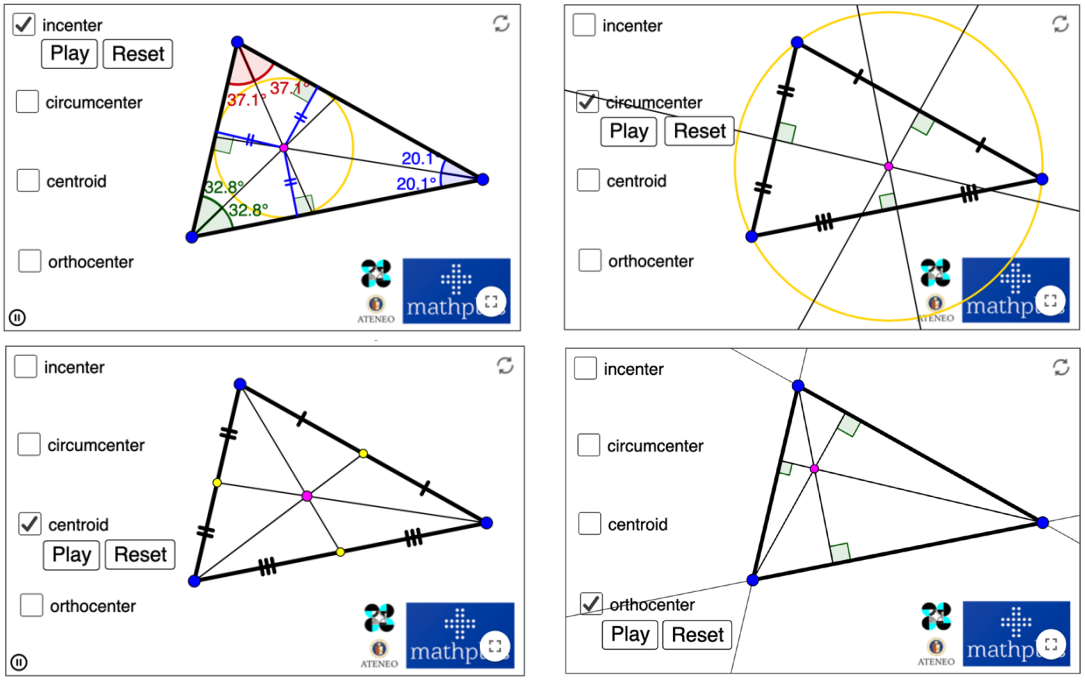


Figure 3.8 Screenshots from the *Geogebra* applet: Bisectors of Triangles

The GOAL of the performance task is for the students (their ROLE as a research team) to present and discuss a proposal to the Chief Executive Officer (AUDIENCE) of a local amusement park, *Enchanted Kingdom*, regarding the best possible location of a snack stand in the park so that it will be strategic to the three most popular rides in the park (SITUATION). The final output (PRODUCT) is a multimedia presentation (using a presentation software, video demo, poster) that showcases i) the location of the snack stand such that it is central to the three main popular rides in the park and how

this was determined, ii) maps and pictures to support the recommendation, and iii) the advantages and disadvantages of the location, supported by sound geometric principles. The multimedia presentation will be one session (in an online environment, this will be one synchronous session) where the students will present the proposal to the teacher.

The performance task is divided into four parts.

1. Students will work in groups. To get students to start thinking about this task, some questions posed could be: “Have you ever visited the amusement park?”, “What was your experience in getting a snack in between rides?”, “Where would be a location that would be "central" to most customers?”. The location of the snack stall is critical in order to attract more potential customers. Using a Google Form, students then conduct a survey among classmates and friends of the top three favorite/popular rides in the park.
2. In order to be guided with regards to the strategic location of the stall in the park, explorations on the definition and properties of the various four (4) centers of a triangle will be carried out using the *Geogebra* app “Bisectors of Triangles” (see Figure 3.8) which will be given to the students. This is a very important part of the task where students explore the various centers by using the “drag feature” of the app. By being able to drag the vertices of a triangle, students develop visualization skills, which are important in processing information. For example, some key concepts and properties that can be explored for the incenter using the *Geogebra* app are: the incenter is the center of the triangle’s inscribed circle; the incenter is always inside the triangle, the perpendicular segments from the center to sides of the triangle serve as radii of the inscribed circle. The incenter can be verified for various types of triangles.
3. For the discussion/analysis part, students will consider the four different types of circle centers when solving this task. They will also need to clearly establish the assumptions that are necessary in order to come to a solution, i.e., how did you define “best site of the stand,” etc.

   In the proposal analysis, the student justifies the appropriate use of the triangle center depending on its application. (i) Is it important that the center be equidistant from the three vertices or sides of the triangle? ii) Should it be that the center, when connected by a line segment to each location (vertex) creates three portions that are equal in area? iii) Should the circle be circumscribed about or inscribed in the circle? iv) Is this choice realistic in a given situation?

   Through the dynamic geometry software, the students can employ trial and error techniques, interpret the behavior of the triangle and its parts to make inferences about it, and test the validity of their conjectures. The students’ conjectures can be communicated through email or Messenger app for quick feedback as they go through the task and seek confirmation of their work.
4. Students will then find evidence of the distance between the food stand and the three rides using Google Earth (see Figure 3.9) or some other application.

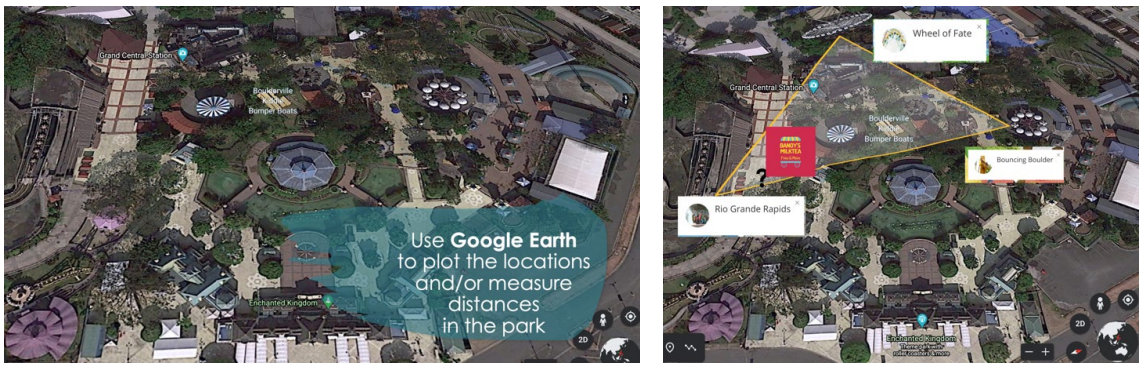


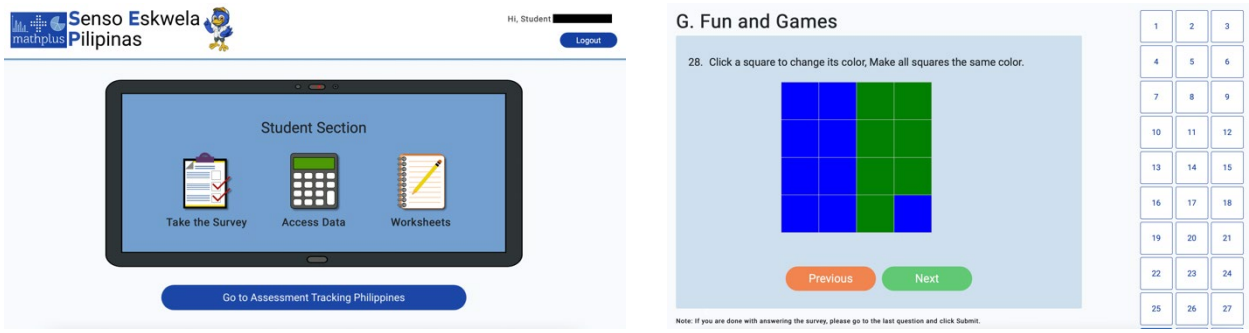
Figure 3.9 Screenshots from the *Geogebra* applet: Bisectors of Triangles

For the STANDARDS part of the GRASPS framework, the following criteria may be used: i) data/assumptions used to arrive at the location of the stand (survey, geometric explorations using the app), ii) geometric principles used to arrive at location, iii) accuracy of the geometric solution, iv) supporting figures/screenshots from app/tables, and v) presentation (video, PowerPoint, poster) and delivery.

3.4 Statistics

The role of context in learning and teaching statistics has consistently been emphasized in the literature. For instance, according to Cobb and Moore [19, p. 801], "Statistics requires a different *kind* of thinking, because data are not just numbers, they are numbers with a context." Moreover, the Pre-K-12 Guidelines for Assessment and Instruction in Statistics Education II (GAISE II) [20, p. 11] indicates that assessments "must require students to use statistical reasoning with context and variability at all stages of the statistical problem-solving process." In this section, in light of the role of context in Statistics Education, the design of performance tasks using data from the web-based application *Senso Eskwela Pilipinas* (SEP) is discussed. The SEP app provides teachers and students not only contextualized but also authentic and relatable data. The benefits afforded for the exploration and transformation of data with digital technology reveals fresh approaches to analyzing statistics [21].

SEP is "an online platform for gathering, storing, and accessing readily available and relatable data for learning and teaching statistics" [22, p. 174]. It is a web application patterned after *CensusAtSchool,* which was first established in the UK in 2000 with the "dual thrust to enliven data handling activities within the classroom while also educating children about the principles and processes involved in conducting a census" [23, p.1]. In SEP (Figure 3.10 (a)), students anonymously log in to the platform (freely available at mathplusresources.com) to answer a 31-item survey (Figure 3.10 (b)) involving questions on different topics such as food, hobbies, and interactive tasks. The students' responses become part of the SEP database, to which they have access. Students and teachers can retrieve the data of their own class/es or random samples from the entire database.



(a) (b)

Figure 3.10 Screenshots from the SEP platform: (a) The landing page for student accounts, (b) A question from the SEP survey

It is anticipated that students' use of data from the SEP database in performance tasks can increase their engagement and interest as "the sense of belonging that participating pupils feel, purely because they know their own responses will become part of a database of responses from their peers, is fascinating and motivating for them" [24, p. 175]. Another clear advantage is the ready access that students have to the data from the SEP database. In situations in which the actual data collection process is difficult or is not integral to the objectives of the task, readily available data will prove to be convenient.

The performance task *From SEP to Infographic* discussed below recognizes the aforementioned role of context and adapts the GRASPS model. It addresses the following Grade 7 Statistics competencies in the Philippine K-12 curriculum [12, pp. 229-230]: "poses real-life problems that can be solved by Statistics, gathers statistical data, organizes data in a frequency distribution table, uses appropriate graphs to represent organized data: pie chart, bar graph, line graph, histogram, and ogive, draws conclusions from graphic and tabular data and measures of central tendency and variability, draws conclusions from graphic and tabular data (and measures of central tendency and variability)."

The task involves the creation of an *information graphics* or *infographics*, which has become more common in recent years. These infographics, which are visual representations of information, are mainly used to present complex data in a form that is more understandable and, at times, more impactful. This performance task allows learners to choose from the varied data available in the SEP database and use these data to create their own infographics. The GOAL is for the learners to retrieve data from the SEP database and organize, process, and analyze these data so that they can create an infographic on their chosen topic.

The students are given the following setting and instructions for the performance task: "In this task, you will play the ROLE of a researcher and writer for a social media page that mainly posts interesting information (e.g., their hobbies, activities, interests) about the Filipino youth. The administrator (or admin) of the social media page is asking the page's researchers and writers to submit new infographics (PRODUCT) that can be posted on the page. Naturally, the admin would like to receive submissions that have the potential for increased engagement from the page's followers (AUDIENCE). Moreover, the infographic should be based on real data, so the admin has advised the use of the SEP database.

As one of the researchers and writers, you are now faced with the following tasks (SITUATION):

1. Determine a suitable topic of the infographics based on the available data in the SEP database.
2. Process and organize the data into graphs or charts.
3. Analyze the data and the graphs/charts.
4. Design and create an infographic, with the graphs/charts integrated in a creative or meaningful way, that presents clear, concise, and interesting information on the chosen topic."

The students are also given the following criteria for grading (STANDARDS): i) topic of the infographic, ii) correctness of graphs or charts, iii) organization of the infographic, iv) design and creativity.

The performance task using the SEP platform offers a number of advantages. First, since the SEP database contains a large amount of readily available and varied data, students are given more space to think creatively in deciding the topic and direction of their performance task. This aspect is enhanced further by the fact that the students also answered the SEP survey themselves. Because the data is relatable to them, the students may easily develop a curiosity towards how their responses compare to others. Moreover, the Access Data feature (Figure 3.11) of the SEP platform offers the students flexibility and allows them to make their own decision/s in terms of i) using their own class' data or a random sample from the database, ii) which questions in the SEP survey they want to work with, iii) which grade level/s they should focus on, and iv) how small or large a sample size they

should use. These allow them to ask themselves questions like “Do we use data only for Grade 7, or Grades 7 to 10? Do we do a comparison between grade levels? Do we try to relate the data from two SEP questions?”

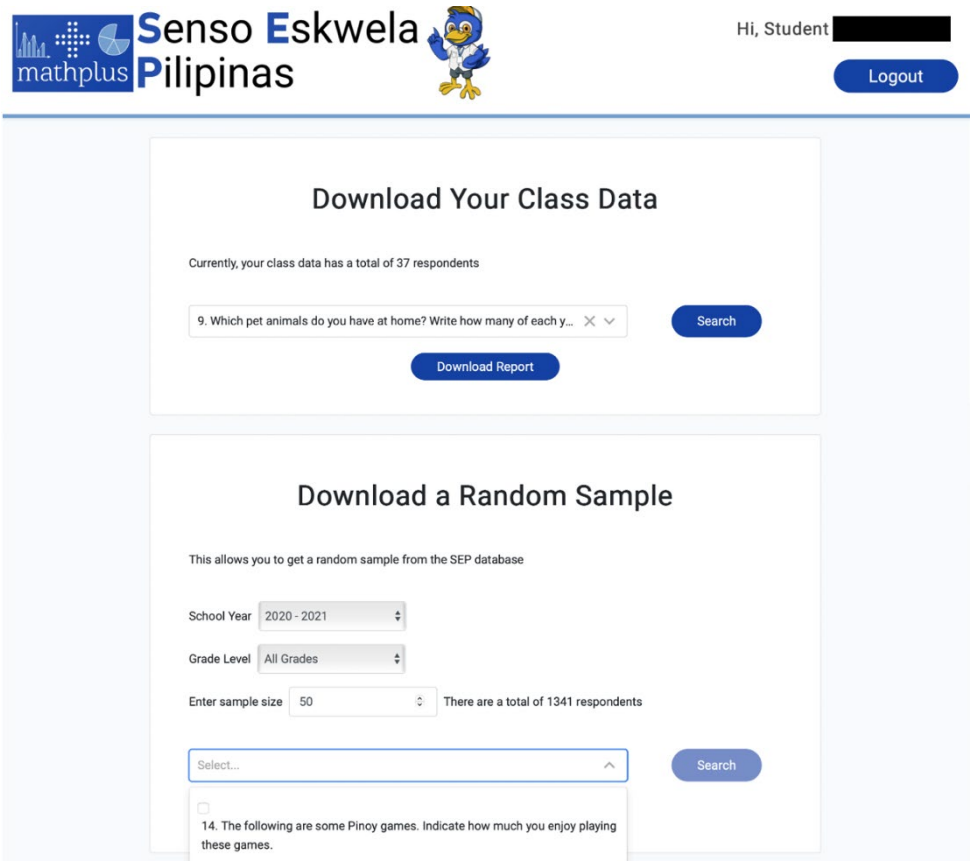


Figure 3.11 The Access Data feature of the SEP platform

Given sufficient time, students will also be able to create multiple infographics, from which they can choose their best work for submission. With the SEP database, this is possible for them without having to repeat the data collection process. Finally, accessing and processing data from the SEP database mirror steps in doing statistical analyses using data from actual databases. Students get raw data from the SEP database, then organize and process them using the skills that they have learned. This results in a more authentic context for the performance task.

Moreover, conforming to one of the thrusts of the Department of Education’s Order No. 31 s.2020 [1], the performance task can integrate competencies in other subject areas. For example, the students may be asked to accompany their infographic with a feature article in English or in Filipino. The article may serve as an assessment for their written communication classes. The design aspect of the infographic may also be used as an assessment for their arts class. Lastly, their output may also be used as an assessment for their Understanding Culture, Society and Politics class if the possible topics of the infographic are restricted appropriately.

The SEP database naturally leads to a number of performance tasks. The infographics task is suitable for Grade 7, but more tasks can be designed to assess more advanced statistical concepts. For example, in *Hypothesis Testing Using SEP Data*, which is designed for Grade 11 classes, students play the role of researchers for a non-profit organization that promotes the health and welfare of

Filipino youth. After performing data analysis and hypothesis tests, they are to prepare a written report for the officers of the organization to help in their planning of programs and campaigns. The complete details of this performance task are available in the SEP website (https://mathplusresources.wordpress.com/senso-eskwela-pilipinas-for-grades-1-11-statistics/).

Finally, the use of SEP in performance tasks offers a possible avenue for doing effective online assessment. In addition to the elements mentioned above, these performance tasks can be easily implemented in an online setting. Teachers can easily communicate the instructions to students and receive students' submissions online (e.g., via learning management systems, email, or official/school-authorized messaging platforms). In the same way, students can conveniently retrieve their data sets from the online SEP database. Moreover, depending on the involved topic/s, students may opt to use freely available online tools (e.g., a spreadsheet application) to organize and process their data.

4. Conclusion and Future Direction

Designing mathematics performance tasks for summative assessments must be adapted to the changes brought by the shift to a blended learning modality of education. The summative assessments in the form of performance tasks described in this article were designed to address the urgent educational difficulties driven by the Covid-19 pandemic and the need of the teachers to create forms of assessment while in a blended learning modality. The design of these tasks considered alignment with curriculum goals so that integrity of assessments is still met.

This study presented the design of mathematics performance tasks that creatively make use of mathematical apps to support student learning. The sample tasks address the need of making assessments in four key content areas: Numbers, Algebra, Geometry and Statistics. This set of tasks can provide teachers in basic elementary, junior and senior high school with creative ideas on designing performance tasks in the future. The designs of all performance tasks discussed in this paper considered the creation of an environment of inquiry and the integration with other subject areas whenever applicable. Further, the task designs for Geometry and Statistics were guided by the GRASPS framework by Wiggins and McTighe [11].

While these apps were designed to help students gain more insight and develop stronger mathematical skills, this paper described how the apps can also be integrated into performance tasks that teachers can assign their students. The performance tasks were presented in the webinar for Philippine teachers *Amplifying Development of Assessment and Performance Task Skills with Mathplus Resources.* Although the performance tasks and apps were designed for Filipino learners, these can assess learning outcomes that are present in many other national curricula as well. As a goal, the authors affirm the recommendations of [6] for an "open assessment repository (OAR) or an education observatory" that addresses the need for challenging but engaging technology-assisted performance tasks. The tasks designed by the authors can form part of this repository.

Acknowledgements The authors would like to thank the Department of Science and Technology-Philippine Council for Industry, Energy, and Emerging Technology Research and Development (DOST-PCIEERD) and the University Research Council (URC), Ateneo de Manila University for the support of the development of the performance tasks and interactive resources discussed in this paper. Deep gratitude is extended to our app developers Mr. Victor Antonio M. Ortega, Mr. Nigel Benedict D. Cargo, Mr. Jose Teodoro P. Lacson, and Sparksoft Solutions Inc. We also thank Mr. Amiel Damian F. Justiniani for the apps' art assets. Mr. Marc Pison and the Logik team for the *SEP.*

We are indebted to Mr. Restituto I. Rodelas, Education Program Supervisor of Department of Education-School Divisions Office Mandaluyong City, for his assistance.

References

[1] Department of Education (2020). Interim Guidelines for Assessment and Grading in Light of the Basic Education Learning Continuity Plan. Retrieved 8/1/2021 from https://www.deped.gov.ph/wp-content/uploads/2020/10/DO_s2020_031.pdf

[2] Black, P. and Wiliam, D. (2009). Developing the Theory of Formative Assessment. *Educational Assessment, Evaluation and Accountability* 21, 5.

[3] Boud, D., Cohen, R., and Sampson, J. (1999). Peer Learning and Assessment. *Assessment and Evaluation in Higher Education*, 24(4), 413-426.

[4] Cusi, A., Morselli, F., and Sabena, C. (2017). Promoting Formative Assessment in a Connected Classroom Environment: Design and Implementation of Digital Resources. *ZDM - Mathematics Education* 49, 755-767.

[5] Pierri, A. (2020). Formative Peer-review Practices in Online Environment to Promote the Undergraduate Students' Mathematical Thinking. *Quaderni di Ricerca in Didattica* 8, 53-60.

[6] Spector, J.M., Ifenthaler, D., Sampson, D., Yang, L., Mukama, E., Warusavitarana, A., Dona, K.L., Eichhorn, K., Fluck, A., Huang, R., Bridges, S., Lu, J., Ren, Y., Gui, X., Deneen, C.C., San Diego, J., and Gibson, D.C. (2016). Technology Enhanced Formative Assessment for 21st Century Learning. *Journal of Educational Technology & Society*, 19(3), 58-71.

[7] Venturini, M., and Sinclair, N. (2017). Designing Assessment Tasks in a Dynamic Geometry Environment. In A. Leung and A. Baccaglini-Frank (Eds.), Digital *Technologies in Designing Mathematics Education Tasks* (pp. 77-98). New York: Springer.

[8] Bakker, A., Cai, J., and Zenger, L. (2021). Future Themes of Mathematics Education Research: An International Survey Before and During the Pandemic. *Educational Studies in Mathematics* 107, 1-24.

[9] Olsher, S., Yerushalmy, M., and Chazan, D. (2016). How Might the Use of Technology in Formative Assessment Support Changes in Mathematics Teaching? *For the Learning of Mathematics* 36(3), 11-18.

[10] Department of Education (2015). Policy Guidelines on Classroom Assessment for the K to 12 Basic Education Program. Retrieved 8/1/2021 from https://www.deped.gov.ph /2015/04/01/do-8-s-2015-policy-guidelines-on-classroom-assessment-for-the-k-to-12-basic-education-program/

[11] Wiggins, G. P., and McTighe, J. (2005). *Understanding by Design (2nd ed.)*. Upper Saddle River, NJ: Pearson Education, Inc. Print.

[12] Department of Education. (2020). K to 12 Most Essential Learning Competencies with Corresponding CG Codes. Retrieved 8/1/2021 from https://commons.deped.gov.ph/K-to-12-MELCS-with-CG-Codes.pdf

[13] Bolyard, J. and Moyer-Packenham, P. (2012). Making Sense of Integer Arithmetic: The Effect of Using Virtual Manipulatives on Students' Representational Fluency. *Journals of Computers in Mathematics and Science Teaching*, 31(2), 93-113.

[14] Khalid, M. and Embong, Z. (2020). Emphasizing Concrete Representation to Enhance Students' Conceptual Understanding of Operations on Integers. *Turkish Journal of Computer and Mathematics Education,* 11(3), 762-773.

[15] Verzosa, D.M.B., De Las Peñas, M.L.A.N., Aberin, M.A.Q., and Garces, L.P.D.M. (2018). App for Addition and Subtraction of Integers. *International Journal for Technology in Mathematics Education*, 25(4), 21-33.

[16] Stephan, M. and Akyuz, D. (2012). A Proposed Instructional Theory for Integer Addition and Subtraction. *Journal of Research on Mathematics Education,* 43, 428-464.
[17] Olive, J., Makar, K., Hoyos, V., Kor L.K., Kosheleva, O., and Sträßer, R. (2010). Mathematical Knowledge and Practices Resulting from Access to Digital Technologies. In C. Hoyles and J.-B. Lagrange (Eds.), *Mathematics Education and Technology-Rethinking the Terrain. New ICMI Study Series, vol 13* (pp. 133-177). Boston, MA: Springer.
[18] Lee, A.M.-S., Wong, K.-L., and Tang, K.-C. (2004). Exploring the Use of Dynamic Geometry Manipulative Tasks for Assessments. In S.-C. Chu, W.-C. Yang, T. de Alwis, and K.-C. Ang (Eds.), *9th Asian Technology Conference in Mathematics* (pp. 252-261). ATCM, Inc.
[19] Cobb, G.W. and Moore, D.S. (1997). Mathematics, Statistics, and Teaching. *The American Mathematical Monthly*, 104(9), 801–823.
[20] Bargagliotti, A., Franklin, C., Arnold, P., and Gould, R. (2020). *Pre-K-12 Guidelines for Assessment and Instruction in Statistics Education II (GAISE II)*. American Statistical Association.
[21] Forbes, S. and Pfannkuch, M. (2009). Developing Statistical Thinking: Teaching and Learning. In R. Averill and R. Harvey (Eds.), *Teaching Secondary School Mathematics and Statistics: Evidence-based Practice* (pp. 93– 128). Wellington: NZCER Press.
[22] De Las Peñas, M.L.A.N, Verzosa, D.M.B., Sarmiento, J.F., Tolentino, M.A.C., Loyola, M.L. (2020). Designing Mobile Apps to Promote Numeracy and Statistical Reasoning. In W.-C. Yang and D. Meade (Eds.), *Electronic Proceedings of the 25th Asian Technology Conference in Mathematics* (pp. 167-182). Mathematics and Technology, LLC.
[23] Connor, D. (2002). CensusAtSchool 2000: Creation to Collation to Classroom. In *Proceedings of the Sixth International Conference on Teaching of Statistics, Cape Town*. Voorburg, The Netherlands: International Statistical Institute.
[24] Davies, N., Richards, K., Aliaga, M., and Nichols, R. (2010). CensusAtSchool. *Significance* 7(4), 175. Retrieved 9/5/2020 from http://onlinelibrary. wiley.com/doi /10.1111/j.1740-9713.2010.00458.x/pdf

Ateneo de Manila University

Archīum Ateneo

Mathematics Faculty Publications | Mathematics Department

2020

Designing Mobile Apps to Promote Numeracy and Statistical Reasoning

Ma. Louise Antonette N. De Las Peñas
Ateneo de Manila University, mdelaspenas@ateneo.edu

Mark L. Loyola
Ateneo de Manila University, mloyola@ateneo.edu

Jumela F. Sarmiento
Ateneo de Manila University

Mark Anthony C. Tolentino
Ateneo de Manila University, mtolentino@ateneo.edu

Debbie Marie Versoza
Ateneo de Manila University

Follow this and additional works at: https://archium.ateneo.edu/mathematics-faculty-pubs

 Part of the Mathematics Commons, and the Science and Mathematics Education Commons

Recommended Citation

Ma. Louise Antonette N. De Las Peñas, Mark L. Loyola, Jumela F. Sarmiento, Mark Anthony C. Tolentino, Debbie Marie B. Verzosa (University of Southern Mindanao), (2020), Designing Mobile Apps to Promote Numeracy and Statistical Reasoning. Electronic Proceedings of the 25th Asian Technology Conference in Mathematics (eds Yang, W.C. & Meade, D.), null, 167-182.

This Article is brought to you for free and open access by the Mathematics Department at Archīum Ateneo. It has been accepted for inclusion in Mathematics Faculty Publications by an authorized administrator of Archīum Ateneo. For more information, please contact oadrcw.ls@ateneo.edu.

Designing Mobile Apps to Promote Numeracy and Statistical Reasoning

Ma. Louise Antonette N. De Las Peñas
mdelaspenas@ateneo.edu
Department of Mathematics
Ateneo de Manila University, Philippines

Debbie Marie B. Verzosa
dmbverzosa@usm.edu.ph
Department of Mathematics and Statistics
University of Southern Mindanao, Philippines

Jumela F. Sarmiento
jsarmiento@ateneo.edu
Department of Mathematics
Ateneo de Manila University, Philippines

Mark Anthony C. Tolentino
mtolentino@ateneo.edu
Department of Mathematics
Ateneo de Manila University, Philippines

Mark L Loyola
mloyola@ateneo.edu
Department of Mathematics
Ateneo de Manila University, Philippines

Abstract: *Even with major advancements in the field of mathematics education, many students still do not attain the learning competencies prescribed by official curricula. This may be partially explained by students' immersion in classrooms characterized by superficial strategies or rote-learning methods. This paper reports on the design of mobile applications (apps) developed by the authors as part of an ongoing project funded by a national government agency and intended to promote structural thinking and statistical reasoning. It describes the general features of the apps, as well as the pedagogical principles upon which the apps' designs were anchored on. These principles are grounded on research and established practices on number sense and statistical learning. Collaborations with the Philippine Department of Education for widespread implementation and sustainability are also discussed.*

1. Introduction

Multinational benchmarks of mathematics learning indicate that many students still had not attained the learning competencies prescribed by official curricula [1]. Possibly this is because mathematics classrooms in several countries promote rules and procedures with little attention to underlying mathematical concepts [2, 3, 4]. Without effective intervention, children can remain weak in basic numeracy or number recognition [5]. For example, in the Philippines, elementary school children's place value knowledge is limited to knowledge of labels (i.e., hundreds, tens, ones) rather

than on quantity [6]. A similar problem occurs in the learning of statistics, where the focus is on computation rather than on statistical literacy or reasoning skills [7].

A potential solution is offered by mobile technology [8], which can provide a sustainable educational alternative for mathematical learning [9]. However, a large number of available apps are basically glamorized worksheets which are mostly of the "drill and practice" type [10]. These apps may just perpetuate the situation where rote learning is the norm. Thus, there is a need to design apps that are informed by theories emerging from the wealth of knowledge gained from research on numeracy and statistical learning. This paper reports on five purposely designed numeracy apps for Grades 1 to 6; and one web app, a statistical database for Grades 1 to 11, to develop these two broad areas of mathematics.

The apps are products of an ongoing supported project "Technology Innovations for Mathematical Reasoning, Statistical Thinking and Assessment" by the Department of Science and Technology-Philippine Council for Industry, Energy and Emerging Technology Research and Development (DOST-PCIEERD). The apps acquired additional relevance due to the urgent educational difficulties driven by the Covid-19 pandemic and the projected adaptations that teachers or learners may face as they transition into blended learning. Although the apps were designed for Filipino learners, they address learning outcomes that are present in many other national curricula as well.

This paper is divided into several parts. The next section describes some general considerations that guided the design of the apps. The third section presents the pedagogical basis for the numeracy apps and the web statistical learning app. The last section presents the collaboration with the Philippine Department of Education (DepEd).

2. Number Sense and Statistical Learning

This section discusses the rationale for designating number sense and statistical reasoning as target educational outcomes of the apps. These two broad areas of mathematics can be considered "big ideas" because they (1) consist of foundational concepts that are necessary for more advanced learning, (2) encompass a network of concepts and strategies, (3) provide an organizational structure for further learning, and (4) are apparent in learners' activity [11]. Thus, support for these "big ideas" or fundamental ideas is called for. Further, many of the existing mathematical apps overlook much of the current research about mathematical learning [10].

There are several countries where reports of poor number sense and statistical reasoning exist (see [12, 13, 14, 15, 16] and the references therein). However, since the apps were designed through a funded project by the Philippine government to develop innovative tools in supporting the implementation of the Department of Education's K to 12 curriculum, it was a priority to design the apps based on published research on the mathematical achievement of Filipino learners. Nevertheless, these apps would also be of interest to mathematics educators and researchers from many other countries with similar educational contexts.

Number sense is often neglected in Philippine classrooms. For example, conceptual place value considers two distinct ideas: *quantity value* (knowing that 45 is 40 and 5) and *column value* (knowing that the 4 in 45 represents 4 tens) [17, 18]. However, Filipino students do not seem to fully acquire these two fundamental concepts because their knowledge is often limited to labels (e.g., in a 3-digit number, the middle digit is called the "tens place") [6]. This outcome suggests that the treatment of place value in Philippine classrooms is on memorizing the labels "hundreds, tens, and ones" rather than on connecting these labels to number quantity (quantity value) or the quantity represented by each digit (column value). Possibly, this poor understanding of place value is the reason why Filipino students cannot apply mental strategies to solve tasks such as 208−10, where regrouping is involved

[6]. In view of this situation, the apps designed in this project focus on conceptual place value. These apps are different from previously designed apps for Filipino students, where traditional place value instruction and the learning of standard algorithms are emphasized.

Statistical learning is another major concern in the Philippines. The Philippine K to 12 program specifies a very rigorous statistics curriculum. The K to 12 program was implemented in 2012, with the spiral approach as one of its new elements. Whereas previously, statistics was taught in only a few grade levels, in the new program, all teachers from Grades 1 to 11 are expected to teach statistics. The problem is, statistics remains one of the most difficult topics to teach, and even local tertiary level textbooks are replete with statistical misconceptions [19]. The database for statistics learning seeks to address this problem by providing teaching guides that can support teachers who themselves have limited background in statistics. Further, in the spirit of Census at School programs in other countries, the database aims to collect authentic data so that Filipino students would find additional relevance in what they are learning and a better grasp of understanding statistical concepts. An additional obstacle is that in the Philippines, statistics is the last topic in the curriculum of each grade level from Grades 1-11. Any delay in curriculum implementation, by calamities such as floods or earthquakes, will mean that statistics is cut short or not taught at all. With the database for statistical learning, students and teachers may continue the learning of statistics through synchronous or asynchronous activities through the worksheets and teaching guides provided within the database.

3. General Design Considerations

The numeracy apps were designed following these guiding principles. The first principle was inclusivity. The apps were intended to be available to as many learners and teachers as possible. Thus, the project team decided that the apps should remain free, requiring only one-time access to the Internet (during download). Even when there is no Internet access, the app must be transferable wirelessly (Bluetooth) or through an external device (such as USB or memory card).

Second, the apps were designed to be ready-to-use or ready-to-play. Students should be able to figure out how to use the apps even with minimal instruction and limited supervision. Further, students can play or use the apps at their own time. Thus, teachers may assign the apps as supplementary material for mathematical learning.

Third, the apps were designed to align with the DepEd curriculum, which encompasses the five strands of mathematical learning, namely, numbers and number sense, measurement, patterns and algebra, geometry, probability and statistics [20]. The project team consulted teachers, math heads, and principals to ensure that the apps are suitable for Filipino learners.

Fourth, the apps were designed to be interactive and dynamic. These were unlike static worksheets where students primarily perform drill and practice. The apps were designed in a game-like environment. The idea is that, by doing better in the game, learners develop a stronger understanding of mathematical concepts. In all grade levels, the apps facilitate exploration, visualization, and justification.

The design of the web statistical app was guided by similar principles. First, the web app was designed to be mobile-ready so that this may be conveniently accessed and used not just on computers but also on mobile devices such as tablets and smartphones. Second, the interface of the app is simple, easy-to-use, and appropriately designed for young children. Moreover, since the web app requires access to the Internet, measures were put in place to address possible interruptions due to intermittent or limited Internet connectivity. Finally, the student worksheets and teaching guides, aligned with the Statistics and Probability competencies for Grades 1-11 in the DepEd curriculum, were prepared to complement the use of the web application.

4. Pedagogical Design

This section discusses the pedagogical basis of the design of the apps. The five numeracy apps are downloadable from https://mathplusresources.wordpress.com/ and the web app is accessible from mathplusresources.com.

4.1 Quick Images

The *Quick Images* app presents some dot patterns and the user has to figure out how many dots are shown (Figure 4.1). It is meant to address the issue that some learners still rely on counting by ones even well into their upper elementary school years. These learners think of numbers primarily as a count sequence starting from one, rather than as a more sophisticated part-whole relationship [21]. For example, they think of "four" as the end of the count sequence 1, 2, 3, 4, without realizing that "four" is the same as three and one or two and two. In [22], a recommended intervention is the use of subitizing (or "instantly seeing how many"). It was noted that certain arrangements of dots can be quickly recognized as a certain number, such as recognizing that one four-dot and one three-dot dice pattern give the number seven. This recognition involves **conceptual subitizing**, or a knowledge of numbers as a composite of parts. Familiar spatial arrangements and five or ten frames were used as organizing features because these can facilitate arithmetic and place value knowledge [22, 23]. It was further recommended that the dot patterns be shown very quickly, so that students are prompted to draw on the part-whole composition of number rather than on the unitary count sequence. *Quick Images* allows the user to choose between three speed levels: slow (2.5 s), medium (1.5 s) and fast (0.5 s).

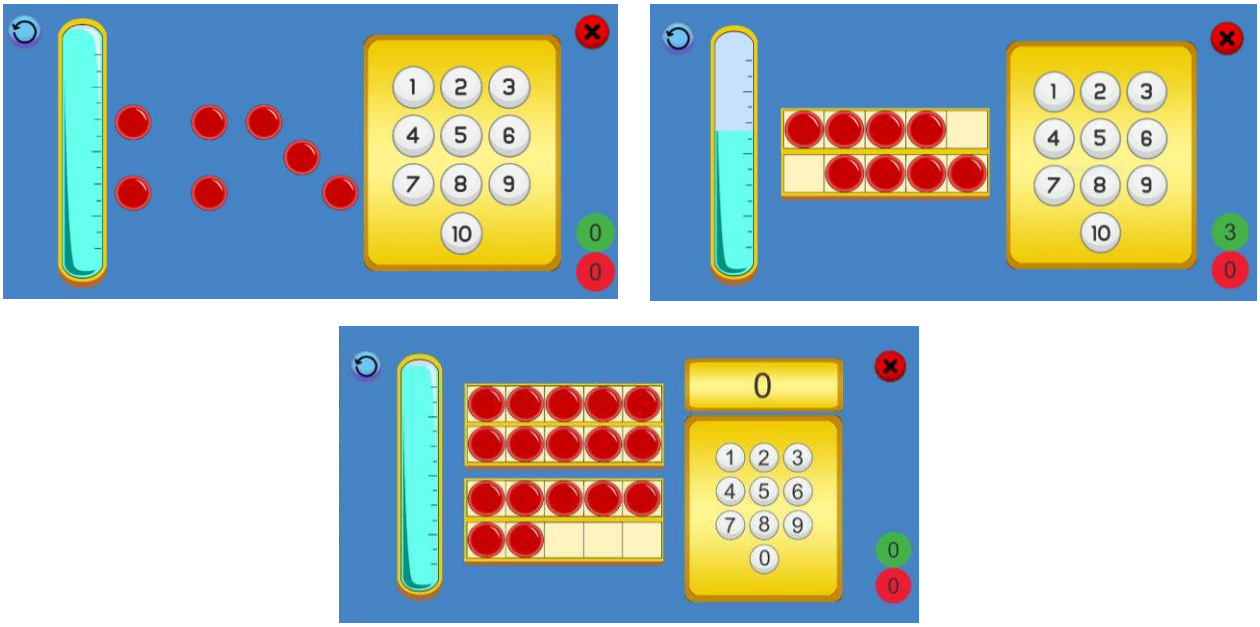


Figure 4.1 Screenshots from *Quick Images*

4.2 Frame Game

The *Frame Game* app (Figure 4.2) was designed to facilitate learners' skill in incrementing or decrementing by 10s, 100s, and so on, which is a foundational piece of knowledge for developing ten-structured thinking [21] and conceptual place value [18]. In *Frame Game*, a target number is given, and users need to press the buttons (±1, ±10) so that the target number is reached. Visual support is provided by a pictorial representation of the current number. As with *Quick Images*, the objects shown in *Frame Game* are arranged in tens to provide an organizational structure in the child's mind.

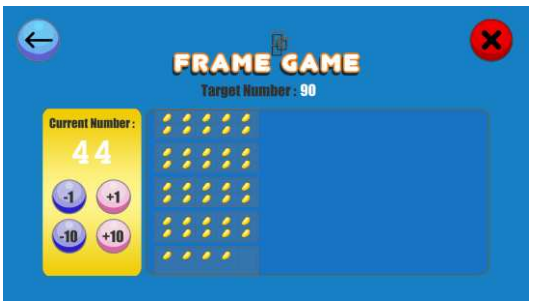
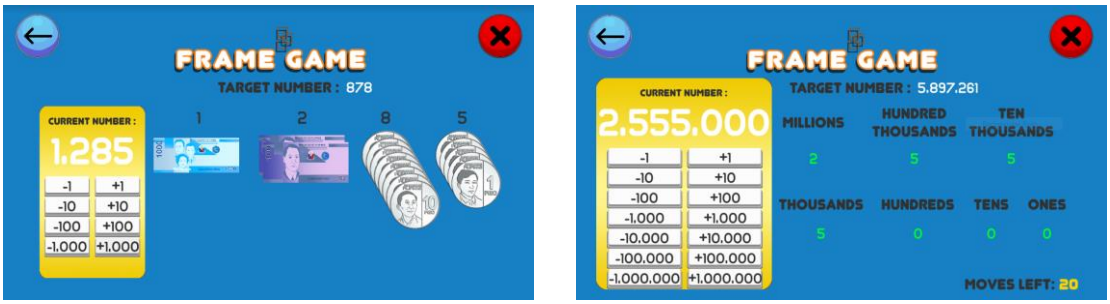


Figure 4.2 Screenshot from *Frame Game*

Frame Game also aligns with several grade levels in the official elementary school curriculum. In this app, the user has an option to choose the number range, from 1 to 100 for Grade 1 through 1 to 10,000,000 for Grade 5. It also addresses various levels of abstraction. Objects are initially shown using pictorial representations of each object (Figure 4.2), then using money (Figure 4.3a), and finally using purely symbolic or digit representations (Figure 4.3b).



(a) (b)

Figure 4.3 Screenshots of higher levels of abstraction from *Frame Game*

4.3 Grid Game

The *Grid Game* app is similar to *Frame Game* in that the user must reach a target number by pressing appropriate buttons. The main visual support is the hundred grid (Figure 4.4). There are also higher levels of abstraction in *Grid Game*. Initially, all numbers can be seen in the grid (Figure 4.4). At a higher level of abstraction, a blank grid is shown (Figure 4.5a). At the highest level of abstraction,

no grid is shown (Figure 4.5b). To address competencies in several grade levels, *Grid Game* offers the user an option to choose the number type: whole numbers (for Grades 1-2), decimals (for Grades 4-5), and integers (for Grades 6-7).

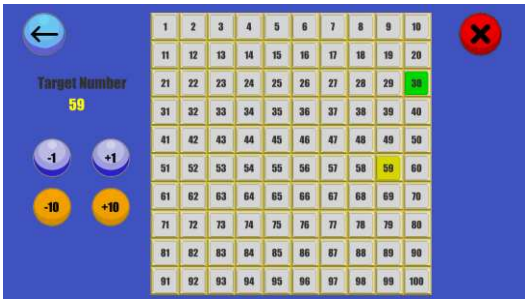
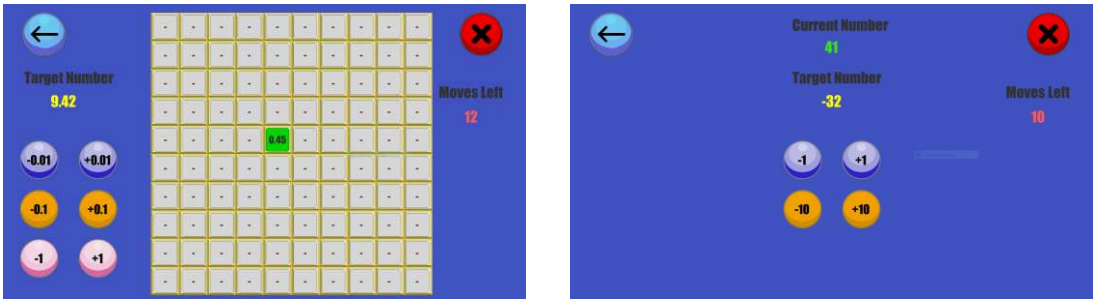


Figure 4.4 Screenshot from *Grid Game*

To ensure that users go beyond unitary counting (e.g., pressing +1 multiple times to go from 24 to 52), both *Frame Game* and *Grid Game* provide a higher level where users are required to apply the most efficient strategy to reach the target number. This is done by indicating the number of allowable moves ("Moves Left" in Figure 4.3b and Figure 4.5). When users run out of moves, they have the option to repeat the same problem and improve their solution.



(a) (b)

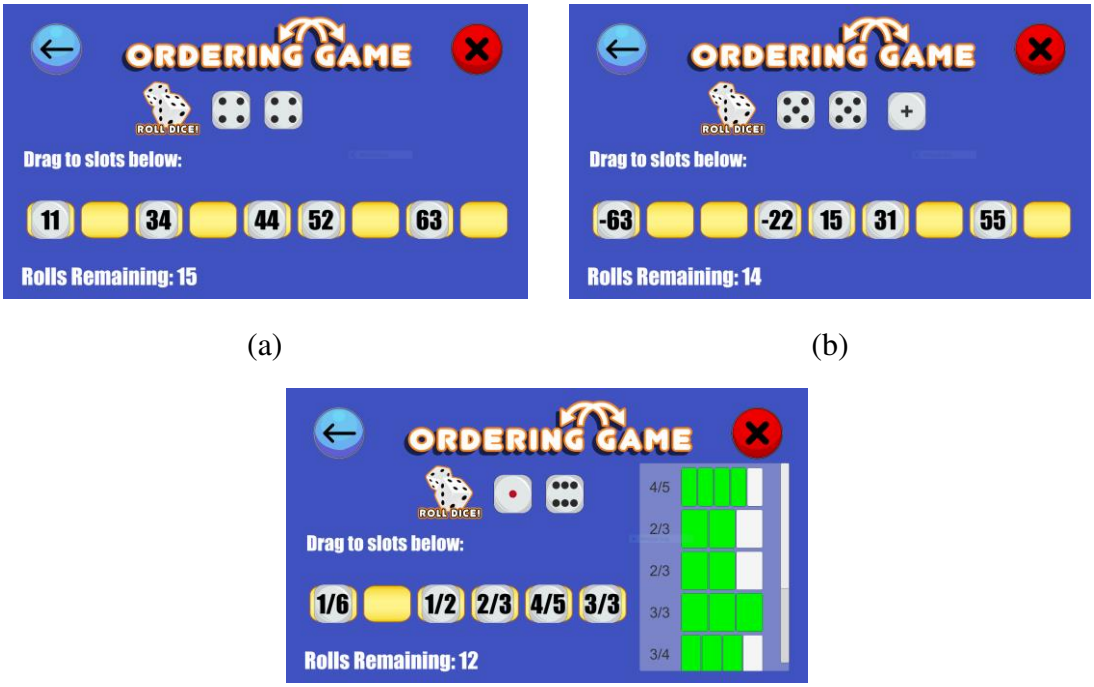
Figure 4.5 Screenshots of higher levels of abstraction from *Grid Game*

4.4 Ordering Game

The design of the *Ordering Game* app (Figure 4.6) was based on the premise that numerical development is hinged on an understanding of number magnitudes; and extending this understanding to increasingly large whole numbers and to a broader set of numbers including integers and rational numbers [24]. A simple proposed intervention is to incorporate activities that utilize the mental number line. It was found that learners from low-income families gained a more precise understanding of numerical magnitude after playing a numerical board game for just four 15-minute sessions [25]. Some resources [26] are already available online, so we developed another game, which was inspired by an activity by Marilyn Burns (cited in [27]).

In *Ordering Game*, users generate numbers by rolling the virtual dice. They place the generated numbers along a row, with the condition that the numbers are increasing from left to right (Figure

4.6). The goal is to fill up the entire row within the allowed number of rolls. The allowed number of rolls can be altered by choosing a level—a higher level allows fewer numbers of rolls before which the user must complete the task. The game becomes more difficult as it progresses because it becomes more likely that a generated number cannot be correctly placed in any of the remaining slots. As with the previous apps, *Ordering Game* also aligns with several grade levels because there are options for number range (e.g., from two- to five-digit whole numbers) and for number type (whole numbers, integers, fractions). In the case of fractions, a visual representation of each fraction is also shown (see Figure 4.6c). The goal is to strengthen the users' images of fraction magnitude. *Ordering Game* also gives feedback to indicate when a user places a number on an incorrect spot along the row.



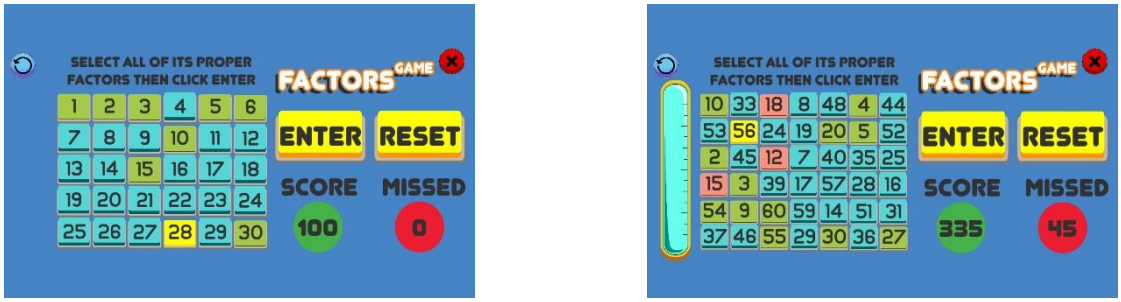
(a) (b)

(c)

Figure 4.6 Screenshots from the *Ordering Game* for (a) whole numbers, (b) integers, and (c) fractions

4.5 Factors Game

The *Factors Game* app was designed so that learners demonstrate understanding of prime and composite numbers in a game setting. There are four levels: 1-30, 1-40, 1-50 or 1-60 and three modes: unlimited time, two minutes or one minute. The player selects a number and then selects all its proper factors. The total score gained is the sum of the number chosen and the correct factors selected. The total missed score is the sum of missed factors and wrong factors selected. The game ends when all the numbers on the grid are selected or missed or the time is up. A summary appears after the game which gives the scored and missed points and correct, wrong or missed factors. The highest score attained by the player is also included in the summary. In the easiest level (Figure 4.7a), numbers 1 to 30 are placed on a 5 x 6 grid arranged in order while in the 1-60 level (Figure 4.7b), randomly selected numbers from 1 to 60, not in order, are placed on the 6 x 7 grid.



(a) (b)

Figure 4.7 Screenshots from the *Factors Game* (a) Level: 1-30 and (b) Level: 1-60

The *Factors Game* is aligned with the official elementary school curriculum. It addresses the need for the learners, from various grade levels, to identify factors and multiples of a given number. Moreover, it also trains the learners to differentiate prime from composite numbers. Without a time-limit, the goal is for the player to choose the factors of the number selected, correctly and avoid having missed points. The time limit will require players to strategize in selecting the numbers. Players will find out that they can score more points by selecting certain numbers.

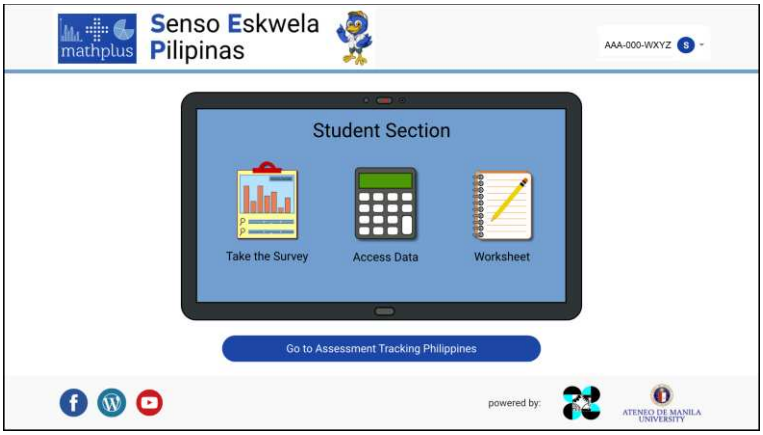
4.6 Senso Eskwela Pilipinas

A web app that has potential in improving statistics instruction in the early school years is the *Senso Eskwela Pilipinas* (*SEP*). The *SEP* is patterned after the international project called *CensusAtSchool*. First established in the UK in 2000, *CensusAtSchool* has the "dual thrust to enliven data handling activities within the classroom while also educating children about the principles and processes involved in conducting a census" [28, p.1]. Since then, *CensusAtSchool* has also been established in other countries like Australia, Canada, USA, New Zealand, and Japan. For instance, the Japanese *CensusAtSchool* project focused to address the observed needs of Japanese statistics teachers and students [29].

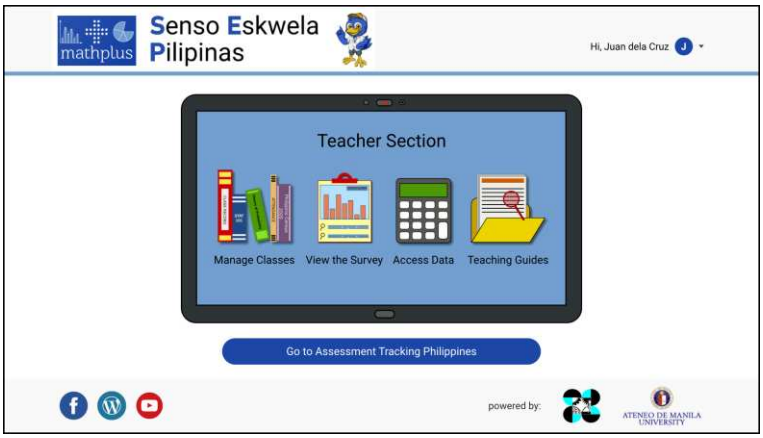
SEP serves as an online platform for gathering, storing, and accessing readily available and relatable data for learning and teaching statistics. In the login page, the user is asked if he or she is a student or a teacher. Grade 1-12 students, who are given access to *SEP* via a code given by their teacher, can answer a 28-question survey about themselves, such as their age, height and weight, favorite food and drinks, hobbies and interests, and others. Some questions involve puzzles and games that can captivate their interest. The students' responses are then stored in the *SEP* database and become part of the data that can be accessed by teachers and students nationwide. Although the same set of questions is used for all grade levels, a broad range of statistics topics can be studied depending on the kind of data the students will retrieve. Student worksheets are also provided to complement the teaching guides for teachers and also to allow students do activities independently (Figure 4.8a). At a basic level, the data can be used to teach students about creating pictographs and other charts or graphs. At a more advanced level, the same data can be used to teach the Central Limit Theorem or hypothesis testing. Moreover, since there are responses from various grade levels and locations, it is possible to do comparative analyses using different statistics topics.

Meanwhile, teachers get access to *SEP* by creating an account for free. The features designed for teachers allow them to manage classes, retrieve their students' survey results, access the data, and view teaching guides (Figure 4.8b). These teaching guides, prepared by the project team, are aligned

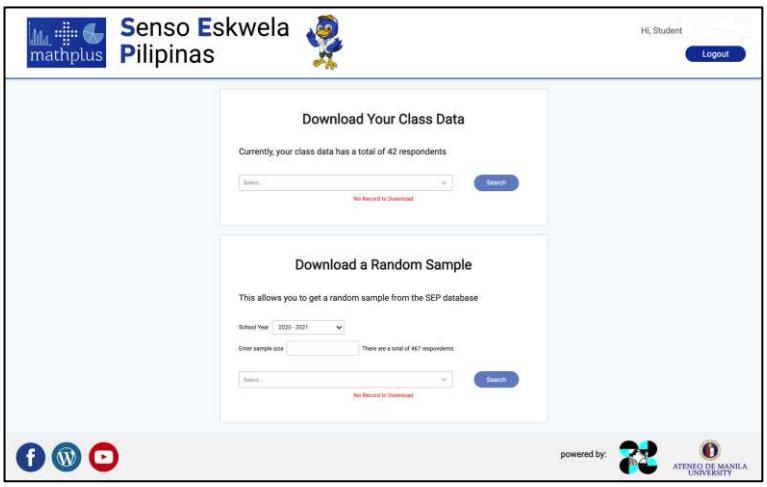
with the official DepEd curriculum. In general, teachers, students, and even unregistered guests may download a random sample from the database to be used for activities or tasks in their teaching or learning of statistics. Moreover, teachers and students can retrieve the data of their own class(es) (Figure 4.8c).



(a)



(b)



(c)

Figure 4.8 (a) Student page, (b) Teacher page, and (c) Students' Access Data page of *SEP*

Similar to the *CensusAtSchool* framework, *SEP* aims to enrich the experience of teachers and students in teaching and learning statistics. In particular, "[t]he sense of belonging that participating pupils feel, purely because they know their own responses will become part of a database of responses from their peers, is fascinating and motivating for them." [30, p. 175] Moreover, the questions in the *SEP* survey have been carefully drafted to appeal not only to the interests of grade school and high school students but also to emphasize experiences that are uniquely Filipino. For example, one question in the survey about games that the respondent likes to play is intentionally focused on Pinoy (Filipino) games (Figure 4.9). Through questions like this, the team aims to maximize the relatability of the *SEP* survey so that students can naturally develop a sense of curiosity about the responses of their classmates or other students elsewhere. In turn, this will hopefully help them formulate statistics questions more naturally and increase their engagement in answering these questions through their statistics lessons.

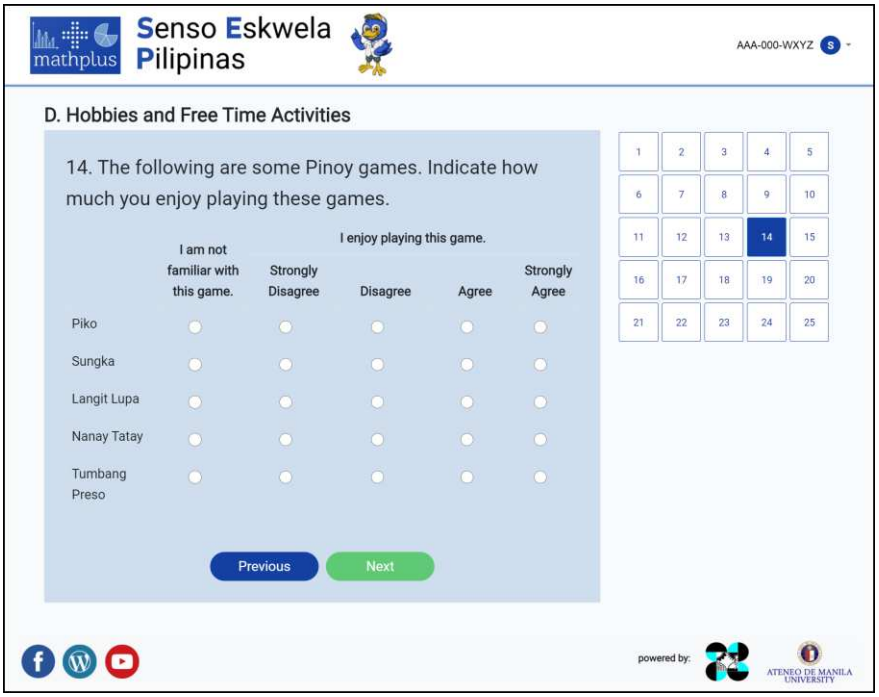


Figure 4.9 An example survey question of *SEP* on Pinoy (Filipino) games

5. Collaboration with the Department of Education

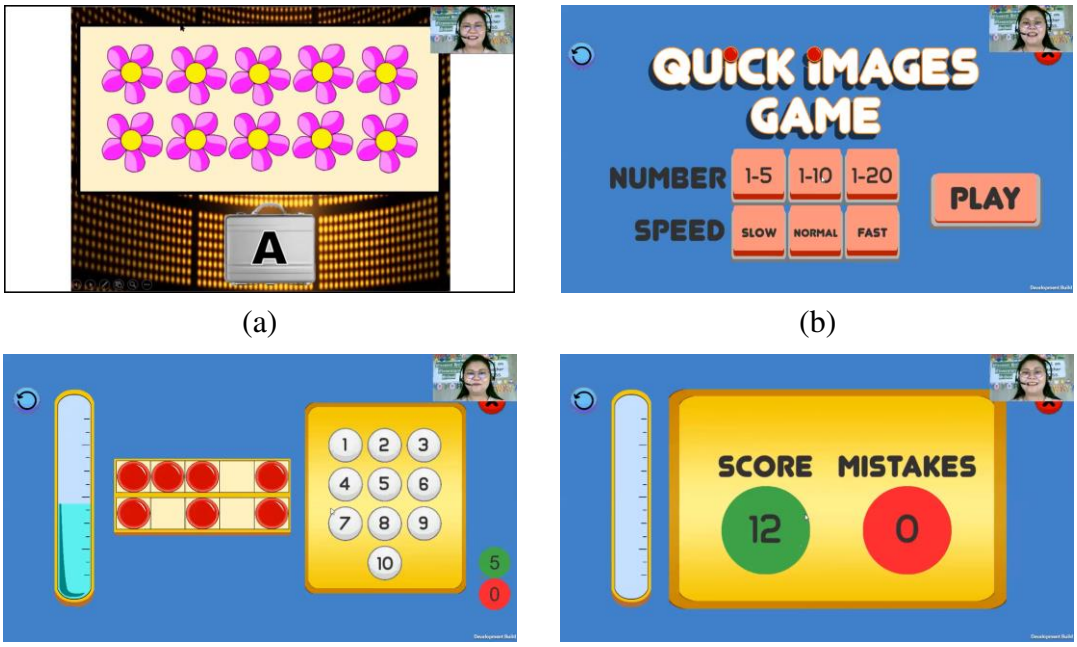
In 2020, the events arising from the Covid-19 pandemic propelled changes in modalities of teaching mathematics in schools in many countries around the world. In the Philippines the Department of Education (DepEd) has announced the shift to blended learning in schools from K to 12 in the school year 2020-2021. Radio, television, online and modular learning are being prepared and updated for use this school year. At the same time, teachers are being trained on utilizing newer platforms and innovative tools to prepare for this new mode of instruction. One of the methods of teaching that authors proposed to the Curriculum and Instruction Unit of the DepEd Central Office was the use of the mathematical apps as additional resources for teachers in blended learning to carry out instruction. One of the biggest opportunities for learning is that the apps can be integrated in the lesson wherein the students may explore and visualize concepts, use real data for exploration, practice drills and exercises remotely, even without much supervision and guidance of teachers or parents.

This approach will also provide students and teachers the opportunity to use their smartphone, tablet or laptop as a tool for mathematical learning. Since the numeracy apps are free and the web app requires no subscription; and the apps run on moderate system requirements, these make the resources very accessible to a wider range of teachers and students. The numeracy apps may be used offline; and will benefit students who do not have the resources and opportunities to engage in online learning due to Internet issues.

As the DepED Central Office is working towards making the apps available in the DepEd learning resources and portal for the information and use of teachers nationwide, the authors already started partnerships with local DepEd divisions in Metro Manila so that teachers can start their training and exposure on the use of the apps. These partnerships produced an ongoing series of webinars to introduce the mathematical apps to teachers and familiarize them with how the apps are aligned with the most essential learning competencies prescribed by the curriculum. While the webinars were carried out in collaboration with local DepEd divisions, these were made available to teachers nationwide. The webinars were streamed live and were also made available shortly by the divisions in their social media channels so teachers could revisit the webinars at their own pace and time. Videos were also created to inform teachers about the apps to supplement the webinars. The apps were discussed in radio programs to reach the teachers in remote areas.

One collaborative effort worth mentioning was the project with Grades 1 to 6 teachers of one local DepEd division where the teachers created their own video lessons with the aid of the numeracy apps. In the video, teachers taught a lesson and introduced the game-based math apps as a means to strengthen the math concepts discussed. This endeavor was very fruitful as it provided other teachers the much-needed material to use for their lessons. Other teachers may use the videos as starting points to create their own lessons.

One of these videos is a 5-min video demonstration [31] of a Grade 1 teacher employing *Quick Images* as a complementary activity to a lesson on number identification. Screenshots of the lecture are shown in Figure 5.1. The video was posted in a city schools division portal and is available for use by teachers who may want to integrate the app in their own blended learning classroom.



(a) (b)

(c) (d)

Figure 5.1 Screenshots of a video lesson integrating *Quick Images*

In the first half of the video, the teacher reviewed the counting numbers from 1 to 10 with the aid of static images of various objects found in school. She guided the students in identifying the number of objects shown in each image (Figure 5.1a). The second half of the video shows the teacher using *Quick Images* to practice or enrich the students' quick number identification skills. She first introduced *Quick Images* and explained its interface and important features (Figure 5.1b) before playing the *Quick Images* app herself (Figure 5.1c). For students who may have missed identifying the number of objects before they disappeared, she occasionally pressed the "show again" button that allows the app to redisplay the objects. Towards the end of the video, the teacher encouraged the students to independently play with the app and send her a screenshot of the page which displays their score (Figure 5.1d).

Teachers who created the videos and prepared lessons using the apps answered feedback/ evaluation forms. They found the apps interactive, easy to use, and can cater to beginning, average and advanced students. Moreover, they believe students will be able to explore and practice mathematical skills through the applications. Some commented that the apps are helpful and adaptable into a blended learning setting because it offered a strategy to make mathematics interesting and accessible when the teacher is not always present as in a physical classroom setting. One teacher commented "The advantage is really to the young learners, as it is appropriate not only in learning math concepts but improves their mood/ attitude in learning."

The apps are currently undergoing field-testing; observations of the apps' contribution to conceptual development would be presented during the conference.

6. Conclusion and Future Direction

In this paper the design of five numeracy apps for various learning competencies in Grades 1 to 6, as well as the framework for a statistical web app for Grades 1 to 11 is presented which is part of a continuing project funded by a national government agency, to create a digital mathematics learning environment for implementation in schools. Studies on Grades 7 to 10 apps also created under the project have shown that the apps offer additional opportunities to learn mathematical content even with minimal supervision [32]. Example, a statistical analysis of the students' scores after using the proving app *Prove It* even for a short period of time showed evidence of a significant improvement in proof-writing [33].

As of this writing, more apps from Grades 1 to 10 intended for other strands in the mathematical curriculum, such as Geometry and Algebra are being created and field tested for refinements.

Initial feedback from teachers and students for the numeracy apps for Grades 1 to 6 indicates their huge potential for mathematical learning. Moreover, the apps have been showcased and discussed by the team in some webinars [34, 35, 36, 37, 38] for grade school and high school educators. The responses of the teachers in these webinars imply their appreciation towards the availability of the apps as well as their interest and intent to use these in their classes. One teacher echoed the value of technology in teaching amidst the pandemic: "With pandemic or not, technology is always there to help us deliver the knowledge [t]hat our learners deserve to receive." Another teacher commented "Instead of playing the usual games offered online, these mathematical apps can be a very good substitute beneficial to young children… with proper guidance and supported with other resource…" while another teacher affirmed that "gamification of the lessons is the new trend in education." In particular, within a week after the webinar *Senso Eskwela at iba pa* [38] where *SEP* was formally launched, more than 50 teachers registered a combined total of more than 350 classes for participation to *SEP*. Moreover, the participants were asked to evaluate the apps *SEP*, *Quick Images, Grid Game, Ordering Game,* and *Frame Game*. Majority of the teachers have indicated that

these apps are easy to use, adaptable to a blended classroom, and can be catered to beginning, average, and advanced students.

The next step will be to examine more closely the educational effectiveness of these apps in facilitating student performance. Studies will also be carried out on how *SEP* impacts statistical learning. Among the planned activities in collaboration with DepEd schools in the next months will be for these purposes.

There is also the assessment database that is currently under construction. The database will store assessment data gathered within (or outside) this project for evaluation and analysis. It is anticipated that the database can be useful to teachers and researchers who seek to improve the mathematical learning of students. Research-based assessments used to track the mathematical performance of thousands of pupils in Australia [39] will inform the implementation of the assessments in the Philippine setting. It is anticipated that the research-based assessments will provide a richer description of the strategies used by Filipino learners than can be gleaned from the more traditional multiple-choice assessments such as the Philippine National Achievement Test (NAT). In particular, these assessments can determine whether Filipino students are merely imitating procedures demonstrated by their teachers or using their critical thinking to solve mathematical tasks.

The authors invite the readers to visit the project website at https://mathplusresources.wordpress.com/ where future updates and refinements to the developed resources will be made available.

Acknowledgements The authors would like to thank the Department of Science and Technology-Philippine Council for Industry, Energy, and Emerging Technology Research and Development (DOST-PCIEERD) and the University Research Council (URC), Ateneo de Manila University for the support of the development of the interactive resources discussed in this paper. Acknowledgement also goes to Dr. Ma. Mercedes T. Rodrigo and the Ateneo Laboratory for the Learning Sciences (ALLS) for providing technical assistance. Deep gratitude is extended to our app developers: Mr. Victor Antonio M. Ortega and Mr. Nigel Cargo (numeracy apps).; Mr. Marc Pison and the Logik team (*SEP*); and app designer Mr. Amiel Damian F. Justiniani. We also would like to acknowledge Ms. Mary Rose E. Villanda, Pleasant Hills Elementary School - School District of Mandaluyong for the video lesson featuring *Quick Images.*

References

[1] OECD (2019). PISA 2018 Results (Volume I): What Students Know and Can Do. Paris: PISA, OECD Publishing. Retrieved 10/4/2020 from https://doi.org/10.1787/5f07c754-en.

[2] Bergqvist, T. & Lithner, J. (2012). Mathematical Reasoning in Teachers' Presentations. *The Journal of Mathematical Behavior*, 31, 252-269.

[3] Cox, R. D. (2015). "You've Got to Learn the Rules": A Classroom-level Look at Low Pass Rates in Developmental Math. *Community College Review*, 43(3), 264-286.

[4] Nag, S., Chiat, S., Torgerson, C., and Snowling, M. J. (2014) Literacy, Foundation Learning and Assessment in Developing Countries: Final Report. *Education Rigorous Literature Review*. United Kingdom: Department for International Development.

[5] Wright, R. J., Martland, J., and Stafford, A. K. (2006). *Early Numeracy: Assessment for Teaching and Intervention. Second Edition*. Thousand Oaks, CA: Paul Chapman Publishing.
[6] Verzosa, D. (2015). Developing the Number Knowledge of Low-attaining First-to-fourth-graders. In C. Vistro-Yu. (Ed.), *Proceedings of the 7th ICMI-East Asia Regional Conference on Mathematics Education* (pp. 217-224). Quezon City: Philippine Council of Mathematics Teacher Educators (MATHTED), Inc.
[7] Reston, E. and Bersales, L. G. (2008). Reform Efforts in Training Statistics Teachers in the Philippines: Challenges and Prospects. In C. Batanero, G. Burrill, C. Reading, and A. Rossman (Eds.), *Joint ICMI/IASE Study: Teaching statistics in school mathematics. Challenges for teacher education. Proceedings of the ICMI study 18 and 2008 IASE round table conference.* Monterrey, Mexico: ITESM.
[8] Goodwin, K. and Gould, P. (2014). Race in the Outback: Investigating Technology Designed to Support Number Development in a Preschool Serving an Under-Resourced Community. In J. Anderson, M. Cavanagh & A. Prescott (Eds.), *Curriculum in focus: Research guided practice (Proceedings of the 37th annual conference of the Mathematics Education Research Group of Australasia)* (pp. 247–254). Sydney, Australia: MERGA.
[9] Kim, P., Buckner, E., Kim, H., Makany, T., Taleja, N., and Parikh, V. (2012). A Comparative Analysis of a Game-based Mobile Learning Model in Low-socioeconomic Communities of India. *International Journal of Educational Development*, 32, 329–340.
[10] Highfield, K. and Goodwin, K. (2013). Apps for Mathematics Learning: A Review of 'Educational' Apps from the iTunes App Store. In V. Steinle, L. Ball & C. Bardini (Eds.), *Mathematics education: Yesterday, today and tomorrow (Proceedings of the 36th annual conference of the Mathematics Education Research Group of Australasia).* (pp. 378-385). Melbourne, Australia: MERGA.
[11] Siemon, D., Bleckly, J., and Neal, D. (2012). Working with the Big Ideas in Number and the Australian Curriculum: Mathematics. In B. Atweh, M. Goos, R. Jorgensen & D. Siemon, (Eds.), *Engaging the Australian National Curriculum: Mathematics – Perspectives from the Field* (pp. 19-45). Online Publication: Mathematics Education Research Group of Australasia.
[12] Courtney-Clarke, M. & Wessels, H. (2014). Number Sense of Final Year Pre-service Primary School Teachers. *Pythagoras,* 35(1)
[13] Sengul, S. (2013). Identification of Number Sense Strategies Used by Pre-Service Elementary Teachers. *Kuram Ve Uygulamada Egitim Bilimleri,* 13, 1965-1974.
[14] Wang, X., & Chen, G. (2010). Survey and Analysis of the Statistical Reasoning among High School Students in China and Dutch Wenfang. *Journal of Mathematics Education,* 2(1), 15-26
[15] Mohamed, M. & Johnny, J. (2010). Investigating Number Sense Among Students. *Procedia - Social and Behavioral Sciences*, 8, 317-324
[16] Yang, D., Li, M., & Lin, Chih-I. (2008). A Study of the Performance of 5th Graders in Number Sense and its Relationship to Achievement in Mathematics. *International Journal of Science and Mathematics Education*, 6, 789-807
[17] Thompson,I. and Bramald, R. (2002) An Investigation of the Relationship between Young Children's Understanding of the Concept of Place Value and their Competence at Mental Addition. Retrieved 9/5/2020 from https://www.atm.org.uk/write/MediaUploads/Journals/MT184/Non-Member/ATM-MT184-14-15-Extra.pdf

[18] Ellemor-Collins, D. and Wright, R. (2011) Developing Conceptual Place Value: Instructional Design for Intensive Intervention. *Australian Journal of Learning Difficulties*, 16.1, 41-63.
[19] David, I. P. and Maligalig, D. S. (2006). Are we Teaching Statistics Correctly to our Youth? *The Philippine Statistician,* 55(3-4), 1-28.
[20] Department of Education (2016). K to 12 Curriculum Guide: Mathematics. Retrieved 8/26/2020 from https://www.deped.gov.ph/wp-content/uploads/2019/01/Math-CG_with-tagged-math-equipment.pdf
[21] Fuson, K. C., Smith, S. T., and Lo Cicero, A. M. (1997). Supporting Latino First Graders' Ten-structured Thinking in Urban Classrooms. *Journal for Research in Mathematics Education,* 28, 738-766.
[22] Clements, D.H. (1999). Subitizing: What Is it? Why Teach it? *Teaching Children Mathematics*, 5(7), 400-405.
[23] Özdem, Ş. and Olkun, S. (2019). Improving Mathematics Achievement via Conceptual Subitizing Skill Training. *International Journal of Mathematical Education in Science and Technology*, 1-15.
[24] Siegler, R. S. and Lortie-Forgues, H. (2014). An Integrative Theory of Numerical Development. *Child Development Perspectives*, 8(3), 144-150.
[25] Siegler, R. S. and Ramani, G. (2008). Playing Linear Numerical Board Games Promotes Low-income Children's Numerical Development. *Developmental Science,* 11(5), 655-661.
[26] Ulbright, J. (n.d.). Arranged Ten Frames. Retrieved 8/29/2020 from https://www.geogebra.org/m/cb4CSBMj
[27] Silva, M. Place Value Activity Package. Retrieved 9/5/2020 from http://www.mathematicshed.com/uploads/1/2/5/7/12572836/place-value-activity-pack.pdf
[28] Connor, D. (2002). CensusAtSchool 2000: Creation to Collation to Classroom. In *Proceedings of the Sixth International Conference on Teaching of Statistics, Cape Town.* Voorburg, The Netherlands: International Statistical Institute.
[29] Aoyama, K., Watanabe, M., and Tamura, Y. (2013) Statistics Learning Environment for Students through Japanese CensusAtSchool Project. *IASE/IAOS Satellite* Retrieved 9/5/2020 from https://iase-web.org/documents/papers/sat2013/IASE_IAOS_2013_Paper_2.1.2_Aoyama.pdf?1402525000
[30] Davies, N., Richards, K., Aliaga, M., and Nichols, R. (2010) CensusAtSchool. *Significance* 7(4), 175. Retrieved 9/5/2020 from http://onlinelibrary.wiley.com/doi/10.1111/j.1740-9713.2010.00458.x/pdf
[31] Mathematics Teacher Association of Mandaluyong. (2020). *Pagbibilang mula 1 hanggang 10* [Video file]. Retrieved 9/5/2020 from https://www.facebook.com/watch/?v=354568545569833&extid=LBsxn6IpsNATl4zx
[32] De Las Peñas, M. L. A. N., Aberin, M. A. Q., Bautista, E. P., Francisco, F. F., Tabares, W. C., and Verzosa, D. M. B. (2018). *Development of Interactive Software and Teaching Guides for Grades 7-10 Mathematics*. Manila: DOST-PCIEERD.
[33] Verzosa, D. M. B., De Las Peñas, M. L. A. N., Aberin, M. A., and Garces, L. P. D. M. (2019). App-based Scaffolds for Writing Two-column Proofs. *International Journal of Mathematics Education in Science and Technology,* 50, 766-778.
[34] DepEd Tayo – Mandaluyong City. (2020). Webinar on the Use of Mathematical Apps for Blended Learning [Video file]. Retrieved 10/5/2020 from https://www.facebook.com/depedmandaluyongcity/videos/295266868265384/

[35] Mathplus Resources. (2020). Math Plus Apps for Blended Learning (Grade School) [Video file]. Retrieved 10/5/2020 from https://youtu.be/7-BIrCT5a3k
[36] GEG Quezon City. (2020). Virtual Orientation on the Use of Math Apps for Blended Learning [Video file]. Retrieved 10/5/2020 from https://www.facebook.com/GEGQuezonCity/videos/571343836879548/
[37] Mathplus Resources. (2020). Math Plus Apps for Blended Learning (High School) [Video file]. Retrieved 10/5/2020 from https://youtu.be/PGo2kq4JQYA
[38] ADMU School of Science and Engineering. (2020). Senso Eskwela Pilipinas at iba pa (Day 2) [Video file]. Retrieved 10/5/2020 from https://www.facebook.com/admusose/videos/767231277154400/
[39] Bobis, J., Clarke, B., Clarke, D., Thomas, G., Wright, R., Young-Loveridge, J., and Gould, P. (2005). Supporting Teachers in the Development of Young Children's Mathematical Thinking: Three Large Scale Cases. *Mathematics Education Research Journal,* 16, 27-57.

Developing Competency in Two-Column Proof-Writing via Interactive Software Applications

Debbie Marie B. Verzosa[†], **Ma. Louise Antonette N. De Las Peñas**[*], **Ma. Alva Q. Aberin**[*], **Len Patrick Dominic M. Garces**[*]

[†]*Mathematics and Statistics Department, University of Southern Mindanao, Kabacan, Cotabato*

[*]*Department of Mathematics, Ateneo de Manila University, Quezon City, Metro Manila*

Abstract

Dynamic tools in geometry act as a platform for student learning by allowing students to manipulate representations, observe the consequences, and formulate conjectures. In line with this direction, educators are now conceptualizing applications that may be used to sharpen students' ability to formulate proofs for geometric statements.

As such, this poster describes software applications designed to facilitate the understanding and writing of proofs for geometric statements on triangle congruence, triangle similarity, and properties of parallelograms. *ProveIt* is an Android-based application that provides users a framework for conceptualizing and thinking about proofs. Another application developed is a GeoGebra-based tool that provides a step-by-step proof alongside relevant visual support. Both applications were designed so that they can be accessed using mobile devices such as smartphones and tablet computers.

Focus group discussions and initial testing of the applications revealed that the applications offered a more engaging medium for learning proof-writing and were capable of facilitating proof-writing skills in geometry.

Introduction

Many mathematics educators are now advocating an increased role of proofs in the mathematics education of students at all levels [1]. Further, there are calls for rethinking the role of proofs from being mere exercises in logical thinking to providing opportunities for students to work like mathematicians—observing patterns, making conjectures, and presenting justification [2]. In learning geometry, dynamic geometry has been a natural platform for students to test geometric conjectures as it allows them to manipulate representations, observe the consequences, and formulate conjectures [3, 4, 5].

This presentation focuses on the two-column proof, the most common proof format in secondary school textbooks in the Philippines, and developing the competency to construct two-column proofs with the aid of interactive software applications. As such, we introduce *ProveIt*, an Android-based proving application for geometry, and a GeoGebra-based application on two-column proofs.

ProveIt

ProveIt is an Android-based mobile application that provides users a visual representation and interactive learning tool in aid of proving theorems and statements involving congruent triangles, similar triangles, and parallelograms. *One of the app's most important features is that it allows the user to select parts of the geometric figures on the visual representation itself, using the mobile device's touch-screen interface.* As opposed to usual textbook exercises on these topics, ProveIt also provides immediate feedback, allowing the user to quickly amend any incorrect responses provided or to understand why the provided response is correct.

The application is subdivided into three main topics: (a) Triangle Congruence, (b) Triangle Similarity, and (c) Quadrilaterals. As of this writing, two levels (Level 1 and Level 2) are available for Triangle Congruence, one level (Level 2) is available for Triangle Similarity, and one level (Level 1) is available for Quadrilaterals.

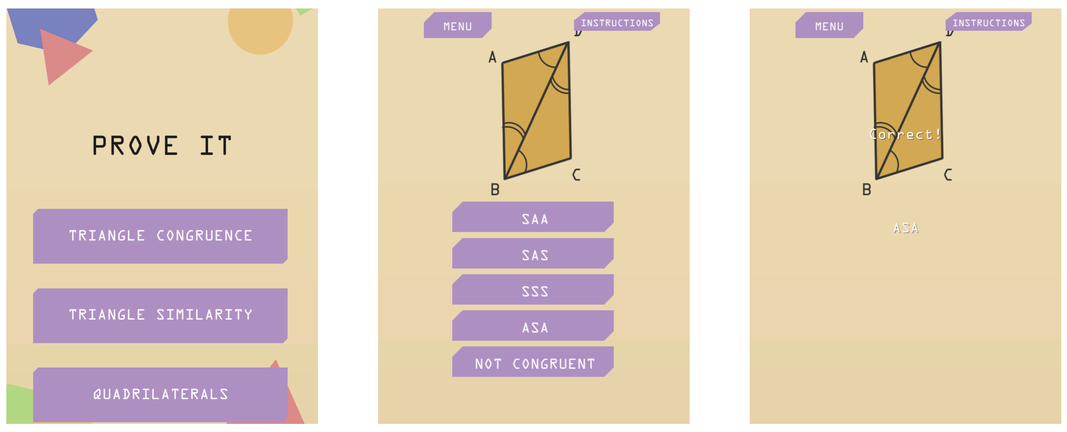


Figure 1. (Left) The main menu of ProveIt; (Center) A sample task in Triangle Congruence Level 1; (Right) the screen confirming a correct response

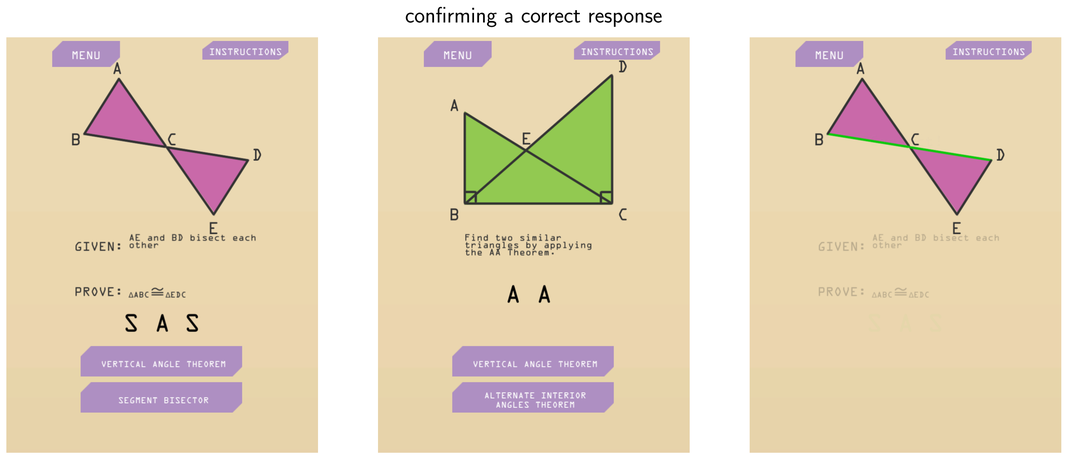


Figure 2. (Left) A sample task in Triangle Congruence Level 2 and (center) Triangle Similarity Level 1; (Right) Highlighted part of the figure after the user has selected it using the touch screen

The application tabulates the number of correct responses the student provided and displays this information after the user has answered 10 problems. Instructions for each problem may also be displayed by clicking the "Instructions" button on the upper-right corner of the screen.

GeoGebra-Based Application for Two-Column Proofs

We have written a number of applets on proving theorems using two column proofs using the dynamic software GeoGebra. These applets provide students a framework for writing proofs by allowing them to visualize given statements of the proof. To navigate the proof, the student clicks the right arrow button. A line of the proof appears and the student writes the answer to the blank. To visualize the statement given, the box provided is ticked.

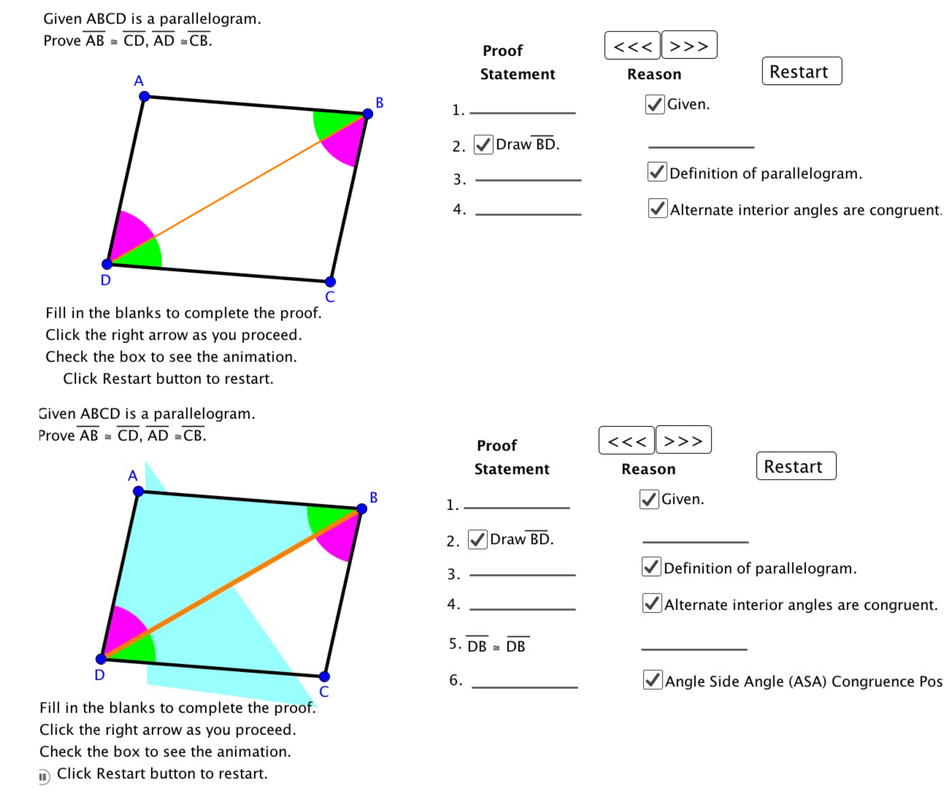


Figure 3. A sample task in the GeoGebra applet for two-column proof and the visual feedback and assistance accompanying the task (captured mid-animation)

A teachers' version was also developed for each GeoGebra applet. This version contains the answers to the two-column proof in the corresponding students' version.

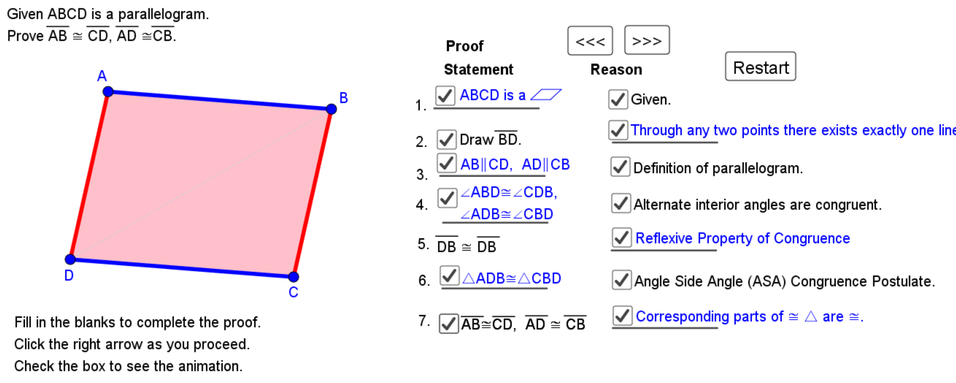


Figure 4. Teachers' version of the GeoGebra applet on parallelograms

Interactive GeoGebra applets were also developed for other topics such as bisectors of triangles, theorems on parallel lines and transversals, and theorems on quadrilaterals.

Evaluation and Discussion

The designed applications can distinguished from existing software in three ways. First the ProveIt and GeoGebra apps were designed to run on mobile devices, which increase access due to its portable nature. Secondly, the design is toward scaffolding learning—because properties are already indicated at the outset, users who are already at an impasse can readily select a property and focus on how to apply this property on objects on the screen. A third distinction between our apps and the existing software is that our apps highlight portions of the proof that have already been achieved.

An initial evaluation of the apps by secondary school students and their teachers through various field tests yielded promising results. Evidence of the app's effectiveness is reinforced by a significant difference in pre- and post-test scores (with post-test scores being higher) of students when assessed on triangle congruence. Our initial evaluation and field testing has shown that even a short session on the use of the ProveIt app can produce some improvement in proving skills.

About the Presentation

The authors would like to thank the **Department of Science and Technology (DOST) Philippine Council for Industry, Energy and Emerging Technology Research and Development (PCIEERD)** for the funding support. This presentation was also possible due to the support of the Department of Mathematics, the School of Science and Engineering, and the Office of the Vice President of the Loyola Schools, Ateneo de Manila University.

The mobile applications and teaching guides developed under the project can be accessed on the **MathPlus Resources** website at `https://mathplusresources.wordpress.com`. Inquiries about the project may be forwarded to Dr. Ma. Louise Antonette N. De Las Peñas (`mdelaspenas@ateneo.edu`), Project Leader.

References

[1] Stylianou DA, Blanton ML, Knuth EJ, editors. Teaching and learning proof across the grades: A K-16 perspective. New York: Routledge; 2009.

[2] De Villiers M. Rethinking proof with Geometers Sketchpad. Berkeley, CA: Key Curriculum Press; 2003.

[3] Wares A. Using dynamic geometry to stimulate students to provide proofs. Int J Math Educ Sci Tech. 2007; 38: 599-608.

[4] Wares A. Dynamic geometry as a context for exploring conjectures. Int J Math Educ Sci Tech. 2018; 49: 153-159.

[5] Marrades R, Gutierrez A. Proofs produced by secondary school students learning geometry in a dynamic computer environment. Educ Stud Math. 2000; 44: 87-125.

Ateneo de Manila University

Archīum Ateneo

Mathematics Faculty Publications | Mathematics Department

2021

Development of a Gamified Number Line App for Teaching Estimation and Number Sense in Grades 1 to 7

Debbie Marie Versoza
Ateneo de Manila University

Ma. Louise Antonette N. De Las Peñas
Ateneo de Manila University, mdelaspenas@ateneo.edu

Maria Alva Q. Aberin
Ateneo de Manila University, maberin@ateneo.edu

Agnes Garciano
Ateneo de Manila University, agarciano@ateneo.edu

Jumela F. Sarmiento
Ateneo de Manila University, jsarmiento@ateneo.edu

See next page for additional authors

Follow this and additional works at: https://archium.ateneo.edu/mathematics-faculty-pubs

 Part of the Science and Mathematics Education Commons

Custom Citation

Verzosa, D.M.V., De Las Peñas, M.L.A.N., Aberin, M.A.Q., Garciano, A.D., Sarmiento, J.F., & Tolentino, M. A. C. (2021). Development of a Gamified Number Line App for Teaching Estimation and Number Sense in Grades 1 to 7. In M.M.T. Rodrigo, S. Iyer, & A. Mitrovic (Eds.), 29th International Conference on Computers in Education Proceedings Volume 2 (pp. 648-656). Asia Pacific Society for Computers. https://icce2021.apsce.net/proceedings/volume2/

This Conference Proceeding is brought to you for free and open access by the Mathematics Department at Archīum Ateneo. It has been accepted for inclusion in Mathematics Faculty Publications by an authorized administrator of Archīum Ateneo. For more information, please contact oadrcw.ls@ateneo.edu.

Authors

Debbie Marie Versoza, Ma. Louise Antonette N. De Las Peñas, Maria Alva Q. Aberin, Agnes Garciano, Jumela F. Sarmiento, and Mark Anthony C. Tolentino

This conference proceeding is available at Archīum Ateneo: https://archium.ateneo.edu/mathematics-faculty-pubs/207

Rodrigo, M. M. T. et al. (Eds.) (2021). Proceedings of the 29th International Conference on Computers in Education. Asia-Pacific Society for Computers in Education

Development of a Gamified Number Line App for Teaching Estimation and Number Sense in Grades 1 to 7

Debbie Marie B. VERZOSA[a*], Ma. Louise Antonette N. DE LAS PEÑAS[b], Maria Alva Q. ABERIN[b] , Agnes D. GARCIANO[b], Jumela F. SARMIENTO[b] & Mark Anthony C. TOLENTINO[b]

[a]*Department of Mathematics and Statistics, University of Southern Mindanao, Philippines*

[b]*Department of Mathematics, Ateneo de Manila University, Philippines*

*dmbverzosa@usm.edu.ph

Abstract: Fraction knowledge is known to be a gatekeeper to more advanced mathematical learning. On the basis of the literature on early number learning, a number line mobile application called *Catch the Carrot* was designed to develop students' knowledge of whole number and fraction magnitude. This paper aims to describe the design of the *Catch the Carrot* app and discusses the rationale for using number lines as representational scaffolds for developing children's understanding of numbers, particularly their estimation and number sense skills. The gamification features of the app, as well as strategies for integration in a classroom are also presented. This app, which utilizes number line representations, is expected to deliver the benefits associated with using number lines in instruction as suggested by the literature. The next step is to investigate the extent to which the gamified number line app promotes number sense and mathematical competence.

Keywords: Mathematical app, number sense, estimation, number line, STEM

1. Introduction

Entry into STEM-related fields requires competence in mathematical reasoning. Fraction knowledge is known to be a gatekeeper to more advanced mathematical learning. In fact, it has been shown that children's knowledge of number magnitude, particularly fraction magnitude, predicts algebraic performance (Booth & Newton, 2012) and various aspects of mathematical competence (Schneider, Merz, Stricker, De Smedt, Torbeyns, Verschaffel, & Luwel, 2018). As such, it is important to cultivate students' knowledge of fraction magnitude to facilitate success in mathematics and reinforce motivation to enter science-related fields. This necessitates knowledge of whole numbers as well, since both whole numbers and fractions are equally important in numerical development (Siegler, 2016).

To support students' critical thinking and problem solving, which are the twin goals of mathematics education as stated in the Philippine K-12 Mathematics framework (Department of Education, 2016), the authors designed a suite of applications, or apps, through the ongoing project "Technology Innovations for Mathematical Reasoning, Statistical Thinking and Assessment" (https://mathplusresources.wordpress.com/about/). This project is supported by the Philippine Department of Science and Technology-Philippine Council for Industry, Energy and Emerging Technology Research and Development (DOST-PCIEERD). The apps are aligned with the Philippine Department of Education's most essential learning competencies that were identified in response to the pandemic (Department of Education, 2020). The apps are free and internet connection is needed only once upon download. The apps can run on mobile phones, tablets, laptops, or computers.

The design of the apps was based on research in mathematical learning. The apps are not electronic versions of textbooks or worksheets whose primary functions are to provide drill and practice. Rather, the apps were designed to promote mathematical reasoning and visual thinking so that children can learn to think by and for themselves. Further, the apps were designed to be engaging, interactive and easy-to-play. Thus, the apps can be used even with minimal supervision, and are very apt for remote

and independent learning of students. As most instruction in elementary and high schools focuses on theory rather than on experiential learning (Vahidy, 2019), these apps were designed to increase student engagement and support STEM learning outcomes.

This aim of the paper is to discuss one particular mathematical app, called *Catch the Carrot*. It discusses the pedagogical basis of the app, followed by its description and game-like factors, and suggestions for integration in the classroom. This app, which harnesses a number line representation of whole numbers, fractions, decimals, and integers, can be used for Grades 1 to 7 mathematics. The app is based on locating numbers on a number line, which has been shown to improve arithmetic knowledge (Maertens, De Smedt, Sasanguie, Elen, & Reynvoet, 2016) and mathematical competence (Schneider et al., 2018).

The app highlights the impact of technological tools in studying mathematics, namely Edu-tainment: the use of tools in a game-like environment to master mathematical concepts, develop estimation skills, and harness the aptitude for visualization and exploration. Educational apps are frequently game-based (Murray & Olcese, 2011), so an important feature of mathematics education apps is that the learning of mathematics is embedded in the playing of the game (Moore-Russo et al., 2015).

2. Pedagogical Basis: Number Line Representation

Children's experience of mathematics typically begins from the act of counting. Connecting the counting process to a spatial representation of these numbers, such as in a number line, is beneficial for further mathematical learning (Wu, 2010). A number line is a particularly useful tool for developing students' understanding of cardinality and the ordinal relationship of numbers (Woods, Geller, & Basaraba, 2018) and mathematical operations (Diezmann & Lowrie, 2006; Wu, 2010). There are various other spatial representations of numbers, but the unidimensional number line has been shown to be more effective in improving number sense. For example, Gunderson, Hamdan, Hildebrand, and Bartek (2019) found that the use of the number line resulted in better performance in magnitude comparison tasks as compared to area or square models. Training on number line estimation has been shown to have a positive effect on arithmetic for young children (Maertens et al., 2016). Further, number line estimation proficiency is associated with multiple aspects of mathematical competence (Schneider et al., 2018).

Another rationale for using number lines is it can illustrate the magnitude of numbers. Number magnitude is an integral concept that underpins knowledge of the real number system (including rational and irrational numbers) (Siegler, Thompson, & Schneider, 2011). Number magnitude knowledge of whole and rational numbers is correlational with and predictive of arithmetic, general mathematical achievement (Siegler, 2016), and algebraic performance (Booth, Newton, & Twiss-Garrity, 2014).

The number line also has an advantage as it can be used to represent all real numbers and the four arithmetic operations (addition, subtraction, multiplication, and division) (Wu, 2010). It is also a natural representation for the mathematical definition of a fraction. That is, the fraction $1/b$ is defined to be a whole divided into b equal parts, and the fraction a/b is defined to be a copies of $1/b$ (Wu, 2010). By contrast, other models are appropriate in only particular contexts. A pie representation can be used to model fraction addition, but it is not appropriate for representing two-thirds divided by three-halves. Ultimately, a fraction has to be learned as a *number*, and not as pieces of a pie.

To summarize, number line tasks promote estimation skills and magnitude knowledge, which are associated with increased mathematical competence. As such, an app based on number line representations is expected to deliver the benefits associated with using number lines in instruction. Further, a number line can represent both whole and rational numbers, as well as the four fundamental operations. For these reasons, the number line provides a consistent instructional medium for mathematical learning over several years.

3. The *Catch the Carrot* App

3.1 App Description

The *Catch the Carrot* app presents a random number and a number line or rectangle on the screen (Figure 1). A player has to estimate the location of the number on the number line or rectangle. The goal is to locate as many numbers as possible within the given time. Depending on the student's Grade Level, s/he may choose to play with whole numbers (Figures 1a and 1b), fractions (Figure 1c and 1d), decimals (Figure 1e) or integers (Figure 1f). There are also multiple given lengths for each number line. For example, Figure 1a shows a number line that starts from 0 to 10, while Figure 1b shows a number line that starts from 0 to 1,000,000. The number lines for fractions also utilize a hybrid unidimensional number line (Figure 1c) for younger students and a pure unidimensional number line (Figure 1d) for older ones. The hybrid unidimensional number line is particularly helpful because the space between the hash marks can draw students' attention to the unit or the whole (Gunderson et al., 2019), which is an important aspect of fraction understanding.

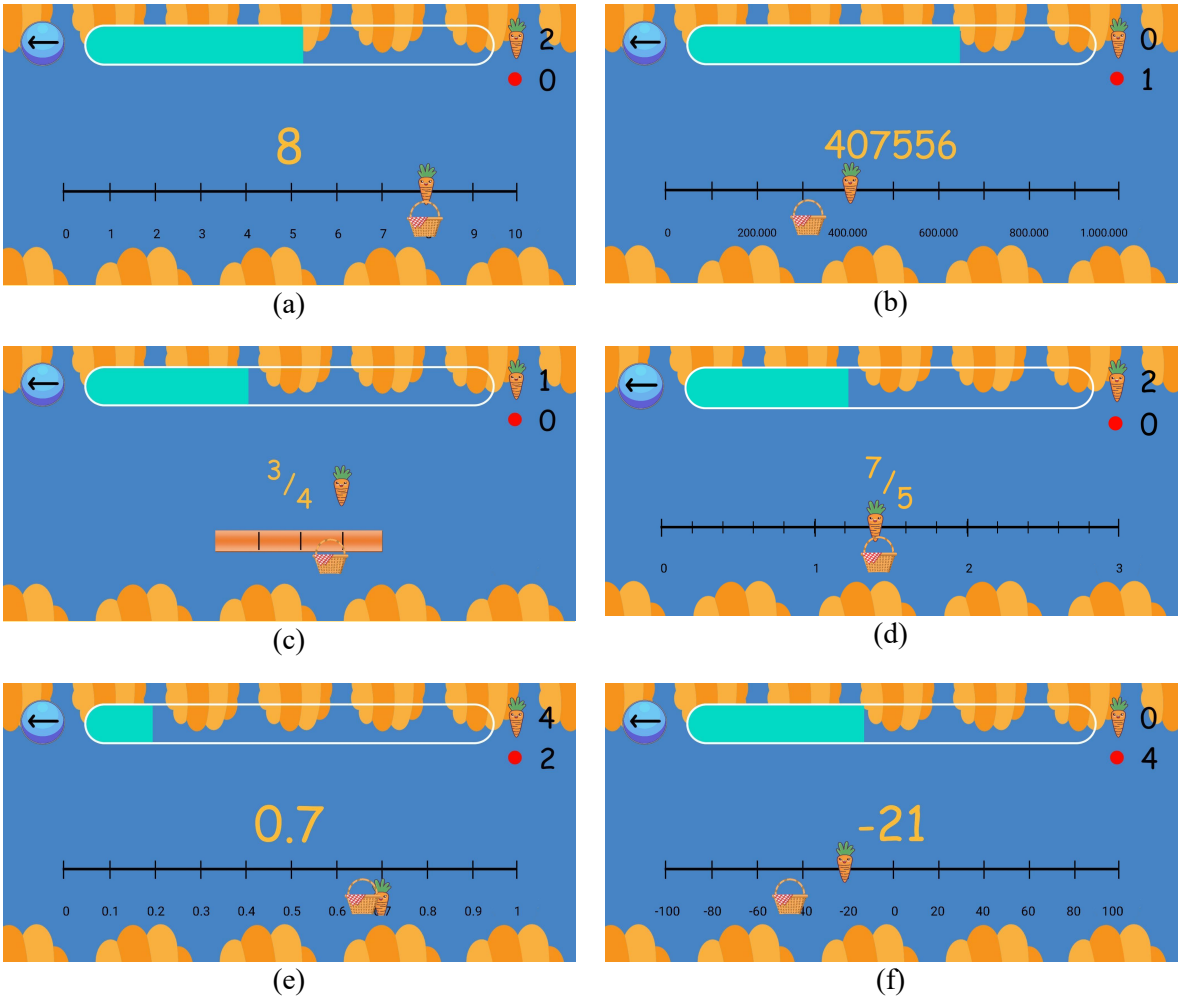
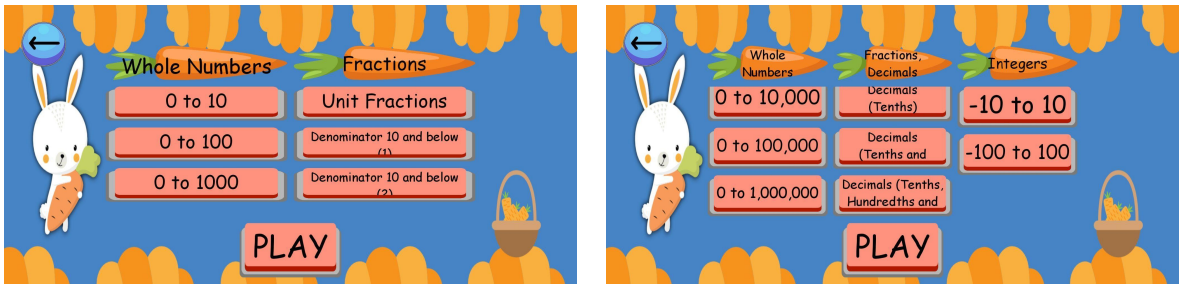


Figure 1. Different Levels of *Catch the Carrot.*

The levels and challenges present in the app are sequenced to align with the type and range of numbers that are specified in the official most essential learning competencies (MELCs) mandated by the Philippine Department of Education (Department of Education, 2020). For example, in Grade 2, a student is expected to learn whole numbers in the range 0 to 1000, as well as fractions less than 1 with denominators 10 and below. Thus, the app provides options "0 to 10", "0 to 100" or "0 to 1000" for whole numbers and "unit fractions" or "denominator 10 and below" for fractions (Figure 2a). On the other hand, the options for Grade 6 include "0 to 1,000,000" for whole numbers, "tenths, hundredths

and thousandths" for decimals, and "–100 to 100" for integers, to align with the expected learning competencies for Grade 6 (Figure 2b). For Grade 7, the official curriculum includes a review of arranging rational numbers on the number line. Thus, the optional levels for Grade 7 include all the prior competencies from Grades 1 to 6. Across all levels, students are expected to be able to locate whole numbers and fractions on a number line, according to the scope of numbers specified in the official curriculum, to develop their skills in estimating number magnitude.



(a) Options for Grade 2 (b) Options for Grade 6

Figure 2. Number Types and Levels.

The *Catch the Carrot* app was designed with the premise that numerical development is dependent on knowledge of estimation and number magnitude (Siegler & Lortie-Forgues, 2014). By playing the game, students learn to anchor their fraction images with fraction magnitude and the definition of a fraction. For example, Figure 1d shows that the fraction 7/5 is equal to 7 copies of 1/5 (or 7 segments of 1/5 away from 0).

The design of the *Catch the Carrot* app was also informed by the theory of representational systems as a key goal of mathematics education (Goldin, 1998). Goldin (1998) proposed a unified psychological model of mathematical learning and problem solving based on a theory of representations. He stated, "the overarching goal should be to foster in students the *construction of powerful, internal systems of representation*" (p. 159; italics in original). In this connection, the app was designed to provide manipulable representations and visual corrective feedback (Bouck, Long, & Park, 2020). For *Catch the Carrot*, the manipulable representation is shown in the way a student can freely adjust the position of the basket along the number line. For visual feedback, the student's answer is verified by the animation of a falling carrot; the answer is correct if the carrot falls into the basket positioned by the student. Through this dynamic feedback, students are allowed to learn from their mistakes and develop a better understanding of number magnitude.

3.2 Game-design Factors

With the intention of maximizing the potential of learning activities, gamification is sometimes employed as an approach to increase students' engagement and motivation (Dichev & Dicheva, 2017). Moreover, gaming can serve to capture children's interests towards studying and develop important skills in STEM education such as analytical thinking, strategizing, and problem-solving (Vahidy, 2019). In addition, given the current online or distance learning set-up in the Philippines coupled with the popularity of mobile games among Filipinos (Elliott, 2020), the gamification approach, especially when using mobile technology, may prove to be timely and apt.

Gamification has been applied in instructional design for elementary mathematics. For example, Hu and Shang (2018) transformed traditional teaching activities for learning elementary mathematics into gamified teaching activities using mobile applications. Following this approach, the development of *Catch the Carrot* involved the gamification of number estimation activities. While traditional activities on number estimation involve board work and paper worksheets, *Catch the Carrot* situates the activities in a game environment with the aim of increasing students' engagement and motivation, and allowing them to play with minimal instruction. Furthermore, the app was designed so that it can be one of the mathematical apps that are "not only aligned with the curriculum but are also creative, fun and provide opportunities to learn" (Miller, 2018, p. 9).

The development of *Catch the Carrot* adheres to the GBL Design Model (Shi & Shih, 2015) that involves 11 interrelated game-design factors: game goals, game mechanism, game fantasy, game value, interaction, freedom, narrative, sensation, challenges, sociality, and mystery. The game-design factors of *Catch the Carrot* are presented in Figure 3, following the layout used by Shi & Shih for the GBL Design Model (Shi & Shih, 2015). Each game design feature is discussed in the succeeding paragraphs.

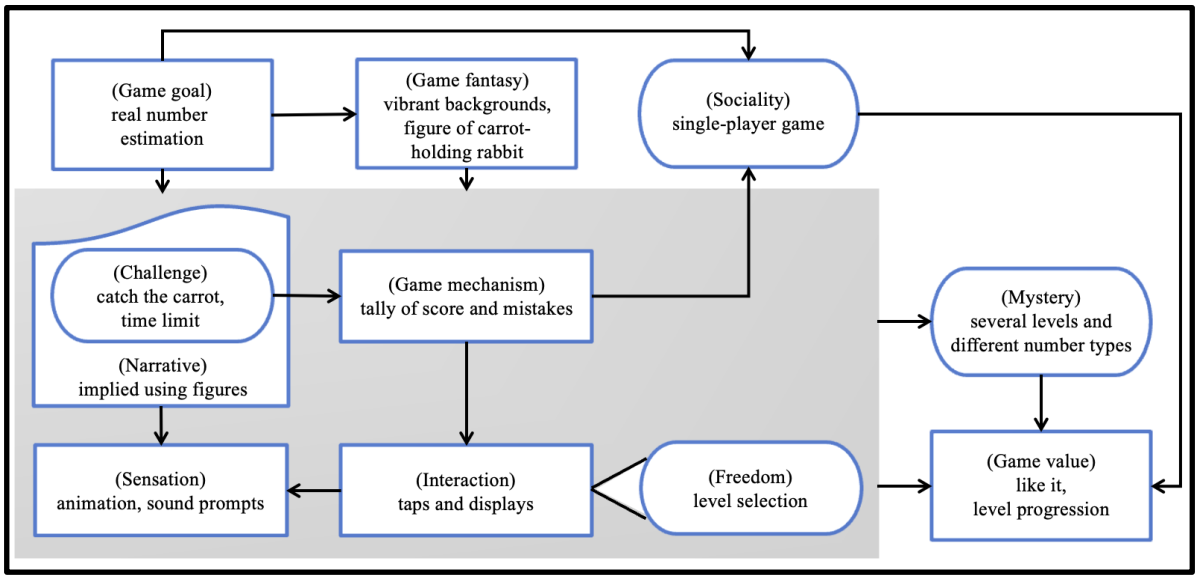


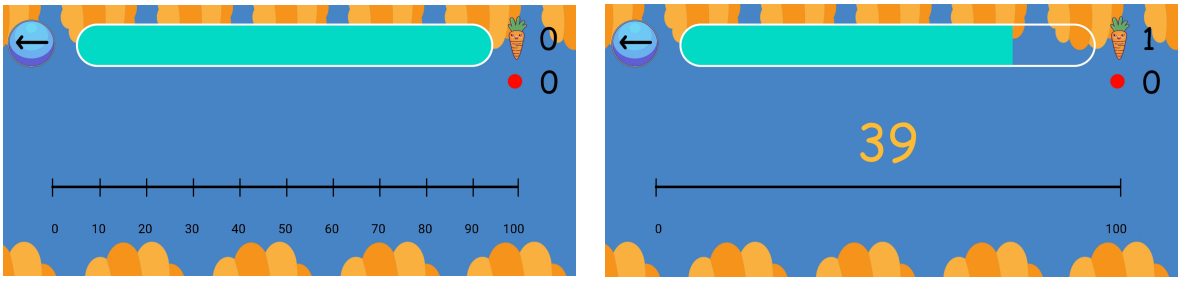
Figure 3. The GBL Design Model for *Catch the Carrot.*

The *game goal* of *Catch the Carrot* is to correctly estimate the location of a given real number in the real number line. This goal was a key consideration in the design of the app as it affected the game's mechanics, context, and structure. The game's *narrative* was designed to be simple. As the game is primarily intended for Grades 1 to 7 students, a complicated or text-heavy context might not be appropriate for the game. The narrative is presented through the figures of a rabbit, a basket, and carrots (Figure 4); that is, the implied narrative of the game is to help the rabbit by placing the basket correctly to catch a falling carrot. The figure of the carrot-holding rabbit and the game's vibrant background were designed to enrich the *game fantasy.*



Figure 4. The home screen of *Catch the Carrot* depicts an implied narrative for the game.

The *challenge* in the game design is demonstrated by showing the markings on the real number line only at the beginning of a round (Figure 5a). Once the game starts, the tick marks along the number line disappear (Figure 5b). Another challenge is the timer, which is shown at the top of the screen (Figure 5a and 5b) as a light blue bar that slowly decreases in length.



(a) Tick marks are shown at the beginning of a round.

(b) Tick marks disappear as the game progresses.

Figure 5. The game screen of *Catch the Carrot*.

The game's *mechanism* is a simple tally, shown in Figure 6, of successful and unsuccessful carrot catches. The score tally is displayed after each round is played. While the mechanism was designed to be simple, it can be effective for both teachers and students alike, for tracking students' progress with number estimation.

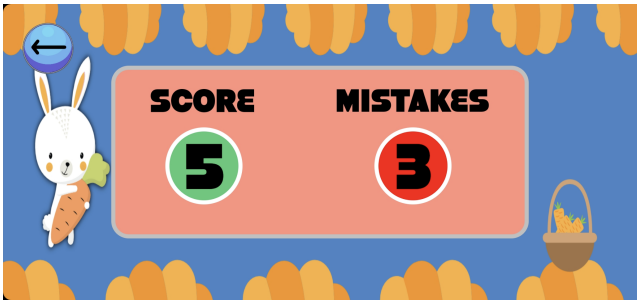


Figure 6. The score tally after one round of *Catch the Carrot*.

For the *interaction* and *sensation* game-design factors, it was important to take advantage of the capabilities of the digital environment without making the game appear or sound too complicated and chaotic. Thus, the game combines simple tap/click interactions with appropriate graphics, animations, and sound prompts. More specifically, after a player estimates the location of a given number on the number line, a basket appears at the tapped location and an animation of a falling carrot is shown. A bell sound is played if the carrot falls into the basket while a buzz sound is played otherwise. To provide the game some *mystery*, the app was designed to have separate levels for Grades 1-7. Within each grade level, the player has the *freedom* to choose the difficulty level based on number magnitudes and number types. The availability of these multiple levels is intended to capture the players' curiosity about the harder levels or the levels involving other number types. In addition, these levels were made available to align with the students' grade level and to give students the freedom to progress to higher levels once they are not anymore challenged by earlier levels.

Sociality was not woven into the game as it was mainly set up as a single-player game. However, with teacher intervention, it is possible to set up different scenarios where students may socialize with each other while playing the game. For instance, a teacher may group students and have the members of each group play the game simultaneously so that they can give their ideas about the correct location to tap. In an online setting, this can be done using breakout rooms with one student sharing his/her game screen.

All the factors mentioned above provide value to the game. In addition, attaining high scores in one level and moving to the next (i.e., level progression) may also add to student motivation. Moreover, the app has also been designed to be easy to understand and to use so that students can play it without the supervision of a teacher or a guardian. This makes the app ideal for asynchronous and independent learning.

4. Integration and Use of *Catch the Carrot*

The design of *Catch the Carrot* app was informed by research on number line estimation and knowledge of number magnitude, which have been shown to improve counting, arithmetic, algebraic knowledge, and general mathematical competence. On this basis, its integration in the elementary mathematics instruction is warranted (Woods et al., 2018). It can be used in presenting a lesson on visualization, representation, and estimation of a number, providing opportunities for practice and assessment. Woods et al. (2018) offered the following steps to integrate number lines in teaching.

Step 1. Determine learning goals

There is a need to determine if *Catch the Carrot* is appropriate and useful in attaining the learning goals. Number lines can model cardinality, ordinality, quantity comparisons, and operations. *Catch the Carrot* can be an effective tool for learning goals that involve these concepts.

Step 2. Plan Ahead

The teacher should prepare in advance the logistical considerations for the use of the app, which is free for download. Teachers should instruct the students on the proper download of the app on an Android device or a desktop. If the app is downloaded on a desktop, the app needs to be played using an Android emulator.

Step 3. Present and Practice

This step involves the presentation of the lesson where the teacher demonstrates how the app works and how it can be played. After this, the teacher may model the process of estimating the fraction (or number) on the rectangle (or number line). The teacher may pose questions such as "Should the fraction be near the left end of the rectangle?" or "When is the location towards the right end?" These guide questions could prevent students from relying on guesswork as a strategy to locate the fraction (or number). As previously suggested, if the online set-up has a provision for break-out groups or rooms, students can work together and collaborate. During this step, students gain opportunities to practice and be familiar with the app.

Step 4. Process and Verbalize

Finally, it is important to process the activity with open-ended questions that can challenge students to explain their thinking and reasoning about the lesson. The teacher can facilitate students' thinking with reflection questions such as, "What helps you locate the position of the fraction?," "How do you know that the fraction is near the right (or left) end of the rectangle?" and "Why do you think the fraction is in the middle portion?"

The teacher may give instructions to the students to use the app asynchronously or outside class time for practice and mastery. If a teacher would like to check the students' progress, s/he may ask them to send screenshots of their outputs over a period of time. A follow-up class discussion can be facilitated in the next synchronous session. During this meeting, the teacher can assess the depth of the students' understanding and determine their difficulties and weaknesses in estimation and visualization of numbers.

An earlier version of *Catch the Carrot*, involving arranging rational numbers in a number line, was part of a set of mathematical apps introduced to high school teachers in a training workshop in October 2018 (De las Peñas et al, 2019). The teachers were asked to give feedback on the apps and the responses were collected at the end of the workshop. Overall, the teachers gave a positive evaluation of the mathematical apps, especially in relation to increasing students' interest in the topic. Moreover, teachers stated that the mobile apps provide a more interactive means of instruction (as opposed to traditional paper-based media). They mentioned that the apps show potential in helping students' retention of lessons. They further responded that the apps provide a novel and interesting way of approaching classroom discussions and help increase student motivation in studying their lessons.

5. Conclusion and Future Direction

This paper described *Catch the Carrot*, a number line app designed to develop estimation and magnitude knowledge. These skills are predictive of algebraic knowledge and facilitate access to more advanced mathematical competencies in STEM fields. The app is an innovation in the form of gamifying the number line to represent number types and magnitudes, which are topics that comprise the curriculum from Grades 1 to 7. The paper discussed the mathematical pedagogies and gamification features that informed the fundamentals of the app creation and offered strategies for its integration in a blended learning environment. The next step is to investigate the effectiveness of the tool in a physical or online classroom, to confirm the anticipated benefits suggested by the literature.

Acknowledgements

The authors thank the Department of Science and Technology-Philippine Council for Industry, Energy, and Emerging Technology Research and Development (DOST-PCIEERD) and the University Research Council (URC), Ateneo de Manila University for the support of the development of the interactive resources for Grades 1 to 11 Mathematics. Acknowledgement also goes to Dr. Ma. Mercedes T. Rodrigo and the Ateneo Laboratory for the Learning Sciences (ALLS) for providing technical assistance. Deep gratitude is extended to our app developers, Sparksoft Solutions, Inc under its Managing Director, Mr. Noel S. Patron.

References

Booth, J. L., & Newton, K. J. (2012). Fractions: Could they really be the gatekeeper's doorman? *Contemporary Educational Psychology*, *37*, 247–253.

Booth, J.L., Newton, K.J., & Twiss-Garrity, L.K. (2014). The impact of fraction magnitude knowledge on algebra performance and learning. *Journal of Experimental Child Psychology*, *118*, 110–118.

Bouck, E.C., Long, H., & Park, J. (2020). Using a virtual number line and corrective feedback to teach addition of integers to middle school students with developmental disabilities. *Journal of Developmental and Physical Disabilities*, *33*, 99-116.

De las Peñas, M.L.A.N., Verzosa, D.M.B., Aberin, M.A.Q., Garces, L.P.D.M., Francisco, F.F., Bautista, E.P., Tolentino, M.A.C., & Tabares, W.C. (2019). Digital simulations for grade 7 to 10 mathematics. *Philippine Journal of Science*. *148*(4), 743-757.

Department of Education. (2016). K to 12 curriculum guide: Mathematics. Retrieved June 1, 2020, from https://www.deped.gov.ph/wp-content/uploads/2019/01/Math-CG_with-tagged-math-equipment.pdf

Department of Education. (2020). *K to 12 most essential learning competencies with corresponding CG codes*. Retrieved July 12, 2020, from https://commons.deped.gov.ph/K-to-12-MELCS-with-CG-Codes.pdf.

Dichev, C., & Dicheva, D. (2017). Gamifying education: what is known, what is believed and what remains uncertain: a critical review. *International Journal of Educational Technology in Higher Education, 14*.

Diezmann, C., & Lowrie, T. (2006). Primary students' knowledge of and errors on number lines. In P. Grootenboer, R. Zevenbergen, & M. Chinnapan (Eds), *Proceedings 29th Annual Conference of the Mathematics Education Research Group of Australasia,* (Vol. 1, pp. 171-178). Canberra, Australia: MERGA.

Elliott, R. (2020). *The Philippines' games market: data and insights*. Retrieved from https://newzoo.com/insights/articles/data-and-insights-on-the-philippines-games-market/#:~:text=Share%3A,Southeast%20Asia's%20overall%20games%20market.

Goldin, G.A. (1998). Representational systems, learning, and problem solving in mathematics. *The Journal of Mathematical Behavior*, *17*(2), 137-165.

Gunderson, E.A., Hamdan, N., Hildebrand, L., & Bartek, V. (2019). Number line unidimensionality is a critical feature for promoting fraction magnitude concepts. *Journal of Experimental Child Psychology, 187*, 1-29.

Hu, R., & Shang, J. (2018). Application of gamification to blended learning in elementary math instructional design. In: S. Cheung, L. Kwok, K. Kubota, LK. Lee, & J. Tokito (Eds), *Blended Learning. Enhancing Learning Success. ICBL 2018. Lecture Notes in Computer Science* (Vol. 10949, pp. 93-104). Springer, Cham.

Maertens, B., De Smedt, B., Sasanguie, D., Elen, J., & Reynvoet, B. (2016). Enhancing arithmetic in pre-schoolers with comparison or number line estimation training: does it matter? *Learning and Instruction*, *46*, 1-11.

Miller, T. (2018). Developing numeracy skills using interactive technology in a play-based environment.

International Journal of STEM Education, 5, 1-11.
Moore-Russo, D., Diletti, J., Strzelec, J., et al. (2015). A study of how Angry Birds has been used in mathematics education. *Digital Experiences in Mathematics Education, 1*, 107–132.
Murray, O.T., & Olcese, N.R. (2011). Teaching and learning with iPads, ready or not? *TechTrends: Linking Research and Practice to Improve Learning*, *55*(6), 42-48.
Schneider, M., Merz, S., Stricker, J., De Smedt, B., Torbeyns, J., Verschaffel, L., & Luwel, K. (2018). Associations of number line estimation with mathematical competence: a meta-analysis. *Child Development, 89*(5), 1467-1484.
Shi, Y.-R., & Shih, J.-L. (2015). Game factors and game-based learning design model. *International Journal of Computer Games Technology, 2015*, 1-11.
Siegler, R.S. (2016), Magnitude knowledge: the common core of numerical development. *Developmental Science*, *19*, 341-361.
Siegler, R.S., & Lortie-Forgues, H. (2014). An integrative theory of numerical development. *Child Development Perspectives*, *8*(3), 144-150.
Siegler, R.S., Thompson, C.A., & Schneider, M. (2011). An integrated theory of whole number and fractions development. *Cognitive Psychology*, *62*, 273-296.
Vahidy, J. (2019). Enhancing STEM learning through technology. *Technology and the Curriculum: Summer 2019.* Lethbridge, AB, Canada: Power Learning Solutions.
Woods, D.M., Geller, L.K., & Basaraba, D. (2018). Number sense on the number line. *Intervention in School and Clinic, 53*(4), 229-236.
Wu, H., (2010). *Understanding numbers in elementary school mathematics.* Providence, RI: American Mathematical Society.

Iyer, S. et al. (Eds.) (2022). Proceedings of the 30th International Conference on Computers in Education.
Asia-Pacific Society for Computers in Education

Development of an App and Videos to Support the Fraction Learning Trajectory from Grades 1-7

Debbie Marie B. VERZOSA[a*], Ma. Louise Antonette N. DE LAS PEÑAS[b], Maria Alva Q. ABERIN[b], Agnes D. GARCIANO[b], Jumela F. SARMIENTO[b], Juan Carlo F. MALLARI[b], & Mark Anthony C. TOLENTINO[b*]
[a]*Department of Mathematics and Statistics, University of Southern Mindanao, Philippines*
[b]*Department of Mathematics, Ateneo de Manila University, Philippines* * dmbverzosa@usm.edu.ph, mtolentino@ateneo.edu

Abstract: Lack of procedural fluency in fractions impedes access to advanced mathematical courses and limits opportunities for entry into STEM-related fields. This paper describes the design and pedagogical basis of the *Moving Fractions* app and supplementary fraction videos for promoting fraction learning. *Moving Fractions* utilizes game-design factors to draw students through a trajectory of fraction learning from part-whole comparisons to a more robust understanding of the measurement concept of fractions. The supplementary video immerses students in a broad range of fraction representations. The app and video are intended to form a fraction learning package for distribution in Philippine schools. Future work involves the gathering of empirical data for validating the expected benefits associated with the application of mathematics education research in the app and video design.

Keywords: mathematical app, fraction, STEM, educational video

1. Introduction

Mathematics plays a fundamental role in STEM education. Students entering STEM-related fields are those who typically perform well in high school mathematics and have positive attitudes towards mathematics (Wang, 2013). Specifically, procedural fluency in fractions was found to improve access to advanced mathematical courses (Ngo, 2018). Students who do not learn fractions properly tend to perform poorly in subsequent mathematical courses (Booth & Newton, 2012), and may demonstrate limited mathematical competencies (Schneider et al., 2018). Unfortunately, many students and even pre-service teachers do not have a robust understanding of fractions (Bansilal & Ubah, 2020). Therefore, early interventions in fraction learning are important to increase opportunities for entry into STEM- related fields.

The design of games and game-based applications (apps) in classrooms has drawn the attention of mathematics education researchers for decades (Larkin, 2013). Games have been shown to increase students' engagement and motivation (Erhel & Jamet, 2013), and increase mathematics performance in various domains such as fractions (Ninaus et al., 2016). This paper discusses the *Moving Fractions* app for PC and Android that has been designed and developed to support the learning of fractions. The app is one of the mathematical resources that has been developed under an ongoing government-funded project (De Las Peñas et al., 2021), one of the goals of which is to strengthen further the mathematical competencies in foundational concepts necessary for more advanced learning. These include place value, number sense and fraction number sense. Aside from the app, mathematical videos were also created for students to help them visualize abstract ideas. In the past two years of the pandemic, these resources have been useful in support of the blended learning modality in schools called for by the Philippine Department of Education (DepEd).

The resources for instance, can be used with minimal supervision, and are ideal for distance learning. An interested reader is invited to visit https://mathplusresources.wordpress.com/ for access to the mathematical resources.

2. Pedagogical Basis: Fraction Understanding

Learning fractions offers substantial challenges for students because traditional instruction does not present fractions in a precise and coherent way (Wu, 2010). Fractions are typically presented as part- whole comparisons (like pieces of pizza), leaving students with an impoverished understanding of fractions (Lamon, 2007). To learn fractions meaningfully, students must learn what fractions are, and how correct procedures can be logically derived from the definition (Wu, 2010). According to Wu (2010), the fraction $1/n$ is defined to be the point on the number line corresponding to dividing the unit segment into n equal parts, and taking the first division point to the right of 0; the fraction m/n is the mth multiple of the fraction $1/n$. Wu argued that a definition based on number lines is more superior than part-whole conceptions of fractions because all basic facts about fractions can be derived from the number line, but not from part-whole knowledge. Some educational systems also present alternative representations such as area and discrete models as a starting point before a number line model is introduced (Alajmi, 2012).

Instruction based on measurement concepts of fractions (i.e., thinking of m/n as m measures of $1/n$; see Kieren, 1980) offers the strongest conceptions of fractions, allowing students to apply their knowledge to new problems and form alternative conceptions of fractions that were not taught (Lamon, 2007). The measurement construct of fractions was further delineated by Wilkins and Norton (2018). They summarized studies involving more than 300 students to propose a hierarchy of fraction schemes, beginning with part-whole to measurement concepts of fractions: (a) part-whole scheme (PWS), (b) measurement scheme for unit fractions (MSUF), (c) measurement scheme for proper fractions (MSPF), and (d) generalized measurement scheme for fractions (GMSF). The PWS and MSUF are loosely inverses of each other– partitioning a whole into n parts versus determining the fractional size of a unit fraction by iterating the unit fraction to produce the whole. The action associated with MSUF involves producing a whole from a proper fraction, and the action associated with GMSF involves producing a whole from any fraction (including improper fractions). The results collectively show that these schemes developed sequentially (i.e., the development of PWS preceded the development of MSUF, and so on).

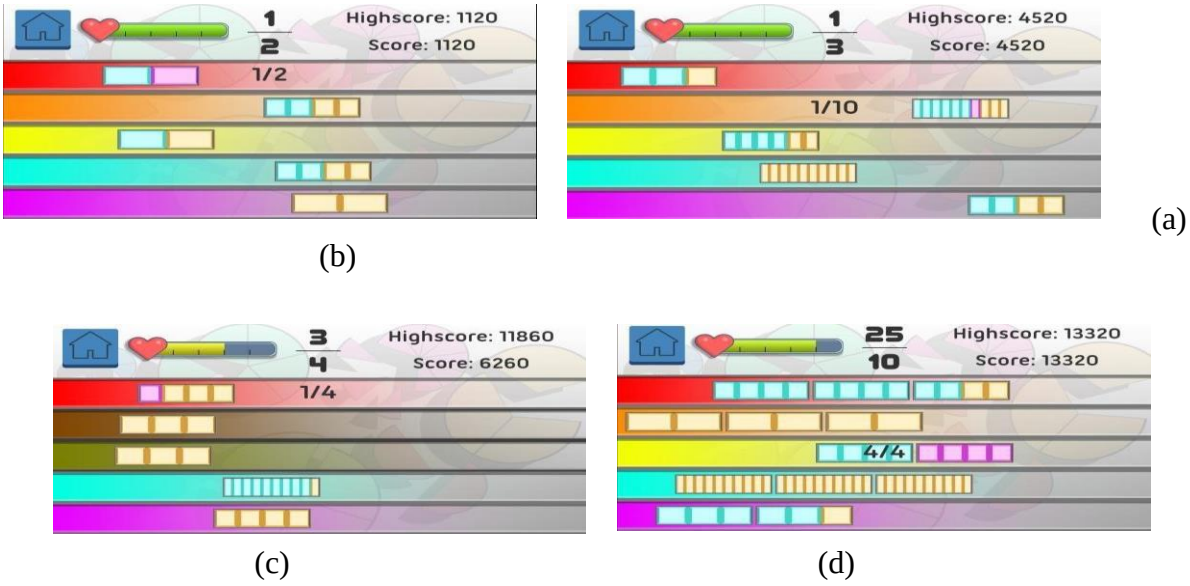
These fraction schemes are aligned with the Philippine DepEd Most Essential Learning Competencies (MELCS) (DepEd, 2020), where students learn fractions in progression: from ½ and ¼ (Grade 1), followed by unit fractions and proper fractions (Grade 2), improper fractions (Grade 3), equivalent fractions and addition and subtraction of fractions (Grade 4). An app and supplementary videos based on this hierarchy are expected to develop a more robust understanding of fractions, compared to learning materials concentrating on part-whole comparisons alone (Lamon, 2007; Wilkins & Norton, 2018).

3. The *Moving Fractions* App

3.1 App Description

In *Moving Fractions*, a learner may choose to play the following levels: ½ and ¼ fractions (Figure 1(a)), unit fractions (Figure 1(b)), proper fractions (Figure 1(c)), improper fractions (Figure 1(d)) and proper, improper, and mixed fractions combined. A fraction is given on the top of the screen, such as 1/3 in Figure 1(b), while partitioned rectangles or fraction bars continuously move to the right. The learner must click on any row where the given fraction can be represented. In Figure 1(b), the fraction 1/3 can be represented by the fraction bars in the first row (with a bar divided into three) or in the third row (with a bar divided into six). If the learner clicks on any of these rows, then the fraction bar on that row would slow down its movement, and the given fraction would be shaded blue. Returning to the example in Figure 1(b), if the learner selects the first row, then the fraction bar would have an additional 1/3 shaded blue, so that the entire rectangle would be shaded blue. If this happens (i.e., a fraction bar is completely shaded), then the bar

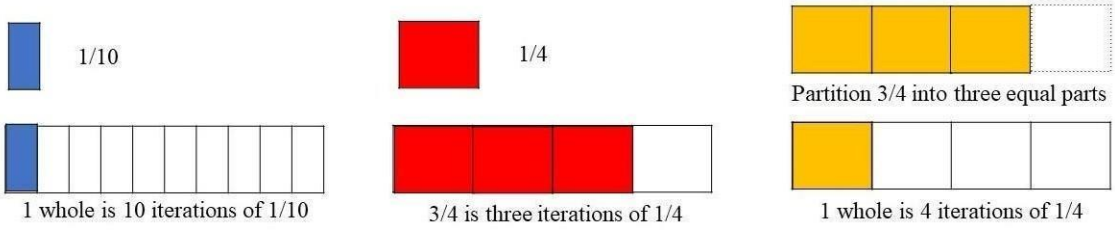
moves back to the left end of the screen. The goal is to delay the forward motion of the fraction bars and prevent them from reaching the right end of the screen by correctly selecting the fraction bars that can represent the given fraction. If a fraction bar reaches the right end of the screen, then that row would be darkened (such as the second and third rows in Figure 1(c)). Each time a row is darkened, the number of "lives" indicated by the scrollbar with a heart on the upper left of the screen decreases by one. The game ends when fraction bars in the five rows reach the right end of the screen. *Moving Fractions* is available as an Android app that can run in Android- powered smartphones or tablets or as a computer app that can be used in Windows personal computers or laptops.



(a) (b) (c) (d)

Figure 1. Screenshots showing four of the five levels in *Moving Fractions*.

In playing the different levels in *Moving Fractions*, learners understand fractions by developing the four schemes proposed by Wilkins and Norton (2018). When learners play the 1/2 and 1/4 level (Figure 1(a)) and the unit fractions level (Figure 1(b)), they see the relative size of a unit fraction to a whole and realize that *n* iterations of 1/*n* produce 1 whole. For example, 10 iterations of 1/10 produce 1 whole, thus developing the MSUF (Figure 2(a)). In playing the proper fractions level (Figure 1(c)), learners see the relative size of a proper fraction *m/n* to a whole and to the unit fraction 1/*n*, thus developing PWS and MSPF (Figures 2(b) and 2(c)). Learners operating with MSPF realize that a whole can be reproduced by partitioning 3/4 into three equal parts and iterating one of those 1/4 parts four times to produce a whole (Figure 2(c)). Similarly, when playing the improper fractions level (Figure 1(d)) learners see the relative size of an improper fraction *m/n* to a whole and to the unit fraction 1/*n*, hence developing GMSF.



(a) (b) (c)

Figure 2. Three of the four fraction schemes are (a) MSUF, (b) PWS and (c) MSPF.

The learning progression from PWS to GMSF was used in the development of other apps, such as in the CandyFactory app (Aslan, Norton, & Balci, 2012). However, *Moving Fractions* has distinct game mechanics that differentiate it from the CandyFactory app. In the CandyFactory app, children learn partitioning and iterating within the context of a company that needs to fill orders of candy that meet certain requirements in terms of its size. *Moving Fractions* demonstrates the fraction schemes more indirectly and through a faster-paced game-like approach. The next section discusses the game- design features of *Moving Fractions* in more detail.

3.2 Game-design Factors

Aligned with the objectives of the ongoing project (De Las Peñas et al., 2021), the authors have been developing mathematical resources that are based on sound pedagogical research and that can prove beneficial even for students who are in distance learning modes. One way this has been achieved is by developing mathematical apps such as *Moving Fractions*. By integrating game-based elements in *Moving Fractions*, it is aimed for learners to be more engaged and interested with studying fractions that may lead to their development of analytical thinking, strategizing, and problem-solving that are important skills in STEM education (Vahidy, 2019). Furthermore, it is envisioned that *Moving Fractions* can be a mathematical resource that is "not only aligned with the curriculum but [is] also creative, fun and provide[s] opportunities to learn" (Miller, 2018, p. 9).

The development of the *Moving Fractions* app as an educational game is also motivated by previous studies where classroom teaching activities have been transformed into game-based versions. Hu and Shang (2018), for example, designed gamified teaching activities using mobile applications.
More recently, Verzosa et al. (2021) developed a game-based version (i.e., an app called *Catch the Carrot*) of traditional number estimation activities for grade school students.

Moving Fractions has been developed in consideration of the 11 interrelated game-design factors in the Game-based Learning (GBL) Design Model of Shi & Shih (2015). These game-design factors are game goal, game mechanism, game fantasy, game value, interaction, freedom, narrative, sensation, challenges, sociality, and mystery.

As previously mentioned, the *game goal* of *Moving Fractions* is, at each turn, to identify the occurrence of a given fraction in five rows each containing one or more partitioned rectangles. The game goal facilitates the learning of fractions through the fraction schemes of Wilkins and Norton (2018). Figure 3 following the layout used by Shi & Shih for the GBL Design Model (Shi & Shih, 2015), further presents the game-design factors as they relate to *Moving Fractions*.

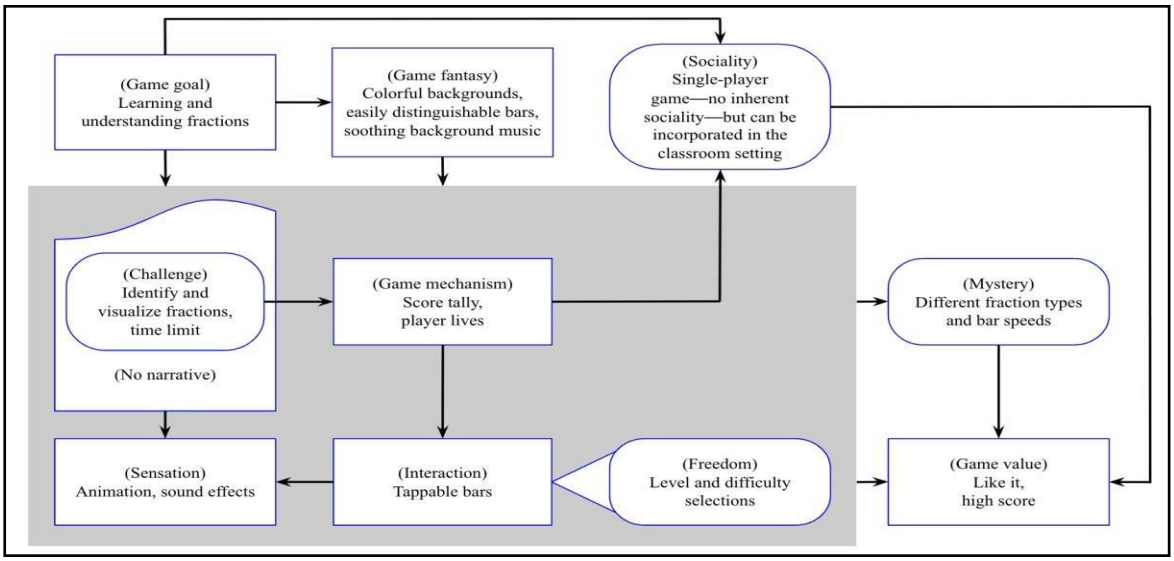


Figure 3. The GBL Design Model for *Moving Fractions*.

The *game mechanism* involves an arcade-type format in which learners must score as much as they can with a limited number of lives. Learners must keep track of the horizontally moving rectangles and quickly tap (*interaction*) correct rows—this increases the game's excitement and replayability.
Moreover, to allow learners to adjust to their current ability levels, the game provides the learners the *freedom* to select the level and difficulty they want to play.

As *Moving Fractions* is intended to be a straightforward replayable game with simple objectives, it has been decided not to integrate *narrative* and *sociability* elements into the game. Instead, the focus is to build the *game fantasy* through colorful backgrounds and soothing music. The animations and sound effects also enhance *sensation* within the game, making it more immersive and exciting for learners. It must be noted, however, that while *sociability* is not built inherently into the game, it may be incorporated when the game is played by multiple learners.

The *game value* depends largely on whether the learner likes the game and is motivated to achieve high scores. This is enhanced by the game's *challenges* that correspond to the increasing difficulty as one progresses through topics on fractions and applying this knowledge to play the game. The availability of multiple levels also adds to the game's *mystery* as learners can become curious about the new elements that can appear in the next levels.

4. Supplementary Fraction Videos

A supplementary tool developed to help promote a deep conceptual understanding of fractions is a series of videos demonstrating various ways that fractions may be represented. These videos are different from existing videos that are available online. First, these are short yet contain three models or representations: the area model, set model and number line model. The use of multiple representations provides a powerful means by which learners can visualize the idea of fractions. Second, the videos also present some questions at the end, to engage the viewers in the thinking process. Third, the text is minimized to lessen the viewers' cognitive demand.

The first video introduces the learner to the concept of ½ and ¼. A representation of these fractions using the area model shows objects (such as a rectangle, circle, triangle, heart, fruit) breaking into two or four equal parts (Figure 4(a)). This helps a learner grasp the idea of these fractions in terms of a part-whole scheme (Wilkins & Norton, 2018). The set model representations of ½ and ¼ shows a set of discrete objects being partitioned into two groups with the same number of objects is shown (Figure 4(b)). Here the learner relates ½ to the notion of segregating one set of objects into two smaller groups with an equal number of elements. Following the definition given above (Wu, 2010), a third representation of the unit fractions is through a number line model. For instance, the portion of the number line from 0 to 1 is shown as being split by a midpoint mark and the fraction ½ is visualized by the animation of a jump from 0 to the midpoint mark (Figure 4(c)). A similar number line model is used to depict ¼.

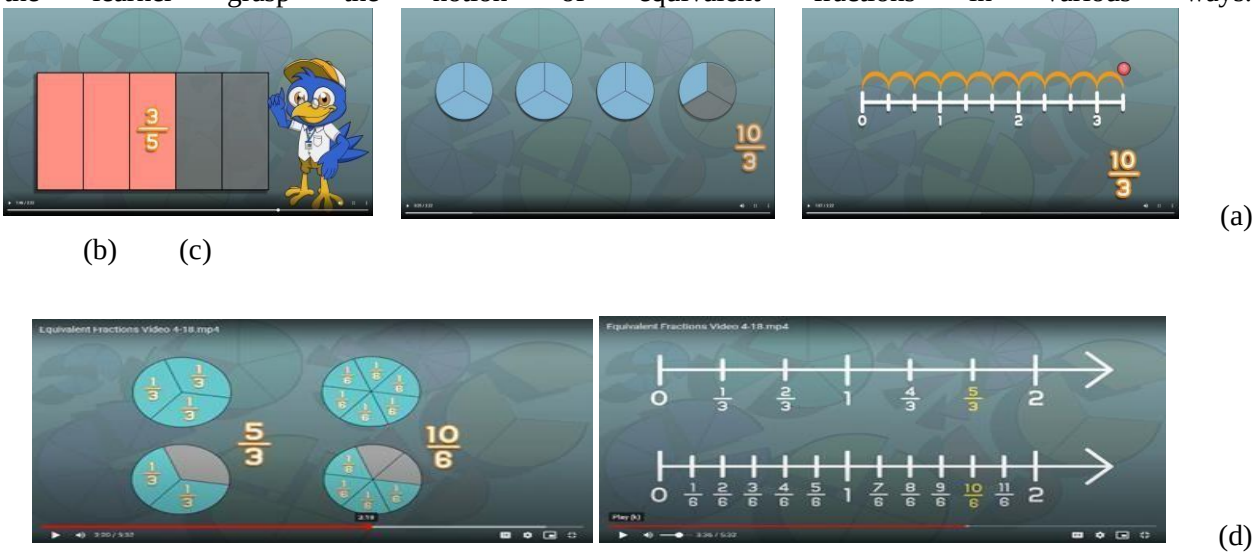


(a) (b) (c)

Figure 4. Representations of ½ using (a) area model; (b) set model; and (c) number line model.

These multiple representations of fractions are reinforced in succeeding videos that depict proper and improper fractions, equivalent fractions, addition, and subtraction of fractions (Figure 5). The concepts demonstrated in the videos build on each other. A video on *Equivalent Fractions* shows the equivalence of the (improper) fractions 5/3 and 10/6 using the area model (Figure 5(d)) and the number line model (Figure

5(e)). The repetitive use of various representations to illustrate other fractions which are equivalent helps the learner grasp the notion of equivalent fractions in various ways.



(a) (b) (c) (d) (e)

Figure 5. (a) Proper fraction ⅗ (area model), (b) improper fraction 10/3 (area model), (c) improper fraction 10/3 (number line model), (d) equivalent fractions 5/3 and 10/3 (area model), and (e) equivalent fractions 5/3 and 10/3 (number line model).

Another video on *Addition and Subtraction* of similar fractions animates the idea of putting together or removing several copies of a unit. For instance, the operation 5/2 + 7/2 is depicted in Figure 6(a) which shows five and six copies of ½ of a papaya, respectively. The sum is shown by putting together the unit pieces to make 12/2 or 6 papayas (Figure 6(b)). The notion of subtraction is similarly made more accessible by animating the removal of unit pieces from a whole (Figure 6(c)).



(a) (b) (c)

Figure 6. Illustrations of (a) 5/2 + 7/2, (b) sum of 5/2+7/2, and (c) 14/5 – 8/5.

5. Integration and Use of *Moving Fractions*

This section describes how Moving Fractions can be utilized as a pedagogical tool based on the RAT (Replacement, Amplification, Transformation) framework (Hughes, Thomas, & Scharber, 2006). The choice of replacement, amplification, transformation or combination of any of this may depend on the classroom context, nature of students and curriculum content and goals.

Based on this framework, the Moving Fractions app can serve as a replacement for the traditional method of teaching fraction understanding without changing content and learning goals in relation to fractions remain the same. For example, the fraction strips or circular regions which are often used by teachers to illustrate and represent fractions are now replaced by the moving fraction bars in the app. The task of representing fractions (unit, proper, improper) remains the same but the static method of presenting

them is now changed to the colorful and dynamic representations of fractions. Such representations can be used by the teacher to discuss relationships between different types of fractions.

Moreover, integration of the app can go beyond the replacement category. The framework suggests that technology can amplify learning. This happens when concepts are presented more clearly and efficiently. The app provides a more precise and accurate representation of fractions than can be done without technology. For example, the fraction ⅕ is difficult to illustrate by hand. But with the app, this fraction is presented accurately. The design of the app can also help students understand different representations of fractions (e.g., ½ is 2/4 or ⅓ is 2/6). Teachers can maximize this feature of the app so that misconceptions about fractions can be addressed.

Finally, technology can involve transformation of instructional methods, learning process, and/or the content. The app may spur a transformation in teachers' instructional practices from being teacher-centric or teacher-controlled to student-centered. By playing with the app, the learning process is transformed as the feedback mechanism heightens student engagement and facilitates opportunities for discovery. The app can be used as a tool for collaborative activity where students can communicate their thinking and experiences in playing the app. Such activity also helps students to work with others and create a positive learning environment. Learning need not come solely from the teacher's input but from the app itself. Further, the game-like environment of the app allows students to create efficient strategies in order to prolong their life in the game.

6. Conclusion and Future Direction

Competency in fractions is identified as an essential skill for entry into STEM-related fields. The *Moving Fractions* app and the supplementary fraction videos were designed to develop learners' knowledge of fractions. The app utilizes game-design principles to concretize the learning trajectory of fraction learning, from part-whole comparisons to a more robust understanding of the measurement concept of fractions. Meanwhile, the fraction videos immerse learners in a range of fraction representations that can extend their initial part-whole understanding of the fraction concepts. The next step is to package the app and videos and gather empirical data on how the fraction learning package can promote fraction understanding.

Acknowledgements

This paper is one of the outputs of the Ateneo Mathplus Resources Laboratory housed at the Department of Mathematics, School of Science and Engineering, Ateneo de Manila University. The authors thank the Department of Science and Technology-Philippine Council for Industry, Energy, and Emerging Technology Research and Development (DOST-PCIEERD) and the University Research Council (URC), Ateneo de Manila University for the support of the development of the mathematical resources for Grades 1 to 11 Mathematics. Acknowledgement also goes to Dr. Ma. Mercedes T. Rodrigo and the Ateneo Laboratory for the Learning Sciences (ALLS) for providing technical assistance. We thank our app developers Victor Antonio M. Ortega, Mr. Nigel Benedict Cargo, Mr. Jose Teodoro Lacson, and Mr. Amiel Damian F. Justiniani for the apps and video art assets.

References

Alajmi, A. H. (2012). How do elementary school textbooks address fractions? A review of mathematics textbooks in the USA, Japan, and Kuwait. *Educational Studies in Mathematics, 79,* 239-261.

Aslan, S., Norton, A., & Balci, O. (2012). CandyFactory educational game for iPad (Version 2.0). Blacksburg: Virginia Tech. https://apps.apple.com/us/app/candyfactory-educational-game-for-ipad/id533213891

Bansilal, S. & Ubah, I.J.A. (2020). The use of cross multiplication and other mal–rules in fraction operations by pre-service teachers. *The Journal of Mathematical Behavior, 58,* 100781.

Booth, J. L., & Newton, K. J. (2012). Fractions: Could they really be the gatekeeper's doorman? *Contemporary Educational Psychology, 37,* 247–253.

De Las Peñas, M. L. A. N., Aberin, M. A. Q., Garciano, A. D., Sarmiento, J. F., Tolentino, M. A. C., Verzosa, D. M. B. (2021). *Annual report (Department of Science and Technology-Philippine Council for Industry,*

Energy and Emerging Technology Research and Development): Mathematical resources for distance learning utilizing community LTE networks and television frequencies.

Department of Education. (2020). *K to 12 most essential learning competencies with corresponding CG codes*. Retrieved July 12, 2020, from https://commons.deped.gov.ph/K-to-12-MELCS-with-CG-Codes.pdf.

Erhel, S. & Jamet, E. (2013). Digital game-based learning: impact of instructions and feedback on motivation and learning effectiveness. *Computers in Education, 67*, 156–167.

Hu, R., & Shang, J. (2018). Application of gamification to blended learning in elementary math instructional design. In: S. Cheung, L. Kwok, K. Kubota, LK. Lee, & J. Tokito (Eds.), *Blended Learning. Enhancing Learning Success. ICBL 2018. Lecture Notes in Computer Science* (Vol. 10949, pp. 93-104). Springer, Cham.

Hughes, J. E., Thomas, R., & Scharber, C. (2006). Assessing technology integration: The RAT – Replacement, Amplification, and Transformation – Framework. In C. M. Crawford, R. Carlsen, K. McFerrin, J. Price, R. Weber, & D. A. Willis (Eds.), *Proceedings of SITE 2006--Society for Information Technology & Teacher Education International Conference* (pp. 1616-1620). Orlando, Florida, USA: Association for the Advancement of Computing in Education (AACE).

Kieren, T. (1980). The rational number construct–Its elements and mechanisms. In T. Kieren (Ed.), *Recent research on number learning* (pp. 125–149). Columbus: OH: ERIC/SMEAC.

Lamon, S. J. (2007). Rational numbers and proportional reasoning: towards a theoretical framework for research. In F. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 629–667). Reston: NCTM.

Larkin, L. (2013). Mathematics education: Is there an app for that? In V. Steinle, L. Ball, & C. Bardina (Eds.), *Mathematics education: Yesterday, today and tomorrow.* Melbourne: MERGA.

Miller, T. (2018). Developing numeracy skills using interactive technology in a play-based environment. *International Journal of STEM Education, 5*, 1-11.

Ninaus, M., Kiili, K., McMullen, J., & Moeller, K. (2016). A game-based approach to examining students' conceptual knowledge of fractions. In R. Bottino & J. Jeuring (Eds.) *GALA 2016* (pp. 37-49). Utrecht: Springer.

Ngo, F. (2018). Fractions in college: How basic math remediation impacts community college students. *Research in Higher Education, 60*(4), 485-520.

Schneider, M., Merz, S., Stricker, J., De Smedt, B., Torbeyns, J., Verschaffel, L., & Luwel, K. (2018). Associations of number line estimation with mathematical competence: a meta-analysis. *Child Development, 89*(5), 1467-1484.

Shi, Y.-R., & Shih, J.-L. (2015). Game factors and game-based learning design model. *International Journal of Computer Games Technology, 2015*, 1-11.

Vahidy, J. (2019). Enhancing STEM learning through technology. *Technology and the Curriculum: Summer 2019.* Lethbridge, AB, Canada: Power Learning Solutions.

Verzosa, D. M. B., De Las Peñas, M. L. A. N., Aberin, M. A. Q., Garciano, A. D., Sarmiento, J. F., & Tolentino, M. A., C. (2021). Development of a gamified number line app for teaching estimation and number sense in grades 1 to 7. In M. M. T. Rodrigo, S. Iyer, & A. Mitrovic (Eds.), *Proceedings of the 29th International Conference on Computers in Education* (pp. 648-656). Asia-Pacific Society for Computers in Education.

Wang, Z. (2013). Why students choose STEM majors: Motivation, high school learning and postsecondary context of support. *American Educational Research Journal, 50*(5), 1081-1121.

Wilkins, J. L. M., & Norton, A. (2018). Learning progression toward a measurement concept of fractions. *International Journal of STEM Education, 5*, 27.

Wu, H., (2010). *Understanding numbers in elementary school mathematics.* Providence, RI: American Mathematical Society.

Ateneo de Manila University

Archīum Ateneo

Mathematics Faculty Publications | Mathematics Department

12-2019

Digital Simulations for Grade 7 to 10 Mathematics

Ma. Louise Antonette N. De Las Peñas
Ateneo de Manila University, mdelaspenas@ateneo.edu

Debbie Marie Verzosa
University of Southern Mindanao

Maria Alva Q. Aberin
Ateneo de Manila University, maberin@ateneo.edu

Len Patrick Dominic M. Garces
Ateneo de Manila University

Flordeliza F. Francisco
Ateneo de Manila University, ffrancisco@ateneo.edu

See next page for additional authors

Follow this and additional works at: https://archium.ateneo.edu/mathematics-faculty-pubs

 Part of the Educational Technology Commons, Mathematics Commons, and the Science and Mathematics Education Commons

Custom Citation

De las Penas, M. L. A. N., Verzosa, D. M., Aberin, M. A. Q., Garces, L. P. D. M., Francisco, F. F., Bautista, E. P., Tolentino, M. A. C., & Tabares, W. C. (2019). Digital Simulations for Grade 7 to 10 Mathematics. Philippine Journal of Science, 148(4), 735-749.

This Article is brought to you for free and open access by the Mathematics Department at Archīum Ateneo. It has been accepted for inclusion in Mathematics Faculty Publications by an authorized administrator of Archīum Ateneo. For more information, please contact oadrcw.ls@ateneo.edu.

Authors

Ma. Louise Antonette N. De Las Peñas, Debbie Marie Verzosa, Maria Alva Q. Aberin, Len Patrick Dominic M. Garces, Flordeliza F. Francisco, Evangeline P. Bautista, Mark Anthony C. Tolentino, and Winfer C. Tabares

This article is available at Archīum Ateneo: https://archium.ateneo.edu/mathematics-faculty-pubs/46

Philippine Journal of Science
148 (4): 743-757, December 2019
ISSN 0031 - 7683
Date Received: 8 July 2019

Digital Simulations for Grade 7 to 10 Mathematics

Ma. Louise Antonette N. De Las Peñas[1]*, Debbie Marie B. Verzosa[1,2], Maria Alva Q. Aberin[1], Len Patrick Dominic M. Garces[1], Flordeliza F. Francisco[1], Evangeline P. Bautista[1], Mark Anthony C. Tolentino[1], and Winfer C. Tabares[1]

[1]Department of Mathematics, School of Science and Engineering,
Ateneo de Manila University, Quezon City, Metro Manila 1108 Philippines
[2]Department of Mathematics and Statistics, University of Southern Mindanao,
Kabacan, Cotabato 9407 Philippines

This article describes a Department of Science and Technology – Philippine Council for Industry, Energy and Emerging Technology (DOST-PCIEERD) project aimed to facilitate the implementation of the mathematical objectives raised by the Department of Education's (DepEd) K to 12 program in the Philippines through the use of innovative digital technologies. In particular, a selection of application software ("apps") were created for Grade 7 to 10 mathematics that covered topics indicated in the five strands outlined in the K to 12 program – namely (1) number, (2) geometry, (3) measurement, (4) patterns and algebra, and (5) statistics and probability. The design of the apps was informed by an amalgamated framework of the Cognitive Theory of Multimedia Learning (Mayer 2005) and Mathematical Theories of Representation (Goldin 1998). The design was informed by how students learn and how students learn mathematics. The project also aimed to design manipulable software that allows learners to construct and grapple with their mental representations of mathematical concepts. This paper describes a selection of the apps designed by the project and how their features were informed by the theoretical framework. It also presents results from pilot studies that demonstrate the apps' potential to increase performance, facilitate conceptual development, and increase learners' engagement.

Keywords: application, K to 12 program, mathematical software, mobile technology, technology in mathematics education

INTRODUCTION

Technology has become an important – if not a necessary – part of the mathematics classroom not only because it enhances learning, but also because the ability to use technological tools is a necessary skill in today's world. The Philippine K to 12 mathematics curriculum framework recognizes that "the use of appropriate tools is needed in teaching mathematics. These include manipulative objects, measuring devices, calculators, computers, smartphones and tablet PCs (personal computers), and the internet" (DepEd 2012). Various computing and graphing devices make lessons more interesting and make exploration and mathematical discovery possible (NCTM 2000). However, the extent and the way technology is used in the teaching of mathematics depend not only on the availability of resources (software, technological tools, infrastructure (Internet), and computer laboratories) but also on the teacher's ability and disposition to use these devices. The primary objective of this project, which was funded by DOST-PCIEERD for the period 2015–2018, was to augment available technological tools for Grade 7 to 10

*Corresponding Author: mdelaspenas@ateneo.edu

mathematics by creating interactive apps. Despite a large number of existing apps in the market, only a very small portion provides opportunities for the user to manipulate objects, form conjectures, and refine ideas (Goodwin and Highfield 2013). Instead, most apps are instructive, founded on "drill-and-practice" pedagogic design.

This paper discusses the design and development process of the apps created under the research project. The scientific framework guiding this process is also presented. The designed apps include manipulable components and visual feedback to guide students towards mathematical learning. The potential effect of adopting these technologies on students' learning outcomes is also discussed. It is hoped that this work will be helpful to researchers who want to develop their own apps using the scientific framework highlighted in this paper.

The apps have the features of mobile technology (MT) and are easily accessible via Android devices, but it is also possible to use these on computers and laptops using a free Android emulator such as Genymotion (https://www.genymotion.com/). To enhance teachers' competency, the software is supported by a set of teaching guides. These teaching guides are linked to the learning competencies prescribed by the current DepEd mathematics curriculum (DepEd 2012). For greater accessibility, the apps will only require an internet connection during installation but not during use. The mobile apps and teaching guides are freely available for download from the MathPlus Resources website (https://mathplusresources.wordpress.com/). A total of 14 Android apps and 41 *GeoGebra* applets were developed under the project.

The developed resources and the project website were launched in a teacher training workshop-seminar attended by Grade 7 to 10 teachers from schools in Metro Manila. Throughout the app development phase, the mobile apps were pilot-tested with public and private school students to gather input for refining the apps. Two mobile apps, namely *ProveIt* and *AlgeOps*, were tested in more depth to assess their impact on students' understanding of specific mathematical concepts. The results of the teachers' evaluation of mobile apps and the testing of *ProveIt* and *AlgeOps* are discussed later in this paper.

Review of Related Literature

Research on the use of technology in the classroom has provided evidence that it is a tool for capacity building of students across levels and disciplines, including mathematics. It improves educational quality by affording more opportunities to develop higher-order thinking skills (Fu 2013). This is further enhanced by the development of dynamic geometry and computer algebra systems. Various studies have explored how to enhance students' proving skills in a range of areas such as geometry, abstract algebra, and calculus using dynamic geometry (de las Peñas and Bautista 2008, 2011). Further, these dynamic tools have also been used to facilitate the understanding of fundamental ideas of calculus (Verzosa *et al.* 2014), to introduce algebra concepts such as the concept of variable and symbolic representations between variables (Mackrell 2012), to facilitate visualization of mathematical concepts (Taka *et al.* 2010), and to develop students creativity (Shelomovskiy and Nosulya 2012).

In recent years we have seen the rise in the use of MT (*e.g.*, smartphones, tablets) in mathematical learning. Accompanying the rising popularity of MT is the growth in the number of mobile apps in mathematics (Larkin 2015) and studies have been carried out to measure their impact on mathematics education (Calder *et al.* 2018, Fabian *et al.* 2018, Larkin and Calder 2016). MT also has the potential to transform the learning experience and enhance mathematics learning opportunities. It is portable, easy to boot up, and can easily fit in one's hand. Mobile apps are more easily accessible and popular among students. Students may thus tend to carry and use them constantly (Norris *et al.* 2011). Among the potential benefits of mobile learning are encouraging learning anytime, anywhere; enabling a personalized learning experience (Shuler 2009); and inducing a positive change in students' attitudes and engagement (Fabian *et al.* 2018). In Latin America, MT (termed pocket school) is used as a sustainable literacy option for underserved indigenous children (Kima *et al.* 2008). MT's ease of operation, allied with the students' interaction being focused primarily on touch and sight, can make their use intuitive for learners (Calder and Campbell 2016). Research studies have also highlighted the effectiveness of MT in other aspects of mathematics learning [*e.g.*, those in Calder *et al.* (2018)].

In the Philippines, the K to 12 Curriculum of the DepEd recognizes that calculators, computers, smartphones, tablets, and PCs can be appropriate tools in teaching mathematics. These can help learners demonstrate understanding and appreciation of key concepts and principles of mathematics as applied in problem-solving, communicating, reasoning, and making connections, representations, and decisions in real life (DepEd 2012).

In the past year, different strategies have been suggested to maximize the use of existing technology in Philippine schools (Bautista and de las Peñas 2006). School administrators have invested in affordable graphics calculator kits for use of faculty and students. Free software packages, such as Wingeom and GRAPES, have provided opportunities for experimentation and discovery with topics in geometry, analytic geometry, trigonometry, and calculus (de las Peñas and Bautista 2008). *GeoGebra* – free dynamic software that has capabilities for geometry, algebra, and statistics – has been a popular choice for

Philippine schools (de las Peñas and Bautista 2011). In recent years, MT has emerged as a tool in studying and teaching mathematics. In 2012, the Science Education Institute and Advanced Science Technology Institute under the DOST launched the project entitled "Technology Package for Student Learning Empowerment: PC Tablet Math Lesson Courseware Development," which involved the design of interactive learning materials in Grade 1 Mathematics that run on tablet PCs (Lee-Chua 2012). To date, there is a courseware involving particular math topics in Grades 1 to 8 that run on Android devices (DOST Courseware may be accessed at www.dost.sei.gov.ph). Studies have also been conducted on the use of MT in high school mathematics (*e.g.*, Etcuban and Pantinople 2018).

Scientific Basis and Framework

The creation of the apps in this study was informed by amalgamating the Cognitive Theory of Multimedia Learning (Mayer 2005) with Mathematical Theories of Representation (Goldin 1998).

Mayer's (2005) Cognitive Theory of Multimedia Learning hinges on theoretical and empirical evidence that learning is maximized when the instructional medium is aligned with how the human mind works. It further argues that the instructional medium is always a reflection of the designer's understanding of how the human mind processes information. As an example in mathematics, computer games that mostly consist of drill and practice sessions reflect the designer's view of the learner from a behavioral perspective. That is, learning is often equated to observable responses to certain tasks or stimuli, with little attention to how a learner's internal representations or mental models facilitate or stifle problem-solving success.

Based on the Cognitive Theory of Multimedia Learning, instructional learning outcomes were designed, informed by how students learn, and – in particular – how students learn mathematics. Mathematical ideas are essentially communicated through representations, which are inventions and reinventions that are indicative of the cultural, semiotic, and technological advancement of a given society (Bu *et al.* 2011). Goldin (1998) proposed a unified psychological model of mathematical learning and problem solving based on a theory of representations. He states, "the overarching goal should be to foster in students the *construction of powerful, internal systems of representation*" (p. 159; italics in original). Also, according to Bu *et al.* (2011), "all mathematical ideas are dynamic in the mind of a mathematically competent person" (p. 23). In this sense, technologies may promote mathematical learning if it provides learners the opportunity to construct and manipulate representations in a way that supports interactions between the learner and the representations themselves.

Despite a large number of commercial or freely available educational software, very few of these allow learners to construct and grapple with their mental representations of mathematical concepts (Goodwin and Highfield 2013). To address this gap, we designed a manipulable software where students can construct representations, test conjectures, reason hypothetically, visualize, analyze, interpret, and discover. A second important feature is the provision of visual feedback. The provision of immediate formative feedback is integrated with a wide range of technology-based tools and game-based apps and is consistent with experiential learning theory (Kiili 2005). Instantaneous feedback allows software or app to act as a tutor (Johnston-Wilder and Pimm 2005), help students modify their conceptions (Avraamidou *et al.* 2015), and increase their motivation (Gros 2015). The feedback should not just inform students that they are correct or not. Instead, the feedback includes animations that demonstrate why a particular response is correct or incorrect.

METHODOLOGY AND DESIGN OF THE APPLICATION SOFTWARE

The team involved in this project consisted of mathematicians and mathematics educators from the Department of Mathematics, Ateneo de Manila University. The process of app development involved the steps given below. This can be compared to existing app development frameworks in the literature, such as Pugh's design process model (Wong *et al.* 2012) and the NCA Model of Design and Development (Nisiyatussani *et al.* 2018). Two types of apps were designed to enhance the teaching and learning of mathematics in Grade 7 to 10 – software apps for Android devices and applets created using the free dynamic geometry software *GeoGebra* (https://www.geogebra.org/).

1. ***Data collection and review of related literature.*** The first few months of the project included a thorough review and survey of related literature on currently available software and new technologies and various methods on the use of technology in teaching mathematics. The DepEd Grade 7 to 10 curriculum in mathematics and prescribed competencies were reviewed and studied. The team also studied the National Achievement Test results that were obtained upon request from the DepEd's National Education Testing and Research Center.
2. ***Identification of topics.*** With the materials gathered in the previous step, the team members identified the grade levels and topics in the different mathematics strands that could be taught and studied with interactive software, which had a huge potential for supporting student learning. Topics that are linked to the least learned competencies (provided by DepEd) were taken into consideration. The apps covered

topics indicated in the five strands of mathematics outlined in the K to 12 Program – namely (1) number, (2) geometry (3) measurement, (4) patterns and algebra, and (5) statistics and probability.

3. ***Conceptualization of design.*** Team members conceptualize the design on a topic or activity that will be implemented on the app. An example is shown in Figure 1 on the design of Triangle Congruence on app *ProveIt.* The teaching guide is also written at this stage.

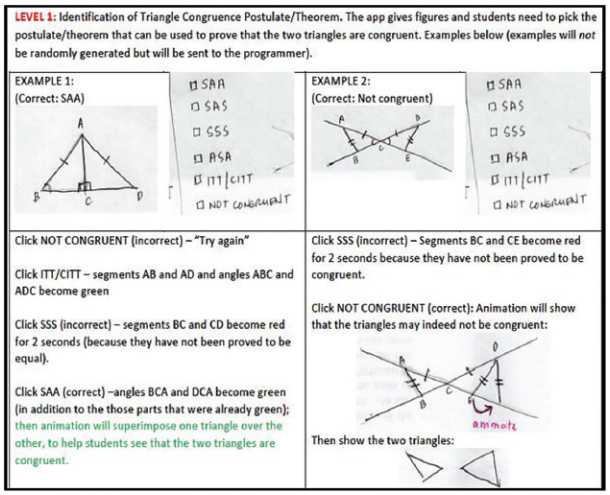


Figure 1. Conceptual design of Triangle Congruence in *ProveIt*.

4. ***Detail of the design and prototyping.*** Another team member writes the idea in the format of a computer program that will be sent to the programmer. This describes the flow of the app. See Figure 2 on suggested commands for the Multiply feature of *AlgeTiles.* A prototype will then be shown by the programmer and, after suggestions and discussions with the team, the app will be created.

a. App generates question
   i. If 1-variable, App generates (ax+b)(cx+d) as follows
      1. generate a and c from the set {-3,-1,0,1,3}
      2. generate b from the set
         a. {-9 to 9-3a} if a >=0
         b. {-9-3a to 9} if a < 0
      3. generate d from the set
         a. {-9 to 9-3c} if c >=0
         b. {-9-3c to 9} if c < 0
   ii. If 2-variables, App generates (ax+by+e)(cx+dy+f) where
      1. generate a and c from the set {-2,-1,0,1,2}
      2. generate b from the set
         a. {-2 to 2-a} if a >=0
         b. {-2-2a to 2} if a < 0
      3. generate d from the set
         a. {-2 to 2-c} if c >=0
         b. {-2-2c to 2} if c < 0
      4. generate e from the set
         a. {-8 to 8-3a-3b} if a >=0 and b >=0
         b. {-8-3b to 8-3a} if a >=0 and b < 0
         c. {-8-3a to 8-3b} if a < 0 and b >= 0
         d. {-8-3a-3b to 8} if a < 0 and b < 0

Figure 2. Suggested flow for the Multiply feature of *AlgeTiles*.

5. ***App delivery.*** The app is delivered by the programmer and presented to the author. The app undergoes revisions after comments from the author and/another team member. The teaching guide is rewritten in response to the full capabilities and limitations of the apps.
6. ***Field test.*** The apps were field-tested on teachers and students from a private and a public high school before finalization. We tested the apps on groups of students from various grade levels – both those who have experience using tablets in their lessons and those who were new to the use of technology in the classroom. The app developers and the team conceptualized the questions to be asked during the field-testing for the teachers and students to ensure that substantial comments can be gathered on the following aspects: (i) functionality and routines present in the app, (ii) ease of use of the apps, (iii) aesthetic look and layout, (iv) retention of user's interest, and (v) ability of the app to address the required learning competency. Questions ranged from the interface and responsiveness of the app to touch, ease of use, clarity of instructions, design, display and colors, and experience when using the app.
7. ***Consolidation of field test results.*** The comments from the field test and other team members are assessed by author and the programmer and incorporated into the app. Though the *GeoGebra* apps were not field-tested, comments from team member/s on *GeoGebra* apps were noted and will be taken into consideration during the refinement process.
8. ***Refinement and finalization.*** The apps undergo further refinement and are finalized. The teaching guides are also finalized. The app and teaching guide are uploaded into a website. Teacher training is conducted on the use of apps and teaching guides.

To ensure that the app design is guided by the indicated scientific framework, team members discussed developments in research on mathematical learning. The discussions included broad theoretical perspectives so that the entrenched behaviorist position, which values observable procedural skill and deemphasizes the development of a learner's internal representations, does not dominate the app design. These discussions were crucial because apps reflect the designer's view of mathematical learning (Mayer 2005). Early on, the team members had to understand that providing learners an opportunity to construct representational systems is a key goal of mathematics education (Goldin 1998). In connection, it was decided that each app needed to provide (1) manipulable representations and (2) visual feedback, as shown in Table 1. These features can facilitate exploration, visualization, and justification. In Table 1, we also describe the novelty of each of the app in accordance with these features.

Table 1. List of mobile apps with descriptions of the manipulable representation and visual feedback.

Subject	App	Competencies[a]	Manipulable representation	Visual feedback	Novelty
Algebra	*AlgeTiles*	Factors completely general trinomials Uses models and algebraic methods to find the: (a) product of two binomials; (b) product of the sum and difference of two terms; (c) square of a binomial; (d) cube of a binomial; and (e) product of a binomial and a trinomial.	Students can use virtual algebra tiles to help them in multiplying (Figure 3a) and factoring (Figure 3b) polynomials. The app simulates the filling in of quadrilaterals using these tiles. Students must fill in the quadrilaterals using the correct counts and sizes of tiles	Students use a button (chk) to check if they have done each phase of the solution correctly; at the end, the student inputs an answer and they can verify if it is correct using the chk button	Distinguishing features include a grid as scaffolding for beginning students, a robust handling of negative numbers, and instant feedback
	LinearX	Finds the solution of linear equations of inequalities in one variable	Students use the buttons so that all boxes (representing the variable) are on one side of the equation. When all boxes are on one side, students figure out how many balls are in each box (assuming each box contains the same number of balls) and enter their result on the number line provided (Figure 3c)	When students enter an incorrect answer, the app shows that the boxes would not contain the same number of balls, as initially required	The app allows students to work on diagrams that represent linear equations. Both correct and incorrect answers are simulated by the app so that students see for themselves why their answer is correct or incorrect; for example, suppose there are 3 unknowns (represented by boxes) and 18 objects that have to be divided equally among the 3 boxes: if the student enters the incorrect answer 5, the app simulates placing 5 balls in each box, and reveals that there are 3 balls that are yet to be placed inside the box
	DrawLine	Graphs a linear equation given (a) any two points; (b) the x- and y-intercepts; and (c) the slope and a point on the line Finds the equation of a line given (a) two points; (b) the slope and a point; and(c) the slope and its intercepts	Students can construct their own lines while choosing parameters such as two points, a slope and a y-intercept, or a point and a slope (Figure 3d)	The students formulate the equation of a line and feedback is given if this is correct or not	The app uses separately the three forms of the equation of a line (two-point, slope-intercept, and point-slope) and allows the student to input one representation given another; feedback is provided as well
Geometry	*Angle Xplore*	Derive relationships of adjacent angles using measurements and inductive reasoning Derive relationships of complementary and supplementary angles using measurements and inductive reasoning	Students are required not just to identify complementary or supplementary angles, but it requires them to construct their own, given a set of angles (Figure 4a)	When students choose two such angles, an animation shows whether the two angles can be used to form a 90- or 180-degree angle; when two angles are neither supplementary nor complementary, this is also displayed in the app.	The app allows the students to construct their own representations of complementary and supplementary angles so that they can deduce the definition and properties of complementary and supplementary angles
	ProveIt Triangle Congruence (Level 2)[b]	Proves two triangles are congruent	Students are shown two triangles, which they must show are congruent. The app provides students two enabling prompts: (1) the triangle congruence theorem/postulate that must be used, and (2) the theorems that are needed so that the appropriate triangle congruence theorem/postulate may be applied (Figure 4b)	If students identify the correct property/ theorem and the correct associated triangle parts, feedback is provided.; if incorrect parts are selected, the app notifies the student and gives a hint about the correct parts	The app provides an interactive visual representation of the triangle congruence statement to be proved; using the mobile device's touch screen, the user can select or highlight parts of the figure when answering the proving exercise; the app also provides immediate feedback so that in the case of an incorrect answer the student can amend his/her answer (Verzosa *et al.* 2019)
	GeoGebra Applet: Two-column proof: prove that the given quadrilateral is a parallelogram	Determines the conditions that guarantee a quadrilateral is a parallelogram	Students are asked to prove the theorem "if a quadrilateral is a parallelogram, then its opposite sides are congruent" using the *GeoGebra* app; to navigate the proof, the student clicks the right arrow button. A line of the proof appears and the student fills in the missing details in the proof (Figure 4c)	By ticking a checkbox beside a step in the proof, the students see a visualization or animation of that step	There is no known app on two-column proofs where visual feedback is given for each line of the proof
	GeoGebra Applet: Intersecting Chords	Proves theorems on intersecting chords of a circle	Students can change the diagrams by dragging the points of the configuration (Figure 4d)	When the students drag a point of the configuration, the lengths of the segments change together with the product of the lengths; moreover, the highlighted triangles help the students prove the relationship between the lengths (Figure 4d)	The applet features scaffolding measures that help the student understand and prove the theorems

Table 1 continuation . . .

Geometry	*Geomatch*	Illustrates triangles-angles and sides; illustrates and applies the properties of the different kinds of quadrilaterals	Students can select from two modes (Triangles or Quadrilaterals) and from three speed modes (Slow, Normal, Fast); triangles or quadrilaterals start to appear on the screen one by one; students need to select the triangle or quadrilateral that satisfies the description, which appears at the lower part of the screen (Figure 4e)	Note that the student has three lives to play; an incorrect selection results in a decrease in the number of lives. The game ends when the screen is filled with 16 shapes	The learning app emphasizes the definitions and characteristics of the various types of triangles and quadrilaterals as well as related theorems involving angles, sides, and diagonals
	GeoGebra Applet: Angles of Elevation and Depression	Describes and illustrates angles of elevation and angles of depression	This app contains three examples, which have accompanying drawings; the student can use built-in functions in *GeoGebra* to draw the line of sight of an observer and measure angles of elevation or depression (Figure 4f)	Upon the use of appropriate functions, the line of sight and angle measures are displayed on the screen	In this *GeoGebra* App, the student is tasked to construct the angle of elevation or depression instead of just seeing these labeled in textbook diagrams; this app is believed to be the first app to focus specifically on angles of elevation and depression
Number	*AlgeOps*	Performs fundamental operations on integers Adds and subtracts polynomials	Students can add or remove red/green boxes and balloons until they obtain the appropriate configuration to solve a given problem (Figure 5a)	After the app animates the fading of positive-negative pairs, the student inputs an answer to the given problem; the app verifies if the answer is correct or not	While the app's main focus is polynomial addition and subtraction, the app can also facilitate students' understanding of integer addition and subtraction via the neutralization model and the number line model (Verzosa *et al.* 2018) The app also provides immediate visual and auditory feedback and applies animations to emphasize the neutralization aspect of polynomial or integer addition and subtraction
	Catch the Carrot	Arranges rational numbers on a number line	Students can freely adjust the position of the carrot basket along the number line (Figure 5b)	The student's answer is verified by the animation of a falling carrot; the answer is correct if the carrot falls into the basket positioned by the student	The app is similar to some existing software that requires students to locate certain numbers on the number line; the novelty of this app is in its availability on Android and mobile devices
	GeoGebra Applet: Absolute Value as Distance	Represents the absolute value of a number on a number line as the distance of a number from 0	Students can drag two numbers a and b along the real number line (Figure 5c)	As the numbers a and b are dragged, the app displays the value of $\|a - b\|$ and how it is computed	This app limits the representation of the points to the one-dimensional number line, unlike most other apps that show distances on a 2-dimensional plane; the aim is to draw attention to the concept of linear distance and its relationship to the absolute value
Statistics and Probability	*Statistics*	Organizes data in a frequency distribution table Uses appropriate graphs to represent organized data: pie chart, bar graph, line graph, histogram, and ogive Calculates the measures of central tendency of ungrouped and grouped data; calculates the measures of variability of grouped and ungrouped data Uses appropriate statistical measures in analyzing and interpreting statistical data Draws conclusions from graphic and tabular data and measures of central tendency and variability	Students can adjust the number of classes to be used in the frequency distribution tables and histograms. In the built-in scenarios, the students may also partition the data into subclasses; importing of custom data is also supported (Figure 6a)	Depending on the student's input, the app displays a histogram, frequency distribution table, and measures of central tendency and variability.	The app features two realistic scenarios that help students learn how to do statistics but also where it can be used The app has a very simple interface, requiring only a few inputs from the user, that allows even first-time users to make the most out of the app
	Counting	Counts the number of occurrences of an outcome in an experiment: (a) table; (b) tree diagram; (c) systematic listing; and (d) fundamental counting principle Finds the probability of a simple event; solves problems involving probabilities of simple events	Students can choose which of the ice cream flavors, cones, and toppings are available (Figure 6b)	The app displays a tree diagram showing all possible combinations of the available ice cream flavors, cones, and toppings	This app is the possibly the first that aims to help students discover on their own the Fundamental Principle of Counting; it features a realistic setting (an ice cream shop) and immerses the students in a scenario (choosing which ice cream flavors, cones, and toppings are available)

[a]Based on the DepEd mathematics curriculum for Grade 7 to 10 (DepEd 2012)
[b]The app has three other modes: Triangle Congruence (Level 1), Triangle Similarity, and Quadrilaterals.

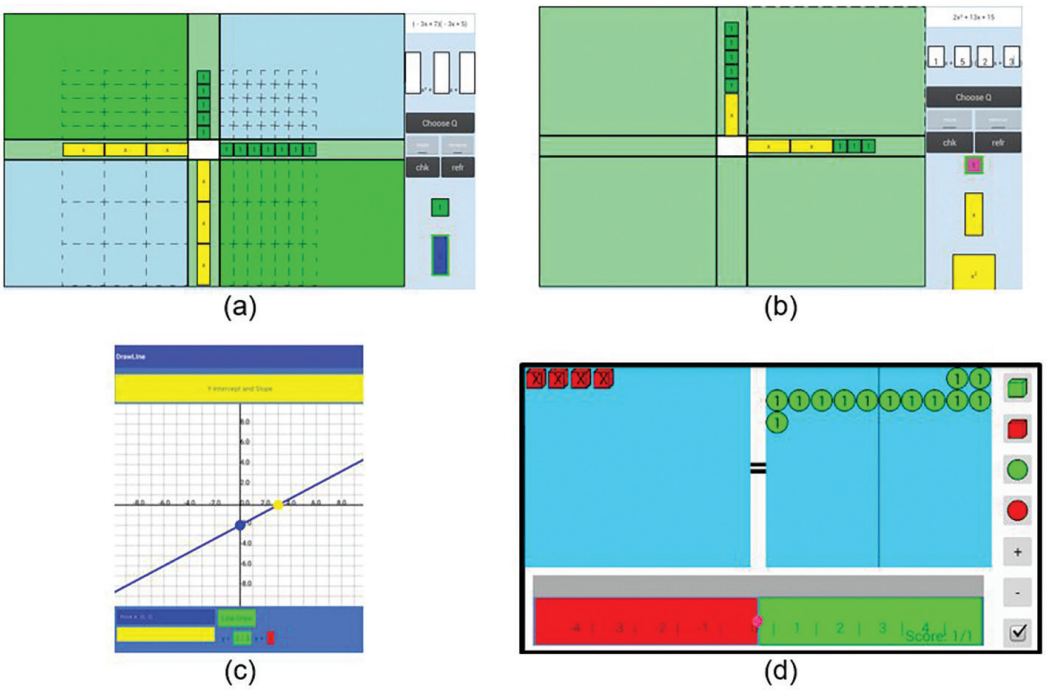


Figure 3. (a) AlgeTiles: Guide gridlines to facilitate the multiplication of two binomials. (b) AlgeTiles: Final phase of a factoring exercise where the algetiles corresponding to the factors of a trinomial are shown. (c) DrawLine: A line is graphed using input y-intercept and slope. (d) LinearX: A linear equation is visualized using four boxes (representing 4x) shown to be equal to thirteen circles.

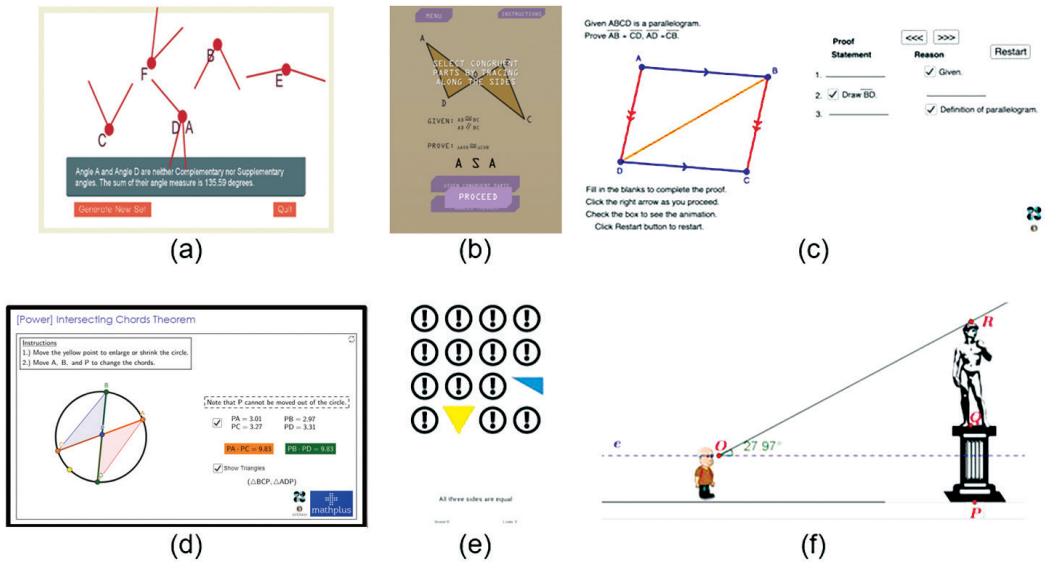


Figure 4. (a) Angle Xplore: A pair of adjacent angles, neither complementary nor supplementary, has been formed by the user by dragging some of the given angles. (b) ProveIt: A prompt showing an instruction to facilitate the solution of the given proving problem. (c) A *GeoGebra* app facilitating a two-column proof of a theorem involving a parallelogram. (d) A *GeoGebra* app illustrating the Intersecting Chords Theorem. (e) Geomatch: As shapes are revealed on the screen, the user has to tap one that matches the description given at the bottom. (f) A *GeoGebra* app where the observer's line of sight is drawn and the angle of elevation to the top of a statue is measured.

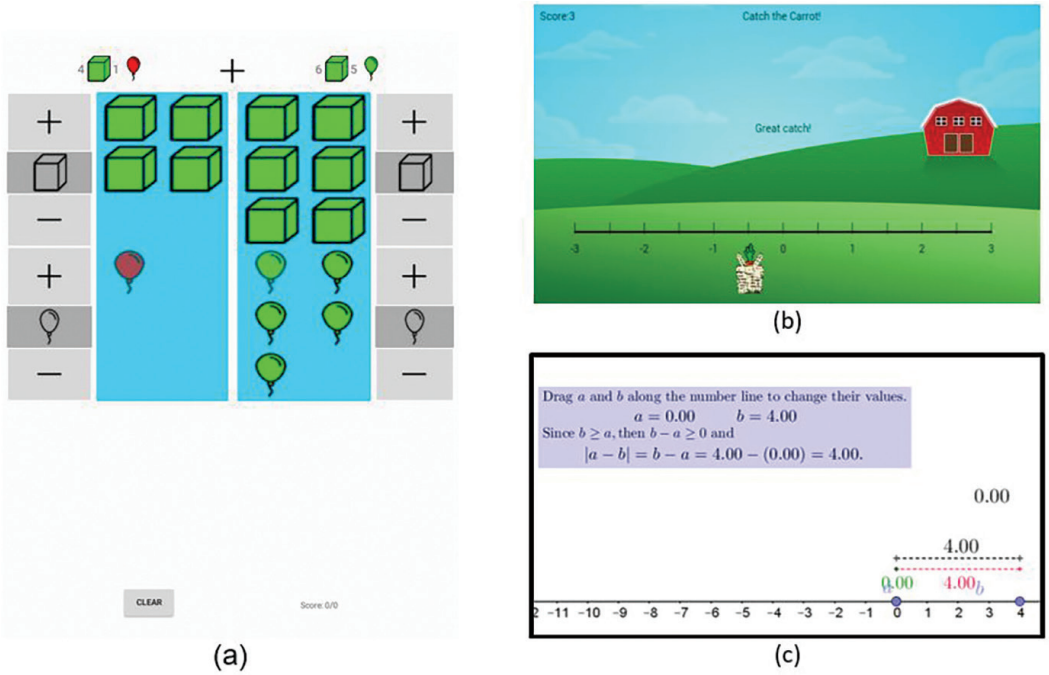


Figure 5. (a) AlgeOps: Addition of two expressions is visualized using boxes and balloons; a pair of balloons (red and green) is shown to fade to signify that they cancel each other. (b) CatchTheCarrot: The basket was correctly positioned at –1/2 so the falling carrot was successfully caught. (c) A *GeoGebra* app for exploring the idea that absolute value can be understood as distance.

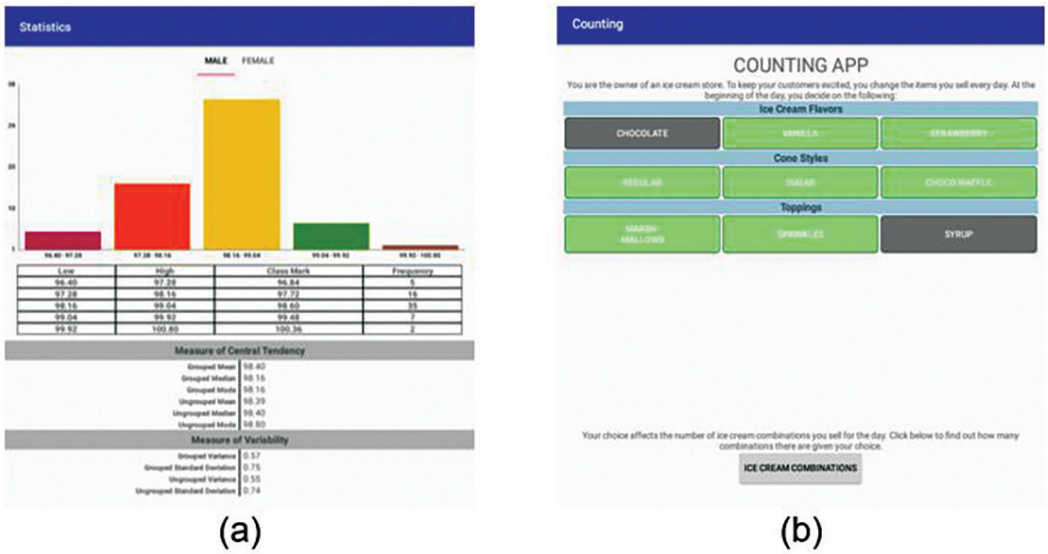


Figure 6. (a) Statistics: After inputting the desired number of classes, the app displays a histogram, a frequency distribution table, and measures of central tendency and variation. (b) Counting: The user makes a selection of which ice cream ingredients are currently available in an ice cream store.

Many of the apps developed focused on providing students a means of visualizing abstract mathematical concepts and operations while also giving them opportunities to explore these on their own. By using these apps, students can deepen their understanding and come up with observations, extensions, or generalizations on their own instead of simply accepting lectured materials. Examples of such apps are *LinearX, AlgeTiles*, *Counting*, *Statistics*, and a *GeoGebra* applet for visualizing the idea of absolute value as distance.

DrawLine and *AngleXplore* are visualization and exploration apps that make use of dynamic geometry, which has been a natural platform for students to test conjectures as it allows them to manipulate representations, observe the consequences, and formulate conjectures. Some *GeoGebra* applets that make use of dynamic geometry were also developed. For instance, one *GeoGebra* app focuses on the Intersecting Chords Theorem while another focuses on Angles of Elevation and Depression.

Apps, such as *CatchTheCarrot*, *GeoMatch*, and *AlgeOps,* not only afford visualization of the concepts but also focus on reviewing and practicing students. To encourage students to use the app even outside classes, these apps were made with a game environment.

Proving in geometry has also been one of the key points focused on the development of the apps since proving is one of the most difficult topics encountered by students. Very few students can be said to have mastered proof writing, even of proofs involving elementary geometric concepts (Senk 1985). In disadvantaged areas in the Philippines, students enter secondary school without having mastered elementary mathematics concepts (Verzosa *et al.* 2017) and, consequently, proofs end up being skipped altogether in favor of easier topics. The app *ProveIt* as well as several *GeoGebra* apps were designed to assist students in understanding and developing proofs for theorems in geometry.

Two of the apps, namely *ProveIt* and *AlgeOps*, were selected for testing for their potential impact on students' understanding of certain topics in geometry (in the case of *ProveIt*) and algebra (in the case of *AlgeOps*). In algebra, we focused on the topic addition and subtraction of integers, since mastery of integer operations forms one of the basic skills in secondary education. In the Philippines, it is quite common for students to reach their last year of high school without knowing how to perform operations on integers (Verzosa *et al.* 2017). *AlgeOps* is a visualization and exploration app that aims to anchor students' understanding of addition and subtraction of integers and polynomials. In particular, the app emphasizes the idea that positive and negative expressions cancel each other as shown by the fading animation of pairs of red and green boxes or balloons. Among the topics in geometry, we focused on proving since it is one of the topics most students have difficulty with. *ProveIt* is an app that provides users a visual representation and interactive learning tool to facilitate the construction of proofs involving congruent triangles, similar triangles, and quadrilaterals. One of the app's most important features is that it allows the user to select parts of the geometric figures on the VMR itself using the mobile device's touch-screen interface. As opposed to usual textbook exercises on these topics, *ProveIt* provides immediate feedback, allowing the user to quickly amend any incorrect responses provided or to understand why the provided response is correct. The results of testing these two apps will be discussed in the next section.

To accompany the mobile apps and *GeoGebra* applets, teaching guides were also developed to optimize the use and benefits of the software and to suggest ways to incorporate these technologies into the lesson. The teaching guides are expected to help teachers design lessons on the mathematical strands (*i.e.*, number sense, algebra, geometry, and probability and statistics) and to promote critical thinking and problem-solving in the classroom. The teaching guides also provide guide questions as prescribed by the topic addressed. In addition to teaching guides accompanying the apps, teaching guides on geometric constructions using a compass and straightedge was also developed. A total of 101 teaching guides were written on selected topics in Grade 7 to 10 mathematics. Note that each mobile app or *GeoGebra* applet may cater to different grade levels and/or topics and may address various learning competencies accompanied by several teaching guides. For example, *ProveIt* may be used in conjunction with lessons on triangle congruence (a topic in Grade 8) and conditions for parallelograms (a topic in Grade 9).

EVALUATION AND DISCUSSION

The apps were presented to 116 teachers from Metro Manila during a teacher training workshop-seminar organized by the researchers. The seminar aimed to disseminate information about the apps and their corresponding teaching guides. The call for participants for this workshop was disseminated through the DepEd local Schools Division Superintendent of cities in Metro Manila and through the research team members' personal network of high school teachers. At the end of the seminar, the teachers were asked to complete a survey to determine their perspective on the apps' usefulness and relevance to the curriculum.

To complement survey results on the apps' potential for increasing engagement and motivation as seen from the teachers' perspective, the research team also performed pilot studies on two of its apps, *ProveIt* and *AlgeOps*, to gain evidence on increased mathematical performance and conceptual development. The team chose to conduct a pilot test for *ProveIt* as we were interested to see the extent to which the app's interactive and visual capabilities can help develop students' competency in writing two-column proofs in contrast to usual textbook-based exercises. The team also launched a pilot study for *AlgeOps* to see how two models for integer operations, namely the neutralization model and the number line model, when implemented via a mobile app can facilitate students' conceptual development in integer addition and subtraction. The two pilot studies and their corresponding research designs are described below, but more details can be found elsewhere (Verzosa *et al.* 2018, 2019).

Teacher's Evaluation of the Apps

The first part of the survey required the teachers to rate their degree of agreement to a given statement (see Table 2 for a list of these statements) on a scale from 1 to 5 (where 5 indicates strong agreement and 1 indicates strong disagreement). The second part asked the teachers the following open-ended questions:

1. Describe your overall experience when using mobile apps.
2. How do you think the apps can be improved?
3. What are the advantages and disadvantages of the apps compared to similar tasks in textbooks and workbooks?

In developing the survey questions for the teacher training-workshop, the research team was primarily interested in the teachers' reception of the mobile apps developed during the project, especially in terms of usefulness in the classroom. Furthermore, their input could be used to further refine the apps, among other uses. As such, the feedback forms distributed during the event asked the teachers for their feedback, comments, and suggestions on the mobile apps only.

Table 2 provides summary statistics on the teachers' responses to the first part of the survey. Representative responses to the second part of the survey are also cited below to supplement the results in Table 2.

Table 2 shows that teachers strongly agreed with most statements presented in the evaluation survey. On one hand, teachers highly rated (with a fair consensus) the mobile apps' interactivity and the ease of use. One teacher highlighted the increasing relevance of mobile technologies in teaching schoolchildren who are nowadays increasingly tech-savvy, saying that "the learners will really participate in class if these apps will be presented because the learners these days can easily adapt to technology." Another respondent echoed this sentiment, saying that "[the apps] are very timely for us teachers. They can enhance our class discussion and are very exploratory and interactive." A similar comment was made by another respondent who said: "These mobile apps are very interactive and are appropriate to our millennial students." Other respondents also noted that the use of mobile apps provides a novel and interesting way of approaching classroom discussions and helps increase student motivation in studying for lessons. One respondent compared the use of mobile apps to the use of textbooks, saying "using a textbook is very traditional; millennial students want an interactive way of learning."

On the other hand, the teachers gave a relatively low rating for the apps' accommodation of students of all ability levels and its adaptability to classroom discussions. One teacher said that the apps are "not for advanced students – they would find it boring." Another teacher mentioned that the apps may also be somewhat confusing for beginner students. Several respondents also highlighted the issue of accessibility of these mobile technologies especially for public school students, citing that not all schools are Wi-Fi-ready and that not all students own a mobile device on which the apps can be used. However, other teachers said that if their students did have access to a mobile device, "students can practice on their own (at home or anywhere) … therefore increasing the students' retention of the lesson."

Table 2 Summary of teachers' degree of agreement to given statements on the apps' applicability to classroom use (n = 116, 1 = strongly disagree, 5 = strongly agree).

Statement	Mean	Median	Mode	SD
The mobile apps are interactive.	4.61	5.00	5.00	0.52
The mobile apps are easy to learn and to use.	4.57	5.00	5.00	0.53
I feel students will be able to practice mathematical skills through the apps.	4.53	5.00	5.00	0.66
I feel students will be able to explore mathematical concepts through the apps.	4.52	5.00	5.00	0.61
The apps can cater to beginning, average, and advanced students.	4.47	5.00	5.00	0.60
The mobile apps can be adapted to my classroom discussions.	4.32	4.00	4.00	0.71

Several respondents also highlighted the visualizations the mobile apps offer and how these aid in increasing students' understanding of mathematical concepts discussed in class. One cited this aspect as an advantage of using mobile apps in classroom discussion, saying that "[the apps] are effective and interactive because of the illustrations and graphics." Another teacher said that "the understanding of the topic will be more concrete since it utilizes manipulatives." The visual aspect is also useful in helping the students explore the topic, with one teacher saying "students can visualize the given activities or exercises, it is good to use in exploratory activities."

Overall, the teachers gave a positive evaluation of the mobile apps, especially in relation to increasing students' interest in the topic and to helping teachers prepare teaching aids more efficiently for their classroom discussions. There was also a strong agreement among the respondents that these mobile technologies are helpful and timely with regards to elevating high school mathematics teaching in an increasingly technology-dependent generation of students.

Increased Performance in Writing Two-Column Proofs

In the study by Verzosa *et al.* (2019), the researchers investigated the use of the *ProveIt* app in relation to students' competency in writing two-column proofs. Non-Android technologies for facilitating two-column proofs require the user to make conjectures based on figures and then enter statements to their proof. By contrast, *ProveIt* is Android-based and allows the user to trace over parts of the figure. In the Triangle Congruence mode of *ProveIt*, a user determines – through logic – a pair of congruent parts in the figure and traces over this pair. Visual feedback is provided through color flashes. The pair of parts becomes green for correct identification, and red if otherwise. The goal is to identify enough pairs of congruent parts so that the displayed triangle congruence postulate (*e.g.*, Side-Angle-Side) is satisfied.

To a sample of 12 private school students with various degrees of mathematical ability (*i.e.*, low-achievers, high-achievers, and average performers, as assessed by their respective teachers), the researchers introduced the *ProveIt* app as an intervention to help students construct two-column proofs for statements on triangle congruence. A pre-test consisting of three proving tasks on triangle congruence was first administered. After the pre-test, the students were given 30 minutes to use the *ProveIt* app, with the time split into periods of guided and unguided use of the app. A post-test – consisting of three proving tasks which were different from, but of the same difficulty level as, the pre-test – was then given to the students after the 30-minute period. The students' responses in the pre- and post-tests were marked using a rubric patterned after Senk (1985).

Using a Friedman two-way analysis of variance, the researchers found a significant improvement in the students' post-test scores when compared to their pre-test scores (p-value $= 0.043$). Further, the improvement tended to be more pronounced among students who already had a preliminary grasp of writing two-column proofs, although this must be investigated further. This preliminary study showed that the *ProveIt* app, even after only a short session on the use of the app, has the potential to improve students' ability to construct two-column proofs.

In a focus group discussion that was conducted after the post-test, the students lauded the interactive and visual elements of the app. Several students commented that using the app was markedly different from the usual textbook experience in that ProveIt requires user inputs via a touch screen. Specifically, students said that the act of tracing line segments and angles to identify them was a novel experience. The students also appreciated the immediate feedback given by the app when they entered their answers in the app and the ease with which the app was used.

Concept Development

A second study (Verzosa *et al.* 2018) presented a quantitative and qualitative analysis of how the *AlgeOps* app facilitated students' understanding of integer addition and subtraction. Earlier technologies involving the addition and subtraction of integers highlight the idea of canceling integers of opposite sign (neutralization model) or the idea of motion on a number line (number line model). The *AlgeOps* app is an integration of these two models. While it was designed to represent addition and subtraction of polynomials, it may also be used for teaching integer addition and subtraction. In the initial screen in the Add mode of *AlgeOps,* a set of red and green objects is presented as a target, with a blank panel below it. The user must use the plus and minus buttons so that the same number of red and green objects is shown on the panel. When the user successfully constructs the target number of objects on the panel, a number line with a highlighted point is shown. The user must move the point to the correct number representing the set of objects on the panel. As the user moves the point along the number line, visual feedback is shown through objects that appear over each number in the number line (see Figure 7).

The study was conducted on a sample of 26 students (Grades 5 to 7, aged 11 to 13 years old) consisting of public and private school students. The pre- and post-tests consisted of eight questions on integer addition and eight

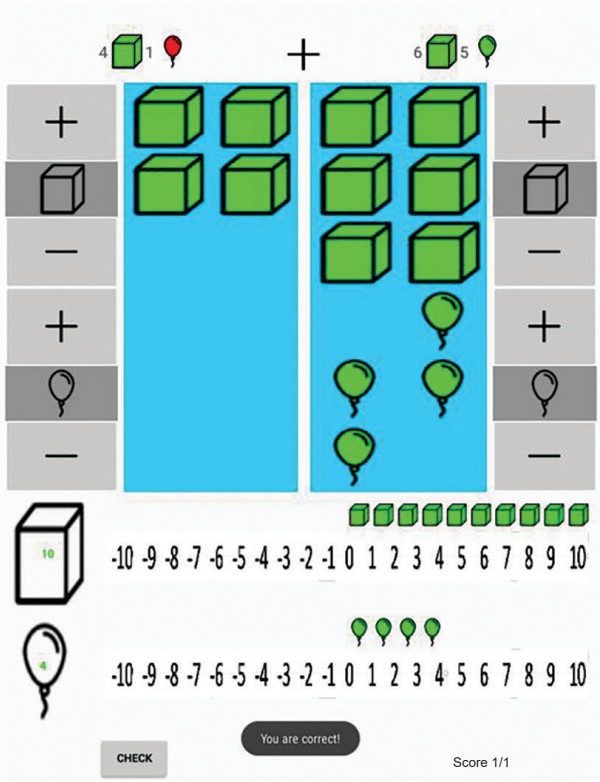


Figure 7. Confirming if the answer is correct in *AlgeOps* "Add" Level 2. The answer entered by the user is visualized using boxes and balloons that appear over the number lines.

questions on integer subtraction. Although different items were used in the post-test, the two tests were of equal difficulty. After the pre-test, students were asked to use and explore *AlgeOps* for 15 minutes, and the post-test was then administered.

The researchers found that post-test scores were generally higher than pre-test scores after a brief guided session on the use of *AlgeOps*. A significant improvement (p-value $= 0.009$) was observed among the public school students in integer addition, whereas a significant improvement (p-value $= 0.045$) was found for integer subtraction among the private school students in the sample. This shows that *AlgeOps* also has the potential to be an effective tool in developing students' competency in integer addition and subtraction.

The quantitative results were complemented with a qualitative analysis to provide an in-depth analysis of the causal processes (Maxwell 2004) behind how the app contributed to conceptual development. A framework informed by instrumental genesis (Guin and Trouche 1999) and a dialogic relationship between voices (Kazak *et al.* 2015) was used to analyze how two students developed their conception integers and integer operations as they used the app.

The two students were asked to use the plus and minus buttons on the app to construct the required number of objects shown on the screen (for example, three red boxes and five green boxes, representing negative and positive integers, respectively). They had no difficulties adding green objects – they simply needed to click the plus sign. However, they had difficulties adding red objects to the panel. When they clicked on the plus sign, only green objects appeared, so they undid their actions by clicking the minus sign. At this point, they still could not conceptualize quantities less than zero, so they stopped clicking the minus sign when there were zero objects on the screen. One student wondered, "How do you change the color?" They clicked other parts of the screen, but still had not succeeded in adding a red object to the panel. Instrumentalization was still incomplete – the users still had not learned the relevant commands in the app. When there were zero objects, one student clicked on various parts of the screen and ended up clicking the minus button and a red box appeared. That child said, "That's it!" and laughs. However, because of the accidental nature of this action, the students still struggled with the next problem. In a succeeding problem, they realized that red objects appeared when they clicked on a minus sign when there were zero objects on the panel. At the moment of realization, they both shouted, "Whoa!" signifying that at this instant, the app became an instrument that was integrated into their mental schemes. This "Whoa!" moment signaled a shift in perspective as the app became a voice that altered their primitive notions of numbers. It was a major turning point – the students took several minutes to answer the first two problems, but their newly learned concept allowed them to complete all subsequent problems more quickly.

CONCLUSION AND FUTURE DIRECTION

Under this project, we designed apps for various learning competencies in Grade 7 to 10 mathematics, guided by the framework of multiple representations and multimedia learning. An investigation of the effectiveness of these mobile apps, such as *ProveIt* and *AlgeOps*, shows the potential of these technologies to improve student learning outcomes in mathematics. The resources we developed have also garnered a positive reception from teachers who stated that these mobile technologies provide a more interactive means of instruction (as opposed to traditional paper-based media) and show potential to help increase students' motivation and retention of lessons. We invite the readers to visit the project website (https://mathplusresources.wordpress.com/) as future updates and refinements to the developed resources will be made available here.

A further investigation may be considered toward the development of digital learning resources on mathematics topics for both primary and secondary levels. Given the feedback gathered from teachers and students during the field-testing and teacher training-workshop, further refinements are being considered for the mobile apps for Grade 7 to 10. In addition, since this project focused on Grades 7 to 10, future work may be considered for Grades 1 to 6 mathematics. This will address the learning of number sense, which is the foundation for higher mathematical learning. The focus on primary mathematics is additionally critical because Filipino children rely on superficially learned strategies from an early age (Verzosa 2015).

The thrust towards the development of learning resources for statistics is timely, as the new K to 12 curriculum specifies a very rigorous statistical curriculum, with many topics not included in the previous basic education curriculum. A framework on mathematical modeling may be considered as the basis of the development of mobile apps on statistics. Specifically, these apps may be designed to take advantage of real-life data to impart a better understanding of statistical concepts and their applications. To this end, developing a database for statistical learning (*e.g.*, Census-at-School, which is implemented in Australia) may also be a worthwhile pursuit in connection with the heavier emphasis on statistics in the new K to 12 curriculum and with the need to build stronger statistical competencies among students.

In addition to developing digital resources for mathematics learning and statistical thinking, future work may also be done in the development of database infrastructure for assessment and statistical reasoning. With a database of student assessment, educators may then measure students' progress across a larger scale and over time.

The creation of resources for Grades 1 to 6, the refinement of the apps for Grades 7 to 10, the development of the database and teaching guides for statistical learning, and the development of an assessment database are the major components of an ongoing project entitled "Technology Innovations for Mathematical Reasoning, Statistical Thinking, and Assessment" being conducted by the researchers. It is suggested that alongside the task of refining the existing apps for Grades 7 to 10, additional studies on the potential impact of these apps for student learning will be carried out. It is further proposed that other apps be developed using the scientific frameworks and methodologies presented in this study. It is also recommended that feedback from teachers regarding the teaching guides accompanying the existing apps be collected in future studies.

There is still a lot of work and questions to be addressed in the study of the use of MT in learning mathematics. While the results of our initial studies yield positive results, further research is needed to help identify which situations and learning environments are most suitable for mobile apps to be most beneficial. There are also other issues, such as the following, that need to be addressed. Should schools in the Philippines allow the use of technology in the study of mathematics in the classrooms in secondary mathematics? Should students bring their mobile phones and other devices? There are also technical considerations. Are schools equipped with wireless internet and mobile devices for teachers to use? Can teachers cope with the technical aspects and difficulties of varying devices? How are teachers being prepared in schools when the field of learning with MT is new, but the technology that supports it is changing very fast?

ACKNOWLEDGMENTS

This work was funded by DOST-PCIEERD with Project No. 03067, 2015.

We also acknowledge the technical assistance provided by the Ateneo Java Wireless Competency Center, SparkSoft Solutions, Inc., Dr. Marlene M. De Leon, Mr. Kyle Kristopher Garcia, Mr. Carlos Oppus, Mr. Daniel Lagazo, Mr. Rey Barcelon, Mr. Mike Syson, and Dr. Eduard Taganap in the creation of the mobile apps. We are also grateful to the administrators of Xavier School, Marikina High School, and CBC Child Learning Center for support during the testing phase of this study. Thanks also go to Mr. Edwin Morales, Mr. Edrian Neverio, and Mr. Janree Gatpatan for the technical support. The authors would also like to thank the Ateneo de Manila University School of Science and Engineering and the Department of Mathematics for their support towards the research project.

We also thank the anonymous reviewers for their constructive comments and suggestions on an earlier version of this paper.

STATEMENT ON CONFLICT OF INTEREST

No potential conflict of interest was reported by the authors.

REFERENCES

AVRAAMIDOU A, MONAGHAN J, WALKER S. 2015. Mathematics and non-school gameplay. In: Lowrie T and Jorgensen R. editors. Digital Games and Mathematical Learning: Potential, Promises and Pitfalls (Vol. 4). Netherlands: Springer. p. 11–34.

BAUTISTA D, DE LAS PEÑAS MLAN. 2006. Strategies in maximizing the use of existing technology in Philippine schools. In: Proceedings of the Eleventh Asian Technology Conference in Mathematics. Chu SC, Hou SH, Yang WC eds. Blacksburg, VA. p. 308–316.

BU L, SPECTOR JM, HACIOMEROGLU ES. 2011. Toward model-centered mathematics learning and instruction using GeoGebra: A theoretical framework for learning mathematics with understanding. In: Model-Centered Learning: Pathways to Mathematical Understanding using GeoGebra. Bu L, Schoen R eds. Netherlands: Sense Publishers. p. 13-40.

CALDER NS, CAMPBELL A. 2016. Using mathematical apps with reluctant learners. Digital Experiences in Mathematics Education 2(1): 50–69.

CALDER N, LARKIN K, SINCLAIR N eds. 2018. Using Mobile Technologies in the Teaching and Learning of Mathematics. Springer.

[DepEd] Department of Education. 2012. K to 12 Curriculum Guides: Mathematics. Pasig City, Philippines.

DE LAS PEÑAS MLAN, BAUTISTA D. 2008. Understanding and developing proofs with the aid of technology. Electronic Journal of Mathematics and Technology 2(3): 261–273.

DE LAS PEÑAS MLAN, BAUTISTA D. 2011. The emerging role of GeoGebra in the Philippines. In: Model-Centered Learning: Pathways to Mathematical Understanding using GeoGebra. Bu L, Schoen R. eds. Netherlands: Sense Publishers. p. 217–230.

ETCUBAN JO, PANTINOPLE LD. 2018. The effects of mobile application in teaching high school mathematics. International Electronic Journal of Mathematics Education 13(3): 249–259.

FABIAN K, TOPPING KJ, BARRON IG. 2016. Using mobile technologies for mathematics: Effects on student attitudes and achievement. Educational Technology Research and Development 66(5): 1119–1139.

FU JS. 2013. ICT in education: A critical literature review and its implications. International Journal of Education and Development using Information and Communication Technology 9(1): 112–125.

GOLDIN GA. 1998. Representational systems, learning, and problem solving in mathematics. Journal of Mathematical Behavior 17(2): 137–165.

GOODWIN K, HIGHFIELD K. 2013. A framework for examining technologies and early mathematics learning. In: Reconceptualizing Early Mathematics Learning. English L, Mulligan JT eds. Dordrecht, the Netherlands: Springer.

GROS B. 2015. Integration of digital games in learning and e-learning environments: Connecting experiences and context. In: Digital Games and Mathematical Learning: Potential, Promises and Pitfalls (Vol. 4). Lowrie T, Jorgensen R eds. Netherlands: Springer. p. 35–53.

GUIN D, TROUCHE L. 1999. The complex process of converting tools into mathematical instruments: The case of calculators. International Journal of Computers for Mathematical Learning 3: 195–227.

JOHNSTON-WILDER S, PIMM D. 2005. Some technological tools of the mathematics teacher's trade. In: Teaching Secondary Mathematics with ICT. Johnston-Wilder S, Pimm D eds. Berkshire, UK: Open University Press. p. 18–42.

KAZAK S, WEGERIF R, FUJITA T. 2015. The importance of dialogic processes to conceptual development in mathematics. Educational Studies in Mathematics 90(2): 105–120.

KIILI K. 2005. Digital game-based learning: Towards an experiential gaming model. Internet and Higher Education 8: 13–24.

KIMA P, MIRANDAB T, OLACIREGUIC C. 2008. Pocket school: Exploring mobile technology as a sustainable literacy option for underserved indigenous children in Latin America. International Journal of Development 28: 435–445.

LARKIN K. 2015. An app! An app! My Kingdom for an app: An 18-month quest to determine whether apps support mathematical knowledge building. In: Digital Games and Mathematical Learning: Potential, Promises and Pitfalls (Vol. 4). Lowrie T, Jorgensen R eds. Netherlands: Springer. p. 251-276.

LARKIN K, CALDER N. 2016. Mathematics education and mobile technologies. Mathematics Education Research Journal 28: 1–7.

LEE-CHUA QN. 2012. Technology Package for Student Learning Empowerment: PC Tablet Math Lesson Courseware Development [Final Report]. Retrieved from http://www.sei.dost.gov.ph/images/ts/icoursewaretab2012.pdf.

MACKRELL K. 2012. Introducing algebra with interactive geometry software. Electronic Journal of Mathematics and Technology 6(1): 96–114.

MAXWELL JA. 2004. Using qualitative methods for causal explanation. Fields Methods 16(3): 243–264.

MAYER R. 2005. Cognitive theory of multimedia learning. In: The Cambridge Handbook of Multimedia Learning. Mayer R ed. Cambridge, UK. p. 31–48.

[NCTM] National Council of Teachers of Mathematics. 2000. Principles and standards for school mathematics. Reston, VA.

NISIYATUSSANI, AYUNINGTYAS V, FATHURROHMAN M, ANRIANI N. 2018. GeoGebra applets design and development for junior high school students to learn quadrilateral mathematics concepts. Journal on Mathematics Education 9(1): 27–40.

NORRIS C, HOSSAIN A, SOLOWAY E. 2011. Using smartphones as essential tools for learning. Education Technology 51(3): 18–25.

SENK SL. 1985. How well do students write geometry proofs? Mathematics Teacher 78: 448–456.

SHELOMOVSKIY V, NOSULYA S. 2012. Development of creativity using 3D Dynamic Geometry System (GInMA). Electronic Journal of Mathematics and Technology 6(1): 75–95.

SHULER C. 2009. Pockets of potential: Using mobile technologies to promote children's learning. New York: The Joan Ganz Cooney Center at Sesame Workshop.

TAKA D, TAKA A, BUDINSKI N. 2010. On visualization problems using the GeoGebra and Scientific Workplace packages. International Journal for Technology in Mathematics Education 17(4): 191–196.

VERZOSA DMB. 2015. Developing number knowledge of low-attaining first- to fourth-graders. Proceedings of the 7th ICMI-East Asia Regional Conference in Mathematics Education. In: Vistro-Yu CP ed. p. 217–224.

VERZOSA DMB, GUZON AFH, DE LAS PEÑAS MLAN. 2014. Using dynamic tools to develop an understanding of the fundamental ideas of calculus. International Journal of Mathematical Education in Science and Technology 45(2): 190–199.

VERZOSA DMB, DE LAS PEÑAS MLAN, ABERIN MAQ, GARCES LPDM. 2018. App for addition and subtraction of integers. International Journal of Technology in Mathematics Education 25(4): 21–33.

VERZOSA DMB, DE LAS PENAS MLAN, ABERIN MAQ, GARCES LPDM. 2019. App-based scaffolds for writing two column proofs. International Journal of Mathematical Education in Science and Technology 50(5): 766–778.

VERZOSA DMB, TULAO-FERNANDO MT, VISTRO-YU CP. 2017. Congruence between context and opportunities for professional development of mathematics teachers in the Philippines. Professional Development of Mathematics Teachers: An Asian Perspective. In: Kaur B, Kwon ON eds. Singapore: Springer. p. 89–96.

WONG CY, KHONG CW, CHU K. 2012. Interface design practice and education towards mobile apps development. Procedia – Social and Behavioral Sciences 51: 698–702.

Engaging Learners through Data: Senso Eskwela Pilipinas

Mark Anthony C. Tolentino
mtolentino@ateneo.edu
Department of Mathematics
Ateneo de Manila University, Philippines

Ma. Louise Antonette N. De Las Peñas
mdelaspenas@ateneo.edu
Department of Mathematics
Ateneo de Manila University, Philippines

Maria Alva Q. Aberin
maberin@ateneo.edu
Department of Mathematics
Ateneo de Manila University, Philippines

Agnes D. Garciano
agarciano@ateneo.edu
Department of Mathematics
Ateneo de Manila University, Philippines

Mark L. Loyola
mloyola@ateneo.edu
Department of Mathematics
Ateneo de Manila University, Philippines

Jumela F. Sarmiento
jsarmiento@ateneo.edu
Department of Mathematics
Ateneo de Manila University, Philippines

Juan Carlo F. Mallari
jmallari@ateneo.edu
Department of Mathematics
Ateneo de Manila University, Philippines

Debbie Marie B. Verzosa
dmbverzosa@usm.edu.ph
Department of Mathematics and Statistics
University of Southern Mindanao, Philippines

Abstract: *The use of real data in teaching and learning statistics has been recommended in the literature. This paper talks about the development and implementation of the Senso Eskwela Pilipinas, the first database in the Philippines that*

builds and provides real data with the intention of making the study of statistics engaging, understandable, relatable, and relevant for Filipino learners.

1. Introduction

The K-to-12 program in the Philippines specifies a very rigorous statistics curriculum, and due to the spiral approach, all teachers from Grade 1 to 11 need to teach statistics. However, statistics remains one of the most difficult topics to teach, and even local tertiary level textbooks are replete with statistical misconceptions [1]. To address this challenge, the authors, under a government-funded project entitled *Technology Innovations on Statistical Reasoning*, created teaching guides that can support Philippine teachers who themselves have limited background in statistics. Further, following CensusAtSchool programs in other countries, a platform named *Senso Eskwela Pilipinas* (SEP) was put together to collect authentic data so that Filipino students would have increased interest and find relevance in learning statistics.

The CensusAtSchool program was first launched in the UK in 2000 with the "dual thrust to enliven data handling activities within the classroom while also educating children about the principles and processes involved in conducting a census" [2, p.1]. With CensusAtSchool, students of different grade levels and from several volunteer schools answer an online survey. The students' responses then become part of a database that students and teachers can access to retrieve sample data for use in different activities for teaching or learning statistics.

The SEP platform follows the general structure of the CensusAtSchool programs while incorporating survey questions that are anchored on unique Filipino experiences and integrating teaching and learning resources that are aligned with the prescribed competencies of the Philippines' Department of Education [3]. Like CensusAtSchool, SEP shares the goal of enriching students' learning experiences in statistics; more specifically, "[t]he sense of belonging that participating pupils feel, purely because they know their own responses will become part of a database of responses from their peers, is fascinating and motivating for them" [4, p. 175].

The importance of situating statistical concepts in contexts that are interesting and relevant to learners has consistently been identified in curriculum documents. The Pre-K-12 Guidelines for Assessment and Instruction in Statistics Education II (GAISE II) indicates that assessments "must require students to use statistical reasoning with context and variability at all stages of the statistical problem-solving process" [5, p. 11]. For example, understanding variability requires students to consider the context in order to recognize whether the variability is due to chance or to controlled conditions [6]. In this way, statistical reasoning is naturally intertwined with context [7]. When students are provided opportunities to work on authentic problems, they learn to develop informal inferential reasoning that works within the given context [8].

The use of authentic data for teaching statistics aligns with constructivist views of learning [9]. Through authentic data, students draw on their prior knowledge to find patterns and make sense of the data through appropriate statistical tools [10]. Authentic data can be gathered from published data sets. Published data can be made more meaningful through data visualization tools such as Gapminder [11]. This tool provides data mostly sourced from the United Nations databank and allows users to generate eye-catching graphs that can highlight multiple variables of interest [12]. Authentic data may also be sourced from students themselves [10]. When students generate their own data, they become more interested and engaged in the analysis [13, 14]. However, it is not easy to collect student data for classroom use. Technology such as Classroom Stats can facilitate the process of data collection [13]. Classroom Stats is a free mobile phone app that enables teachers to send questions to students and see the corresponding descriptive and inferential statistics online and in real time. SEP is designed to add to existing sources of real data that can be used for statistics.

The Analysis, Design, Development, Implementation and Evaluation (ADDIE) [15] approach was adapted in the design, development, and delivery of SEP. This is an instructional system design framework widely used to create effective educational courses and training programs.

This paper reports on the development and implementation of SEP. SEP is freely available and is accessible at https://mathplusresources.com/.

2. Development of Senso Eskwela Pilipinas (SEP)

The first three phases of the ADDIE approach as these relate directly to SEP are elaborated in this section. The discussion of the last two stages is deferred to the succeeding sections of this paper.

2.1 Analysis

In the analysis phase, existing CensusAtSchool platforms were studied. Different aspects apt for a version for Filipino teachers and students were identified.

First, it was determined that SEP must be a student-centered platform through which numerical and categorical data can be accessed. While there is certainly no shortage of accessible data (e.g., from textbooks and online resources), most of these are foreign or unfamiliar to Filipino learners. Thus, SEP was envisioned to have a single survey that can gather data that are: (i) relevant and relatable to the students; (ii) pedagogically and contextually appropriate, and (iii) suitable to the student's academic level. For (i), the survey questions must involve topics that resonate with elementary/high school students or topics that are related to unique Filipino experiences. For (ii) and (iii), the gathered data should be easy to integrate in lessons and student activities aligned with the Philippine Department of Education's Most Essential Learning Competencies (MELCs) in Grades 1 to 11 statistics [3]. In addition, a needs assessment was performed with a number of teachers from different public schools to identify least mastered competencies in statistics. Then, learning activities would be developed to address these competencies.

In the spirit of the CensusAtSchool programs, it was also deemed important for students to be directly involved in the building of the SEP database so that they would have an appreciation for the data-gathering process and a more natural connection with the data sets. Thus, students should be able to answer the SEP survey after which their responses, stripped of any personal identification, would go into the SEP-wide database that would be readily accessible to them.

In terms of accessibility, SEP should be accessible through a website, to ensure efficient and timely recording of responses into the database. To maximize the number of potential users, it was also decided that SEP must be accessible using different devices such as smartphones, tablets, laptops, or desktop computers.

Through the SEP platform, teachers would have easy access to a large volume of authentic data that could be used in different activities for teaching statistics. However, planning and developing these learning activities might not come very easily. Thus, preparing models or examples of such learning activities were also decided to be an integral component of the development of SEP. More specifically, teaching guides and accompanying student worksheets, aligned with the MELCs especially the least-mastered ones, also had to be developed and made available on the platform. The availability of teaching guides and student worksheets would allow teachers and students to engage in statistics learning activities conveniently, immediately, and effectively. Additionally, teachers might choose to modify the available activities or use these as a basis to create their own learning activities.

2.2 Design and Development

The design phase of SEP was divided into two major activities: i) identifying statistics topics; and ii) creating survey questions along with teaching guides and accompanying student worksheets. For Grades 1 to 6, the topics identified were: constructing and reading pictographs, collecting and organizing data, and probability. For Grades 7 to 10, the topics identified were: creating an infographic or a simple questionnaire, creating a stem and leaf plot, experimental probability, organizing and analyzing data using graphs, ungrouped and grouped data, centrality measures, variability measures, and measures of position. For the Grade 11 Probability and Statistics subject, the topics identified were discrete random variable, central limit theorem, hypothesis testing, confidence intervals for population means or proportion, scatter plot, and best fit line.

Given these topics and the aforementioned requirements on the data to be collected, a total of 31 survey questions were formulated and classified into seven categories: *Myself in Numbers*, *More About Myself*, *Food and Drinks*, *Hobbies and Free Time*, *Traveling and Commuting*, *Study and Sleeping Habits*, *Fun and Games.*

As conceptualized, some of the questions were drafted based on topics that could resonate well with the respondents. For example, Question 9 ("Which pet animals do you have at home?"), Question 13 ("On the average, how much times in a day do you spend playing video games?"), and Question 16 ("In which of the following social networking/media sites do you have an account?") relate to topics familiar with the younger generation. Meanwhile, other questions were aimed to engage the students better by alluding to uniquely Filipino experiences. For instance, Question 14 pertains to students' enjoyment of or familiarity with outdoor games that are commonly played by Filipino children. Question 15, on the other hand, asks about students' familiarity with Filipino superheroes that have been featured in TV shows, movies, or other media. There are also interactive questions aimed to pique students' interest further. For example, students are assigned tasks (e.g., clicking all squares of a particular color in a diagram as quickly as possible) and various metrics on the task completion are recorded [16]. For additional details about the questions, the SEP survey may be fully previewed at https://mathplusresources.com/guest/survey. With this complete 31-question SEP survey, it was aimed that students would develop increased interest and connection to the different SEP datasets. Consequently, it was envisioned that students would ask deeper statistical questions about these datasets and will be more motivated in processing and analyzing them.

For the second major activity under the design phase, a functional specification document for SEP was drafted. This document contained the intended capabilities of the platform, the process flows that would govern it, and its initial appearance. Figure 2.1 summarizes the key information from the document. A more detailed discussion of the features pertaining to user types and process flows are found in the next section.

The initial appearance and underlying interface of SEP were designed using the software Balsamiq Wireframe [17]. The art assets, on the other hand, were created using the open-source vector graphics software Inkscape [18]. Prototypes were then used to test the process flows and were explained to the web developers for the intended functional capabilities of the platform. The use of both software was substantive so that more time and resources were spent on the pedagogical aspects of instructional design rather than on the technical side of web-based applications development. User tests were carried out with some teachers and students shortly after. Revisions on the design and functionalities of the platform were carried out from following the user tests.

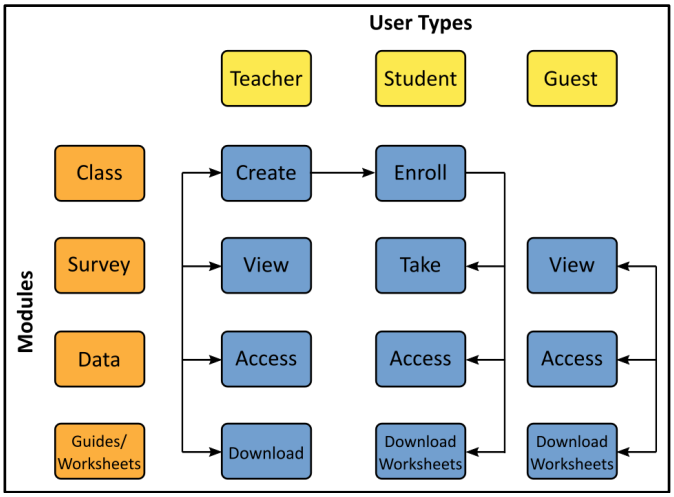
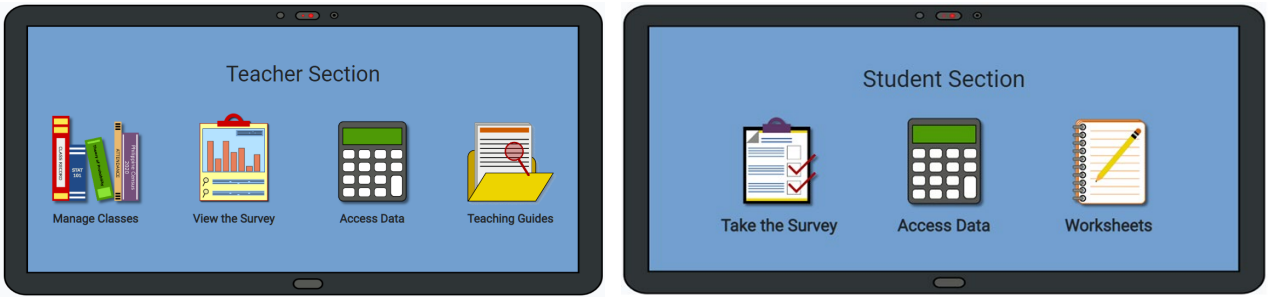


Figure 2.1 User types, modules, and process flows in SEP

3. Features of SEP

As evident from Figure 2.1, the SEP platform has three types of SEP accounts: *teacher*, *student*, and *guest*. Each account has well-defined allowable actions and degrees of accessibility to each of the four main modules (i.e., class, survey, data, and guides/worksheets) of the platform. The modules/features accessible to the teacher and student account types are visually summarized in Figure 3.1. Guest accounts have access to View the Survey, Access Data (limited), and Worksheets.



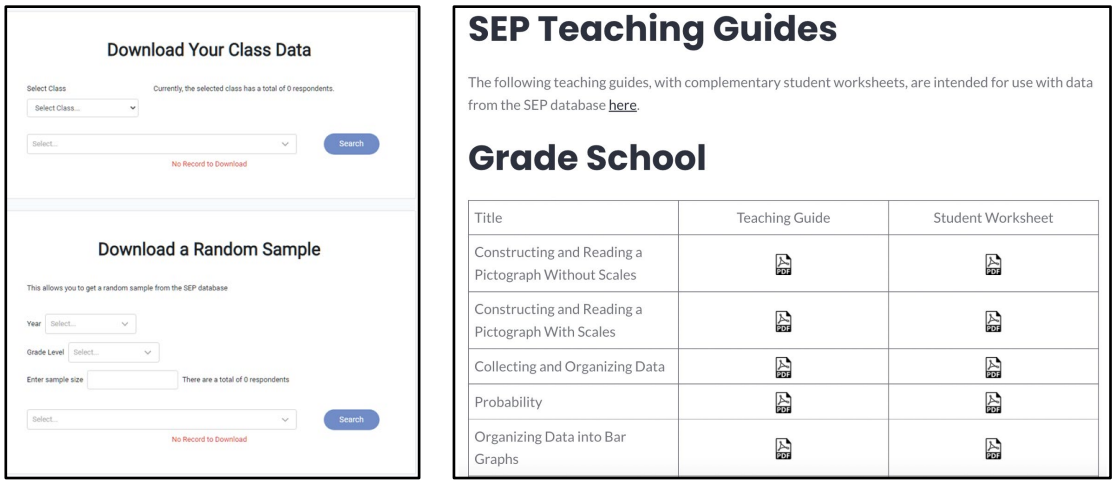
(a) (b)

Figure 3.1 Main SEP menus showing the features available to (a) teachers and (b) students

A teacher account may initiate the main process flow of SEP after creating a new class in the *class* module. This module will generate a set of student class codes. A student with a class code can then enroll in the said class by entering the code in the platform. Each code is unique to a student and is used by SEP to identify the student. Upholding privacy laws, SEP does not ask or store any personal information during students' enrollment and login. Once logged in, the student can answer the SEP survey in the *survey* module. The system allows the student to submit one complete set of responses to the survey. Once submitted, SEP stores the student's responses into the database permanently, unless the class in which the student belongs is deleted by the teacher. Any data stored in SEP can now be accessed in the *data* module by any type of user account. Note, however, that a *guest* account can only access a random set of data, unlike a *teacher* or *student* account which is given access to their respective class's data. Moreover, a *teacher* or *guest* account is only allowed to view the survey. While such a user may enter responses to the survey questions, these will not be stored in the SEP

database. Lastly, all user types are allowed to download the student worksheets. However, only a *teacher* account can access and download the teaching guides. Figure 3.2 shows the data retrieval interfaces of SEP and learning activities for the grade school level.

Teachers, as previously mentioned, may download data (Figure 3.2(a)) on particular questions from their own class/es or as random samples from the entire SEP database. For random samples, year and grade levels as well as desired sample size may be specified. The data set will be downloadable as a single spreadsheet.



(a) (b)

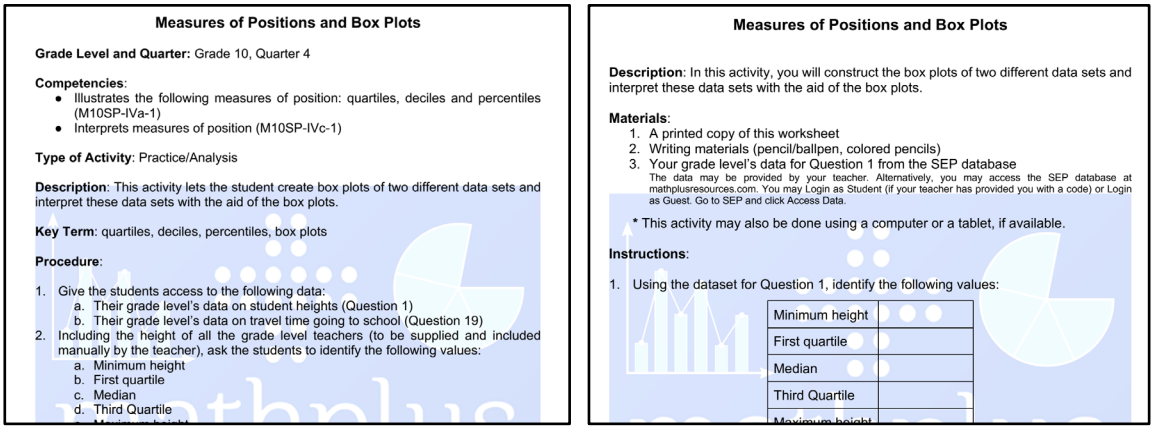
Figure 3.2 (a) Data retrieval interfaces of SEP; (b) SEP learning activities for grade school

As shown in Figure 3.2(b), each SEP learning activity is drafted as a teaching guide and a student worksheet. Each teaching guide indicates the grade level, the competencies covered (based on MELCS prescribed by the Department of Education), type of activity (instruction, practice, assessment, exploration), brief description of the activity, key terms, and the procedure. The procedure consists of a step-by-step guide on how to implement the activity. This usually starts with identifying the SEP data to be used. A student worksheet gives students precise instructions for implementing learning activities using SEP data. Each worksheet contains a description of the activity, materials needed, and step-by-step instructions that are parallel to the procedure given in the corresponding teaching guide. For students who are doing independent work (i.e., without explicit instructions from the teacher), a worksheet also briefly describes the process of data retrieval from the SEP database. In most cases, a student worksheet will also contain fields/boxes for student responses. Figure 3.3 shows an example of a teaching guide and a student worksheet for the topic "Measures of Positions and Box Plots," written to address one of the least learned competencies (measures of position) based on feedback from some teachers. Appendices 1 and 2 present two example activities that make use of SEP data. Some of the SEP learning activities also correspond to performance tasks, which are meant to be a more comprehensive and integrative form of assessment; these are discussed in more detail in [19].

4. Implementation and Deployment of SEP

SEP was formally launched and made accessible to the public last September 24-25, 2020 in a webinar entitled *Senso Eskwela at Iba Pa* [20]. The first day was devoted to an introduction to SEP, providing information on access, use of SEP's features, and integration of SEP into the classroom at

different grade levels. The second day showcased three guest teachers, who shared their insights about using SEP in classes at the grade school, junior high school, and senior high school levels. Since the launch, SEP has been used in some classes in public schools under particular DepEd School Divisions.



Measures of Positions and Box Plots

Grade Level and Quarter: Grade 10, Quarter 4

Competencies:
- Illustrates the following measures of position: quartiles, deciles and percentiles (M10SP-IVa-1)
- Interprets measures of position (M10SP-IVc-1)

Type of Activity: Practice/Analysis

Description: This activity lets the student create box plots of two different data sets and interpret these data sets with the aid of the box plots.

Key Term: quartiles, deciles, percentiles, box plots

Procedure:

1. Give the students access to the following data:
   a. Their grade level's data on student heights (Question 1)
   b. Their grade level's data on travel time going to school (Question 19)
2. Including the height of all the grade level teachers (to be supplied and included manually by the teacher), ask the students to identify the following values:
   a. Minimum height
   b. First quartile
   c. Median
   d. Third Quartile

(a)

Measures of Positions and Box Plots

Description: In this activity, you will construct the box plots of two different data sets and interpret these data sets with the aid of the box plots.

Materials:
1. A printed copy of this worksheet
2. Writing materials (pencil/ballpen, colored pencils)
3. Your grade level's data for Question 1 from the SEP database
   The data may be provided by your teacher. Alternatively, you may access the SEP database at mathplusresources.com. You may Login as Student (if your teacher has provided you with a code) or Login as Guest. Go to SEP and click Access Data.

* This activity may also be done using a computer or a tablet, if available.

Instructions:

1. Using the dataset for Question 1, identify the following values:

Minimum height	
First quartile	
Median	
Third Quartile	

(b)

Figure 3.3 A sample (a) teaching guide and (b) student worksheet

In addressing lack of internet connectivity among students, it was suggested to teachers to provide pre-downloaded datasets for learning activities in classes. These data sets can be shared to students through offline modes such as Bluetooth or portable storage devices. While this workaround will require careful planning by the teacher, it will allow the students to use authentic SEP datasets for their activities without needing to go online. The authors have taken advantage of the use of pre-downloaded SEP data sets in deploying learning materials via offline content distribution channels, such as in a community LTE Network. The deployment of probability and statistics learning packages that use pre-downloaded SEP data sets are further reported in [21].

A similar approach was utilized in partnership with Project REIINN (Resilient Education Information Infrastructure for the New Normal) of the Philippines' Department of Science and Technology - Advanced Science and Technology Institute (DOST-ASTI). The primary objective of Project REIINN (https://rcast.asti.dost.gov.ph/) is to study and to pioneer the use of a digital datacasting framework to distribute teaching resources. In March 2022, the authors have set up probability and statistics learning activities in REIINN's EdukasTV digital datacasting platform (Figure 4.1) that was deployed to two public elementary schools.

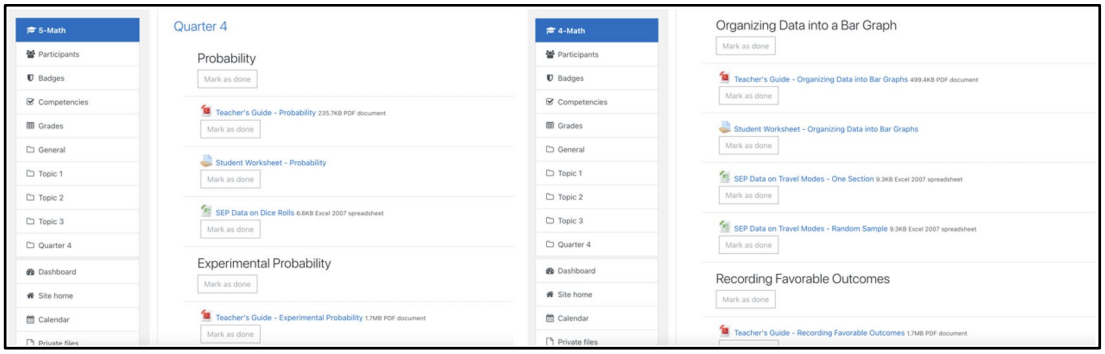


Figure 4.1 Screenshots of Probability and Statistics learning activities, with pre-downloaded SEP data sets, in Project REIINN's EdukasTV platform

5. Initial Integration and Evaluation

To gather initial feedback on the potential effects of SEP on learning statistics, SEP was integrated into two Grade 11 classes of Probability & Statistics during the 3rd quarter of SY2020-2021. One class consisted of students under the Science, Technology, Engineering and Mathematics (STEM) track and the other section consisted of students from Accounting, Business and Management (ABM) track. There were 39 students from the STEM track (18 (Male), 21 (Female)); and 28 students from the ABM track (11 (Male), 17 (Female)). Due to the COVID-19 pandemic, the two classes were in distance learning mode. The teachers of these two classes mainly used Messenger Classroom to communicate with the students about the lessons and the use of SEP. Google Meet was used occasionally to further discuss and explain concepts. Moreover, Google Forms were utilized for formative assessment and for gathering feedback about the SEP.

Aside from the use of SEP, activities that made use of SEP data (from the two classes) were also integrated into the lessons (i.e., introduction to probability distributions and discrete random variables). Some of the students' actual output from the SEP activities are shown in Figure 5.1.

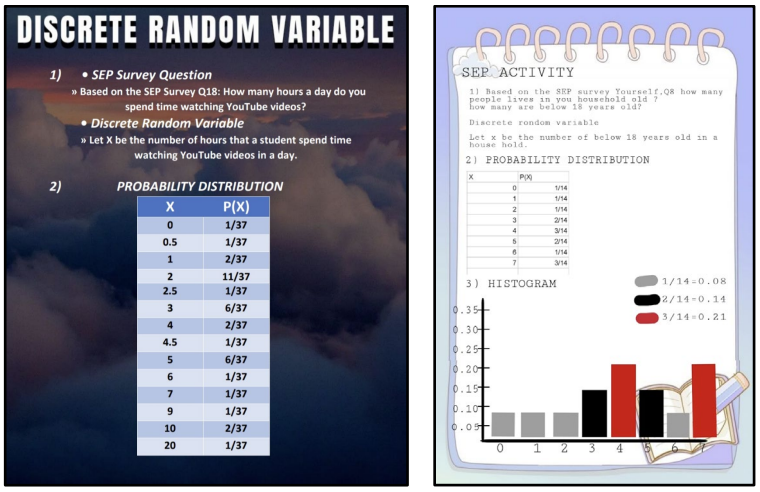


Figure 5.1 Students' output in the SEP activities integrated into their lessons.

The usage of SEP was high for both sections. Overall, **89.55%** of the students were able to completely answer the SEP survey. This resulted in the two sections having a sufficient amount of their own class's data that they could use for examples and activities in their Statistics & Probability subject. At the end of the quarter, a voluntary feedback gathering phase was implemented in which approximately 75% of the students were able to participate. The results showed that about 75% of the STEM students and 67% of the ABM students found the process of accessing and answering the SEP Survey easy and convenient. Moreover, the majority of students from both sections reported positive and favorable feedback in terms of relatability of data, interest in analyzing their own class's data, and usability in teaching and learning concepts in statistics. Some students even indicated that they could already think of possible research topics based on the SEP data for their thesis projects.

6. Conclusion and Recommendations

With the goal of enriching students' experiences and increasing their engagement in learning statistics, the authors have developed and deployed *Senso Eskwela Pilipinas* (SEP), a freely

accessible platform that allows for the collection, storage, and access of authentic, relevant, and relatable data from students all over the Philippines. SEP aims to motivate students in various grade levels to deal successfully with data as they analyze not just any arbitrary datasets, but those that are contextually relevant to them, thereby increasing their interest and engagement in the learning process.

While SEP has been developed based on the existing CensusAtSchool programs, SEP has been designed to gather interesting data that can resonate with students based on their uniquely Filipino experiences and their interests as children or teenagers. In addition, the survey design takes into consideration the most essential learning competencies (MELCs) prescribed by Philippines' Department of Education as well as the least learned competencies that have been reported from a needs assessment performed with some public schools.

Given that improving statistical literacy among students may likely result from well-planned activities given by the teachers, the SEP platform also includes teaching guides and accompanying student worksheets. The materials provide examples of learning activities that use different SEP data. The teaching guides may also provide teachers with ideas to innovate or create new tasks that are appropriate for their objectives.

The enhancement of data collection together with the creation of meaningful learning activities for students is work that is still being pursued. More innovations are in the pipeline for SEP, such as extending the material for undergraduate level statistics, updating existing survey questions, and creating a mobile app version. Furthermore, it is worthwhile to study the extent to which SEP improves the learning competencies and skills of students in the areas of statistics and probability.

Acknowledgements This paper is one of the outputs of the Ateneo Mathplus Resources Laboratory housed at the Department of Mathematics, School of Science and Engineering, Ateneo de Manila University. The authors would like to thank the Department of Science and Technology-Philippine Council for Industry, Energy, and Emerging Technology Research and Development (DOST-PCIEERD) and the University Research Council (URC), Ateneo de Manila University for the support in the development of SEP. Further, we thank Logik, our partner developer headed by Mr. Marc Pison. We also thank the following teachers who helped during the development and deployment of SEP: Mr. Resty Rodelas (Education Program Supervisor in Mathematics, DepEd School Division Office, Mandaluyong City), Mrs. Mary Rose E. Villanda, Mr. Reynaldo C. Collado Jr., Mr. Danilo R. Si, Mrs. Alona A. Ramirez, & Mr. Karl Gadwin G. Dela Cruz.

References

[1] David, I.P. and Maligalig, D.S. (2006). Are We Teaching Statistics Correctly to our Youth? *The Philippine Statistician,* 55(3-4), 1-28.

[2] Connor, D. (2002). CensusAtSchool 2000: Creation to Collation to Classroom. In *Proceedings of the Sixth International Conference on Teaching of Statistics, Cape Town.* Voorburg, The Netherlands: International Statistical Institute.

[3] Department of Education. (2020). K to 12 Most Essential Learning Competencies with Corresponding CG Codes. Retrieved 7/20/2022 from https://commons.deped.gov.ph/K-to-12-MELCS-with-CG-Codes.pdf

[4] Davies, N., Richards, K., Aliaga, M., and Nichols, R. (2010). CensusAtSchool. *Significance,* 7(4), 175.
[5] Bargagliotti, A., Franklin, C., Arnold, P., and Gould, R. (2020). *Pre-K-12 Guidelines for Assessment and Instruction in Statistics Education II*. American Statistical Association.
[6] Pfannkuch, M. (2011). The Role of Context in Developing Informal Statistical Inferential Reasoning: A Classroom Study. *Mathematical Thinking and Learning*, 13(1-2), 27-46.
[7] Cobb, G. (2007). One Possible Frame for Thinking about Experiential Learning. *International Statistical Review*, 75(3), 336–347.
[8] Dierdorp, A., Bakker, A., Eijkelhof, H., and van Maanen, J. (2011). Authentic Practices as Contexts for Learning to Draw Inferences beyond Correlated Data. *Mathematical Thinking and Learning*, 13(1-2), 132-151.
[9] Libman, Z. (2010). Integrating Real-life Data Analysis in Teaching Descriptive Statistics: A Constructivist Approach. *Journal of Statistics Education*, 18(1), 1-23.
[10] Neumann, D.L., Hood, M., and Neumann, M.M. (2013). Using Real-life Data when Teaching Statistics: Student Perceptions of this Strategy in an Introductory Statistics Course. *International Journal for Statistical Education*, 12(2), 59-70.
[11] Gapminder, *Gapminder website,* accessed 29 July 2022, <https://www.gapminder.org/>
[12] Le, D.-T. (2013). Bringing Data to Life into an Introductory Statistics Course with GAPMINDER. *Teaching Statistics,* 35(3), 114-122.
[13] Childers, A.F. and Taylor, D.G. (2020). Making Data Collection and Analysis Fun, Fast, and Flexible with Classroom Stats. *PRIMUS,* 31*,* 91-98.
[14] Chottiner, S. (1991). Using Real (Intimate) Data to Teach Applied Statistics. *The American Statistician*, 45(2), 169.
[15] Branch, R.M. (2009). *Instructional Design: The ADDIE Approach*. New York: Springer.
[16] De Las Peñas, M.L.A.N, Verzosa, D.M.B., Sarmiento, J.F., Tolentino, M.A.C., and Loyola, M.L. (2020). Designing Mobile Apps to Promote Numeracy and Statistical Reasoning. In W.-C. Yang and D. Meade (Eds.), *Electronic Proceedings of the 25th Asian Technology Conference in Mathematics* (pp. 167-182). Mathematics and Technology, LLC.
[17] Balsamiq Wireframe [Computer software]. (2008). Retrieved from https://balsamiq.com/
[18] Inkscape 1.0 [Computer software]. (2020). Retrieved from https://inkscape.org/
[19] De Las Peñas, M.L.A.N, Verzosa, D.M.B., Aberin, M.A.Q., Garciano, A.D., Sarmiento, J.F., and Tolentino, M.A.C. (2021). Designing Performance Tasks in Mathematics Using Technological Tools. In W.-C. Yang, D. Meade, and M. Majewski (Eds.), *Electronic Proceedings of the 26th Asian Technology Conference in Mathematics*. Mathematics and Technology, LLC.
[20] ADMU School of Science and Engineering. (2020). Introducing the New Mathplus Apps: Senso Eskuwela Pilipinas at Iba Pa [Video file]. Retrieved 07/27/2022 from https://www.facebook.com/ateneoSOSE/videos/322177298847306
[21] De Las Peñas, M.L.A.N, Aberin, M.A.Q., Garciano, A.D., Mallari, J.C.F., Sarmiento, J.F., Tolentino, M.A.C., and Verzosa, D.M.B., (2022). Deployment of Mathematical Resources to a Philippine High School through a Community LTE Network. Manuscript submitted for publication.

Appendix 1. SEP Activity on Mean

One activity that uses data sets from Senso Eskwela Pilipinas (SEP) is called *Advantages and Disadvantages of the Mean*. Students are asked to compute the mean of and make graphical representations of different data sets from SEP. The activity aims to help students get insights on how well the mean can serve as a measure of central tendency. The activity is outlined below.

1. Students are asked to compute the means for the following two data sets:
    a. Age in months (Question 4);
    b. Length of hand (Question 5).

    Sample data sets from SEP (random samples) are presented in Tables 1 and 2.

Table 1 Random sample of size 28 for SEP Question 4 (Age in months)

195	196	190	188	192	196	198	189	194	192	192	194	199	194
189	200	190	194	201	187	197	195	196	200	187	200	199	186

Table 2 Random sample of size 28 for SEP Question 5 (Length of hand, in cm)

17.5	16	15	17.5	16	16	19	21	18	19	11.5	17.5	20.5	22
17	19.5	20	18.5	19	19	18.5	16.1	18	19	19.8	19	18	18

2. For each of the two datasets above, students are instructed to make a graphical presentation of the data together with the mean they have computed.
    One possible graphical representation is to plot the data points and the mean on the real number line.
3. Students are then asked: "Based on your graphical presentations, how effective is the mean as a measure of central tendency for the two data sets?"
    The two data sets are expected to have small variance; thus, students are expected to notice that the mean is an effective centrality measure.
4. Students are then asked to repeat #1 and #2 for a data set on Magic Age (Question 31).
    A sample data set from SEP (random sample) is presented in Table 3.

Table 3 Random sample of size 28 for SEP Question 31 (Magic Age)

17	18	19	17	17	289	18	16	18	17	16	16	18	18
18	15	17	18	15	17	18	15	19	19	17	18	18	18

5. Students are instructed to remove the largest data point from their Magic Age data set. They then repeat #1 and #2 for this new data set.
6. Based on #4 and #5, students are asked the same questions as in #3.
    The Magic Age data set is expected to have extreme values. Thus, students might notice how the mean is not very near to many of the values because it is increased by the extreme value/s. Moreover, removing the largest data point from a Magic Age data set will significantly affect the mean. This can help students realize the sensitivity of the mean to extreme values.

Appendix 2. SEP Activity on Confidence Intervals

Another activity that uses data sets from Senso Eskwela Pilipinas (SEP) is called *Confidence Interval for the Population Mean*. Students are asked to answer the question "Do Grade 11 students sleep for at least 8 hours, on the average, on a weekday?" To answer this question, the first compute the sample mean and sample standard deviation of a data set then they proceed to construct confidence intervals on the population mean. The activity is outlined below.

1. Students are first asked to generate a random sample of size 20 from the SEP population of Grade 11 students for SEP Question 24: "How many hours a day do you spend sleeping during a weekday?"
   A sample data set from SEP (random sample) is presented in Table 4.

Table 4 Random sample of size 20 for SEP Question 24 (Hours of sleep)

5	9	8	7	4	7	6	6	5	6	7	7	7	8	8	4	8	7	5	6

2. Students are asked to compute the sample mean (point estimator) and sample standard deviation. They are then asked to answer the question "Do Grade 11 students sleep for at least 8 hours, on the average, on a weekday?" based on these.
3. Students are then instructed to construct a 95% confidence interval for the (SEP) population mean. They are then asked to answer the same question as in #2. Moreover, they are specifically asked "What does the confidence level of 95% imply? What does the interval represent?"
4. Optionally, students may be asked to repeat steps #1-#3 but this time, using a random sample of size 40 and then constructing a 99% confidence interval.
   A sample data set from SEP (random sample) is presented in Table 5.

Table 5 Random sample of size 40 for SEP Question 24 (Hours of sleep)

5	6	8	5	7	7	7	8	6	8	8	8	7	6	8	6	8	8	8	8.5
7	6.5	5	4	10	6	8	8	7	6	9	8	5	7	7	5	6	6	5	7

Enhancing Trigonometry Learning through a Mobile App

Maria Alva Q. ABERIN[a,*], Ma. Louise Antonette N. DE LAS PEÑAS[a], Agnes D. GARCIANO[a], Juan Carlo F. MALLARI[a], Jumela F. SARMIENTO[a], Mark Anthony C. TOLENTINO[a], & Debbie Marie B. VERZOSA[b]
[a]*Department of Mathematics, Ateneo de Manila University, Philippines*
[b]*Department of Mathematics and Statistics, University of Southern Mindanao, Philippines*
*maberin@ateneo.edu

Abstract: A mobile app called *Trigomatch* is designed for students to gain an in-depth understanding of trigonometric concepts in an engaging and dynamic learning environment. A description of the app as well as the pedagogical framework and game design principles that form the backbone of the app are discussed.

Keywords: Mobile app, trigonometry, trigonometric ratios

1. Introduction

Trigonometry is one of the topics in high school mathematics that involves developing skills in algebra and geometry, which is essential in studying pre-calculus and calculus. Moreover, knowledge of trigonometric functions is needed for understanding topics in Newtonian physics, architecture, surveying, and many branches of engineering (Weber, 2005).

Oftentimes, students have difficulties in problem solving in trigonometry. One reason is due to lack of motivation (Nurmeidina & Rafidiyah, 2019). Difficulty is also due to understanding the concepts of trigonometry and its applications (Nanmumpuni & Retnawati, 2021). Understanding multiple representations of trigonometric functions can also be challenging to students (Marchi, 2012). Students have difficulty in the presentations of the topic: the unit circle and the right triangle approach (Jenkins, 2022).

Studies in the literature have shown the benefits of technology in studying trigonometry. Bedada and Machaba (2022) showed that teaching trigonometric functions with the aid of GeoGebra (https://www.geogebra.org/) has significant effects on the achievement of students. Similarly, Rahman et al. (2016), using an interactive GeoGebra Learning Module, showed that students developed positive motivation in studying trigonometry. Emerging technologies such as *TrigReps* (Bornstein, 2020) and *Tangible User Interface (TUI)* (Zamorano Urrutia et al., 2019) proved to have the potential to facilitate learning of trigonometry. The representations offered by *TrigReps* (Bornstein, 2020) assisted the students in connecting their modifications to the algebraic representation with the subsequent modifications to the graphical representation. Moreover, *Tangible User Interface (TUI)* (Zamorano Urrutia et al., 2019) provides a pedagogical experience that prioritizes physical manipulation-based exploration and encourages intuitive and group learning.

Taking advantage of the benefits of technological tools (Haleem et al., 2022) to build knowledge, understanding, and skills in trigonometry and to increase engagement and learning motivation in students, we designed a mobile app called *Trigomatch.* Designed through an interactive game with interactive feedback, the app focuses on the understanding of fundamental concepts in trigonometry such as reference angles and trigonometric ratios.

2. Pedagogical Basis

Drawing on the work of Hans Freudenthal, Martín-Fernández et al. (2022) distinguished between trigonometry from the perspective of elementary geometry and goniometry,

corresponding to the right triangle and unit circle approaches presented by Jenkins (2022). In the former, angles are absolute and static, and are determined by the sides of a right triangle in a non-oriented plane. In the latter, angles are dynamic, and determined by a rotation of half-lines in an oriented plane. They found that transitions between these two systems of thought are not straightforward–even pre-service teachers cling to their conceptions of a static angle and barely know how to draw oriented angles in a circle.

A possible intervention for facilitating understanding of trigonometry is through the presentation of trigonometric representations through interactive computer-aided instruction (CAI). Jenkins (2022) argued that when visual representations of trigonometry are presented using interactive technology, they are more accurate and allow the learners to proceed at their own pace. Further, because of the accuracy of these presentations, they can be used to reinforce images in a person's memory, which is not the case for hand-drawn images which tend to be distorted. While she found that students can learn as well from CAI as from traditional instruction, the use of CAI allows the teachers to differentiate instruction and personalize tasks for their students.

In common with computer-aided technology described by Jenkins (2022), the *Trigomatch* app presents visual representations of basic trigonometric concepts to allow students to extend their understanding of trigonometry from the right triangle to the unit circle approach. The app is interactive in the sense that it presents the concept with feedback mechanisms in a game-like environment. More details on the app are provided in the next section.

3. The *Trigomatch* App

3.1 App Description

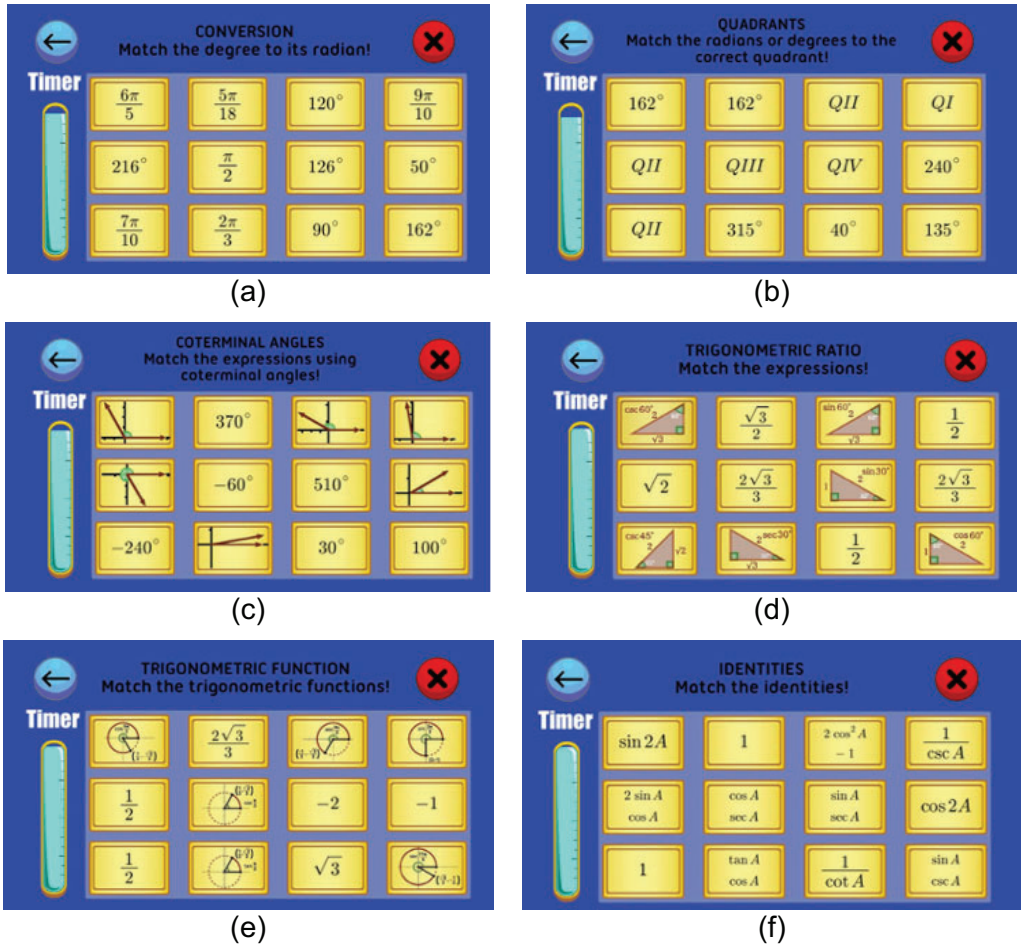
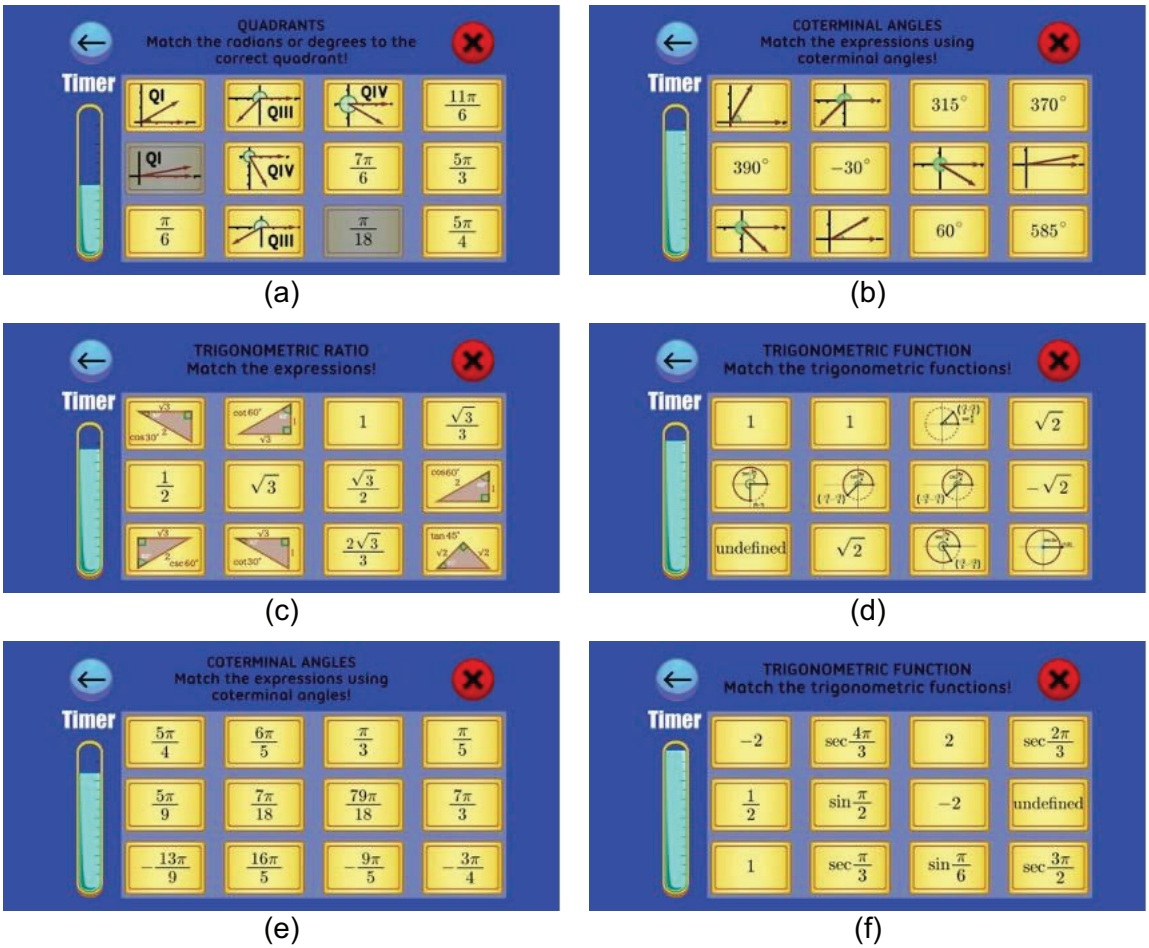


Figure 1. Screenshots showing the six options in *Trigomatch*.

In the *Trigomatch* app, six (6) different options can be played: *Conversion* (Figure 1(a)), *Quadrants* (Figure 1(b)), *Coterminal angles* (Figure 1(c)), *Trigonometric Ratio* (Figure 1(d)), *Trigonometric Function* (Figure 1(e)), and *Identities* (Figure 1(f)). Moreover, learners may choose to display 12, 20, or 36 cards at a time and then match two equivalent representations or trigonometric expressions shown by the cards. The objective is to locate every pair that matches within the allotted time. The game's time aspect feature has the potential to motivate and thrill the learners while they play.



(a) (b) (c) (d) (e) (f)

Figure 2. Screenshots showing the different levels, *with* or *without pictures.*

3.2 Game-design Factors

One of the challenges that students face in learning trigonometry is the presence of several trigonometry facts, conversions, and identities that they need to memorize. These include, among others, conversions between radians and degrees, the trigonometric function values for special angles, and the sum, product, half-angle, and double-angle identities. While these can be learned through rote memorization or drill-and-practice, students usually tend to find such tasks tedious and uninteresting. Thus, a game-based learning approach has the potential to support students' learning of these trigonometry topics. With game-based learning, we aim to increase students' motivation and engagement by situating the learning of trigonometry within a game that can create high situational interest (Hidi & Renninger, 2006).

In the design and development of *Trigomatch*, we took into consideration 11 crucial game-design factors identified in the Game-based Learning (GBL) Design Model of Shi and Shih (2015). Figure 3 presents a summary of how these 11 game-design factors have been applied for *Trigomatch*.

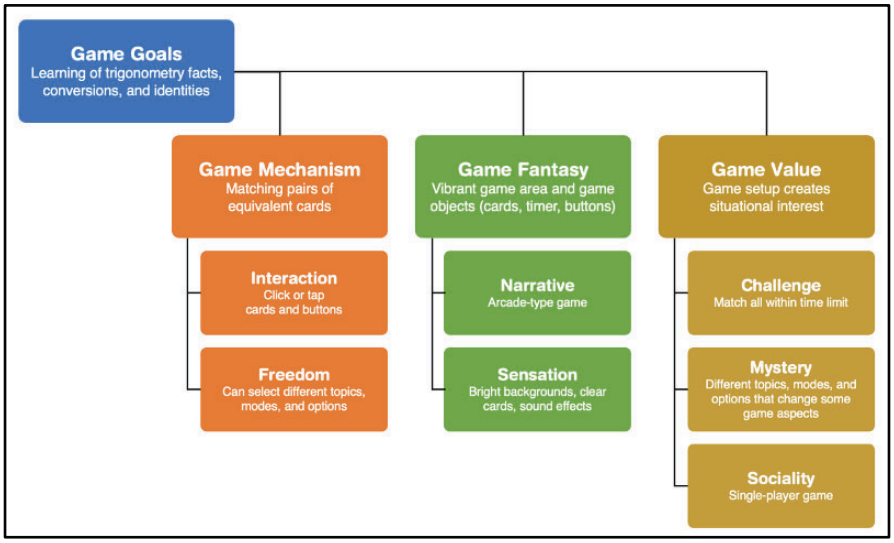


Figure 3. Game-design factors (Shi & Shih, 2015) for *Trigomatch*

The first game-design factor is the game goal, which significantly influences all the other game-design factors. In the case of *Trigomatch*, the game goal is a learning objective focused on trigonometry topics. The game mechanism is then decided towards achieving the game goal. As previously described, the mechanics of *Trigomatch* involve matching pairs of cards (akin to some memory games) that contain equal or equivalent trigonometric expressions. Learners interact with the game through simple click or tap actions on game buttons and cards. An important feature of the game mechanism is adaptivity (Plass et al., 2015) in the sense that learners have the freedom to select from different topics, modes, and options. This allows learners to align their playing experiences with their current level or interest with respect to the learning content. The availability of these different topics, modes, and options also contributes to the game's mystery, which in turn can keep learners interested in progressing in the game. The main challenge in *Trigomatch* is to match all the cards within the time limit. As part of adaptivity, learners can disable the timer if they are still beginning to learn the topics.

Trigomatch is envisioned to be a simple arcade-like game with relatively short rounds that can be played repeatedly. Thus, the game fantasy and narrative are kept simple and are delivered mainly through the game area and game objects such as cards, timer, and buttons. In terms of sensation, *Trigomatch* has display and sound prompts during gameplay. Moreover, as the game is also intended for independent use, *Trigomatch* has been designed to be a single-player game. However, teachers can use the app as part of group activities in the classroom, face-to-face or virtual. All the features of *Trigomatch* contribute to its game value, which is oriented towards creating situational interest among learners.

4. Integration and Use of *Trigomatch*

This section describes how the *Trigomatch* can be used as a pedagogical tool based on the RAT (Replacement, Amplification, Transformation) framework (Hughes et al., 2006). Using this framework, the app can replace the traditional method of providing examples and exercises for students to solve without changing the learning goals of a lesson. The printed worksheets or exercises from a textbook have been replaced by a set of cards shown in a more appealing game-like setting. This function can help teachers offer exercises in an engaging and enjoyable manner.

Playing the app can enrich or amplify a student's experience in solving problems involving concepts in trigonometry. The design of the app (*with or without pictures*) gives students the option to work at just the correct degree of difficulty and includes components that can excite and push them to work on various levels.

Finally, the *Trigomatch* app may encourage teachers to shift from teacher-centered to student-centered instruction. This is achievable because the app was made to be simple enough for students to use and comprehend without much help from a teacher. Students who are learning asynchronously or remotely can thus utilize the app. Task redefinition is another way that transformation might take place. For instance, utilizing the app, one may convert individual work into group activities. This allows the students a chance to interact and instruct one another on the problem-solving strategies needed to match two trigonometric representations displayed on the cards.

5. Conclusion and Future Outlook

Advanced mathematics and other allied disciplines, such as engineering and physics, depend heavily on trigonometry. However, many students struggle to understand and apply trigonometric principles and often lack the motivation to learn these. The *Trigomatch* app presented in this paper is a tool that aims to help students gain a better understanding of trigonometric concepts and reinforce the skill of recognizing equivalent trigonometric representations. Game design principles were employed in the app's design to keep students motivated and engaged. By playing the app, students can recall trigonometric ratios, conversions, identities and coterminal angles in an enjoyable and productive way. The option to display pictures as one plays the app is a useful feature especially for learners who may benefit from the visual representations of trigonometric ideas.

This study is work in progress. Opportunities for further research on how *Trigomatch* is used by students and how effective it can be to improve their trigonometric skills is currently being pursued. An experimental study with pretest and posttest research design is going to be used to accomplish this. Furthermore, the app may be expanded by adding other options such as trigonometric functions and their graphs or using the laws of sines and cosines. This will help build on the skills that students have learned and allow them to advance in other topics.

Acknowledgements

This paper is one of the outputs of the Ateneo Mathplus Resources Laboratory housed at the Department of Mathematics, School of Science and Engineering, Ateneo de Manila University. The authors thank the Department of Science and Technology-Philippine Council for Industry, Energy, and Emerging Technology Research and Development (DOST-PCIEERD) and the University Research Council (URC), Ateneo de Manila University for the support of the development of the mathematical resources for Grades 1 to 11 Mathematics. Acknowledgement also goes to Dr. Ma. Mercedes T. Rodrigo and the Ateneo Laboratory for the Learning Sciences (ALLS) for providing technical assistance. We thank our app developers Victor Antonio M. Ortega, Mr. Nigel Benedict Cargo, Mr. Jose Teodoro Lacson, and Mr. Amiel Damian F. Justiniani for the apps and video art assets.

References

Bedada, T. B., & Machaba, F. (2022). The effect of GeoGebra on STEM students learning trigonometric functions. *Cogent Education*, *9*(1), 2034240.

Bornstein, N. (2020). Teaching transformations of trigonometric functions with technology. *Journal of Interactive Media in Education, 2020*(1), 1-9.

Haleem, A., Javaid, M., Qadri, M. A., & Suman, R. (2022). Understanding the role of digital technologies in education: A review. *Sustainable Operations and Computers*, *3*, 275-285.

Hidi, S., & Renninger, K. A. (2006). The four-phase model of interest development. *Educational Psychologist*, *41*(2), 111-127.

Hughes, J. E., Thomas, R., & Scharber, C. (2006). Assessing technology integration: The RAT – Replacement, Amplification, and Transformation – Framework. In C. M. Crawford, R. Carlsen, K. McFerrin, J. Price, R. Weber, & D. A. Willis (Eds.), *Proceedings of SITE 2006--Society for*

Information Technology & Teacher Education international conference (pp. 1616-1620). Orlando, Florida, USA: Association for the Advancement of Computing in Education (AACE).

Jenkins, R. (2022). *Impact of interactive computer-aided instruction in learning trigonometry in a high school precalculus course.* [Doctoral dissertation, Franklin University]. https://fuse.franklin.edu/docpub/71

Marchi, D. (2012). *A study of student understandings of the sine function through representations and the process and object perspectives* [Doctoral dissertation, The Ohio State University]. http://rave.ohiolink.edu/etdc/view?acc_num=osu1343253667

Martín-Fernández, E., Rico, L., & Ruiz-Hidalgo, J. F. (2022). Conversions between trigonometric representations by pre-service secondary school teachers. *PNA (Revista de Investigacion en Didactica de la Matematica), 16*(3), 237-263.

Nanmumpuni, H. P., & Retnawati, H. (2021). Analysis of senior high school student's difficulty in resolving trigonometry conceptual problems. *Journal of Physics: Conference Series*, *1776*, 012012.

Nurmeidina, R., & Rafidiyah, D. (2019). Analysis of students' difficulties in solving trigonometry problems. In N. Ishartono, M. S. Adhantoro, Y. Sidiq, & Y. Sulistyono (Eds.), *Proceedings of the 4th progressive and fun education international conference*, *Profunedu 2019*. EAI.

Plass, J. L., Homer, B. D., & Kinzer, C. K. (2015). Foundations of game-based learning. *Educational Psychologist*, *50*(4), 258-283.

Rahman, M. H. A., & Puteh, M. (2016). Learning trigonometry using GeoGebra learning module: Are under achieve pupils motivated? *AIP Conference Proceedings*, *1750*, 040001.

Shi, Y.-R., & Shih, J.-L. (2015). Game factors and game-based learning design model. *International Journal of Computer Games Technology, 2015*, 1-11.

Weber, K. (2005). Students' understanding of trigonometric functions. *Mathematics Education Research Journal*, *17*(3), 91–112.

Zamorano Urrutia, F. J., Cortés Loyola, C., & Herrera Marín, M. (2019). A tangible user interface to facilitate learning of trigonometry. *International Journal of Emerging Technologies in Learning*, *14*(23), 152-164.

Educational Psychology
Vol. 29, No. 6, October 2009, 729–745

Routledge
Taylor & Francis Group

Factors influencing Filipino children's solutions to addition and subtraction word problems

Debbie Bautista*, Michael Mitchelmore and Joanne Mulligan

School of Education, Macquarie University, Sydney, Australia

(*Received 25 April 2009; final version received 12 August 2009*)

Young Filipino children are expected to solve mathematical word problems in English, which is not their mother tongue. Because of this, it is often assumed that Filipino children have difficulties in solving problems because they cannot read or comprehend what they have read. This study tested this assumption by determining whether presenting word problems in Filipino or reading them aloud to children in either language facilitated solution accuracy. Contrary to the initial hypothesis, reading word problems aloud did not seem to improve student performance ($p > 0.10$). In contrast, presenting word problems in Filipino significantly improved solution accuracy ($p < 0.0001$) and led to differences in error patterns – children were less likely to use an inappropriate arithmetic operation when problems were presented in Filipino. However, the language of the problem had minimal effects on the more difficult Compare problem type. Finally, the benefits of using Filipino were more pronounced for low-achieving students who may have lower proficiency in English than their high-achieving peers ($p < 0.01$).

Keywords: arithmetic; English as a second language; comprehension; primary

Although many Filipino children may begin school with little exposure to English (Young, 2002), they are expected to learn mathematics and solve word problems in English (Department of Education Bureau of Elementary Education, 2003). Learning mathematics in general, and solving word problems in particular, poses difficulties, given that large-scale assessments show that many students are not proficient in the language. The Philippine National Achievement Test (NAT) is one such example. It assesses Grade 6 and fourth-year high school students (Grade 10) from public schools in five subjects – mathematics, science, English, Filipino, and social studies. Results from the 2005–2006 NAT show that the majority of fourth-year high-school students have low mastery in English (National Statistical Coordination Board, 2007).

Scores in the same achievement tests also indicate a lack of mastery of mathematics. Internationally, the 2003 Trends in Mathematics and Science Study (TIMSS) showed that the Philippines ranked twenty-third out of 25 countries in Grade 4, and forty-first out of 45 countries in Grade 8. In addition, the 2004 Executive Report on the Philippine performance in TIMSS (cited by Carteciano, 2005) stated that in the five years since 1999, only seven of the 15 Philippine regions demonstrated improvements on skills

*Corresponding author. Email: debbie.bautista@students.mq.edu.au

ISSN 0144-3410 print/ISSN 1469-5820 online
© 2009 Taylor & Francis
DOI: 10.1080/01443410903254591
http://www.informaworld.com

and competencies in mathematics. Language barriers and comprehension problems were among the 'what-else-is-new problems' mentioned in the same report.

This paper reports the first part of a larger study which aims to design an intervention programme focused on improving performance in solving addition and subtraction word problems for young Filipino children studying in public schools in the National Capital Region (NCR), the second lowest-performing region in the Philippines (National Statistical Coordination Board, 2007). Because Filipino schoolchildren solve word problems in English, a language typically encountered only in school, it is often assumed that children cannot solve word problems either because they cannot read or comprehend what they have just read. However, little research has been conducted in the area of word problem solving among Filipino public school children to support this assumption. It is necessary initially to conduct a study to investigate the difficulties faced by Filipinos as they solve basic addition and subtraction word problems. We aim to do this by studying whether embedding problems in printed text, or presenting problems in English contributes to solution difficulties. Findings and insights gained from this initial effort will provide indicators of the pedagogical needs of students taking part in a proposed intervention programme.

Background

Word problems form an integral part of mathematics curricula, and have always been well represented in both classroom and large-scale assessments. In spite of children's extensive exposure to word problems, children still find word problems difficult to solve. Students may fail to solve word problems even though they can solve corresponding problems given in purely numeric format (Cummins, Kintsch, Reusser & Weimer, 1988). This phenomenon has been confirmed by studies on Filipino students as well (Bernardo, 2002; Brawner et al., 1999).

In contrast, several studies have provided instances where arithmetic or algebra word problems were easier to solve than equations (Carraher, Carraher, & Schliemann, 1987; McNeil, Uttal, Jarvin, & Sternberg, 2009; Zhou & Lin, 2001). For example, Hughes (1986) found that English nursery school children were better at solving addition and subtraction questions when these were embedded in hypothetical situations. Similarly, Koedinger and Nathan (2004) showed that eighth- and ninth-grade students were better able to solve algebra word problems than symbolic problems. It should be noted, however, that empirical studies which show that word problems are easier than problems in numeric format either involved children in at least the upper primary grades or were carried out orally using clinical interviews.

Carpenter and Moser's (1984) seminal work on children's problem solving strategies and the extensive research which followed (De Corte & Verschaffel, 1987; Fuson, 1992; Verschaffel, Greer, & De Corte, 2007) elucidate the difficulties students find in solving word problems. To solve word problems, children must process the text, recognise the problem structure, construct an internal representation of the problem and, on the basis of this representation, select and execute a solution plan (De Corte & Verschaffel, 1991; Mayer & Hegarty, 1996; Mulligan & Mitchelmore, 1997).

The problem-solving process described above presumes that a student approaches a word problem in a meaningful way. However, not all students solve problems by devising a solution based on their representation of the situation (Mayer & Hegarty, 1996). Some students utilise 'superficial coping strategies' (Verschaffel & De Corte, 1997, p. 86) such as selecting an operation based on certain keywords (such as

'altogether' and 'more'). They may also select the arithmetic operation they are most familiar with, or the operation most recently discussed in class (Sowder, 1988; cited by Reed, 1999). These superficial coping strategies may have been fostered by the belief that all word problems have one correct answer which can be obtained by some arithmetic operation (Greer, Verschaffel, & De Corte, 2002). For example, young children often answer the nonsense problem 'There are 26 sheep and 10 goats on a ship. How old is the captain?' by performing an arithmetic operation on the two given numbers (Verschaffel, Greer, & De Corte, 2000).

One of the most widely used classifications of word problems is the one proposed by Riley, Greeno, and Heller (1983), who distinguished between Change, Combine, and Compare problems. Change problems involve an action which results in an increase or decrease in quantity. A Change problem with an unknown change quantity is 'Bong has 6 mangoes. He gave Kim some mangoes. Now Bong has 2 mangoes left. How many mangoes did he give Kim?'. Combine and Compare problems relate to static relations between quantities. Combine problems involve sets and subsets. For example, a Combine problem with an unknown result set is 'Lisa has 3 pencils. Rosa has 4 pencils. How many pencils do they have altogether?'. Compare problems concern two compared quantities. The problem 'Tino caught 6 crabs. Nato caught 2 crabs. How many more crabs did Tino catch than Nato?' is an example of a Compare problem with an unknown difference set.

The Combine and Change problems have been shown to be capably solved by second- and third-graders, and even by Kindergarten children when aided by concrete materials (Lean, Clements, & Del Campo, 1990; Riley et al., 1983). Compare problems have been shown to be the most difficult problem type. Empirical data (Bernardo, 1999; Riley et al., 1983) and computer models of word problem solving (Briars & Larkin, 1984; Cummins et al., 1988) show that there is a tendency to answer one of the given numbers in the problem. Some researchers contend that Compare problems are difficult because children may not have the linguistic knowledge required to comprehend comparative language (Cummins et al., 1988; Stern, 1993). Thus, children may ignore comparative terms such as 'more than' (De Corte & Verschaffel, 1986) and interpret the question 'How many more crabs did Tino catch than Nato?' as 'How many crabs did Tino catch?', for which a given number in the problem would be a correct answer. Others argue that the difficulties are due to the solver's deficient understanding of the logical principles, or 'invariants', of addition and subtraction (Vergnaud, 1998). For many children, the connection between comparison situations and subtraction has not been well-established (Nunes & Bryant, 1996). They may initially view subtraction only as removing objects and fail to possess an understanding of an invariant of subtraction – that it could be a means of quantifying the difference.

Factors influencing word problem solution

Some crucial factors in solving word problems are a child's language and reading abilities. Interdisciplinary research has shown that children with language difficulties are outperformed by children with no language or mathematical difficulties on word problems, but not on problems presented in purely numeric or visual format (Jordan, Levine, & Huttenlocher, 1995). Similarly, children with both mathematics and reading deficits performed significantly worse on word problem tasks than students with deficits in mathematics only (Jordan & Hanich, 2000).

For students learning mathematics in a second language, how the language impacts on their learning needs to be considered. A large amount of research has documented the difficulties these students encounter when solving word problems (Adetula, 1990; Latu, 2005). For example, English language learners (ELLs) in the USA score lower in mathematical word problem tests than their non-ELL peers (Abedi & Lord, 2001). Because ELLs may be unfamiliar with particular words, they may find word problems written in English difficult, more so if the problem is linguistically complex (Martiniello, 2008). Research has also shown that tests administered to students with very low English language proficiency suffer from low reliability, particularly for items with complex language structures (Abedi, 2002; Abedi, Hofstetter & Lord, 2004). Fully bilingual students, on the other hand, have been shown to be adept at solving problems even when these were given in their second language (Clarkson, 2007; Whang, 1996). Nevertheless, proficiency in the language in which the problem is presented is necessary for problem solving competence (Morales, 1994).

Research aims

The aim of the overall study was to improve word problem solving performance in the Philippine context. This was attempted by trialling strategies which may help students solve word problems. The literature suggests ways by which word problems may be more accessible for second language learners or students with reading difficulties. One strategy is to present problems in the students' first language (Adetula, 1990). Another strategy is to modify the problem to reduce linguistic ambiguities (Abedi & Lord, 2001; De Corte, Verschaffel, & De Win, 1985). A third strategy is to read problems aloud for students to minimise deficiencies in reading comprehension skills (Meloy, Deville, & Frisbie, 2002). Bernardo (1999) found that Filipino children were more successful in solving addition and subtraction word problems when these were given in the child's first language or when problems were reworded so as to make the relations in the problem more explicit. However, there is no existing study on whether reading word problems aloud for students would facilitate word problem solution for Filipino children.

This study investigated the influence of three factors on word problem solution. First, we replicated to some extent Bernardo's research on the first language advantage. Since not all empirical studies show that the use of students' first language facilitates problem solution (Morales, 1994; Oviedo, 2005), one should be cautious in generalising Bernardo's findings which were based on students from a single school. Replicating his study strengthened findings and enabled other factors for word problem solving performance to be considered.

Second, we determined whether a read aloud accommodation leads to better problem solving performance for young Filipino students. Third, since students' general academic achievement may interact with the possible influence of language and a read aloud accommodation (Bernardo, 1999; Moyer, Sowder, Threadgill-Sowder, & Moyer, 1984), we also considered student achievement, as determined by an external measure. Our analysis was based on students' solution accuracy across all conditions and on an analysis of student errors for each problem type, using categories defined by Bernardo (1999) and Cummins et al. (1988).

This study used a pencil-and-paper test even though evidence points to some drawbacks of relying on written tests for assessing children's arithmetic knowledge (Clements & Ellerton, 1995). Since this study aimed to investigate how children solve problems in the particular Philippine context, it was necessary to assess their

performance when given a written test as part of a large group, which is how Filipino children are typically assessed in school.

Method

Participants

The participants were 75 second- and third-grade students (aged 7–12 years; mean age: nine years two months; 38 girls and 37 boys) from a public school in Quezon City. Quezon City showed the best performance in the NAT administered after the school year 2007–2008 among all cities and municipalities in NCR. In this school, students were streamed into different sections based on the average grade obtained in all subjects (including mathematics and English) the previous year. Section 1 consists of top-performing students, followed by Section 2, and so forth, until Section 6. Although the test was originally intended only for Grade 3 students, the principal requested that students from the best Grade 2 section also take the test. While the researcher was allowed to administer the test to whole classes, the existing set-up in the classroom, where two students share a single desk, prompted the researcher to request from the principal a random sample of students instead. The principal informed the researcher that students in the mid- to lower sections (Sections 3–6) were mostly homogeneous in terms of their academic performance. Nevertheless, she informed the teachers to select a random sample of students from their classes. The teachers assured the researcher that they selected students with different achievement levels in mathematics based on the available assessment data.

The final sample consisted of 15 students from Grade 2 Section 1 and 60 Grade 3 students. Of the 60 Grade 3 students, 20 came from Section 1, 20 from Section 3, 1 from Section 4, and 19 from Section 6. Children were randomly chosen from Sections 3 and 6. However, the children with the highest general average were chosen from Section 1 upon the request of the principal who was concerned with how the best students in Section 1 would perform in this word problem test as part of her efforts to assess and improve the school's performance. For the purposes of analysis, the students from Section 1 in Grades 2 and 3 were classified as high-achieving, the others as low-achieving.

At the time of the test, the participants had received instruction in addition, subtraction, multiplication, and division of whole numbers. The main difference between second- and third-graders with respect to these operations was in the magnitude of the numbers used, as determined by the Philippine Basic Education Curriculum (Department of Education Bureau of Elementary Education, 2003). Word problems are also included in the curriculum for the purpose of applying the skills learned in carrying out arithmetic operations. It is recommended by the curriculum that students solve word problems by stating what is asked, the given facts, and the operation to be used; it was also recommended that they write the number sentence and the solution before giving the final answer. Further, the textbook the children used presents all addition problems as a 'put together' situation, while all subtraction problems are presented as 'take away' situations.

Instruments and procedures

The written test consisted of three problem types adapted from the initial work of Riley et al. (1983), and described in the Background section. These problems were

selected on the basis of previous research and their suitability for the current research context. With respect to difficulty, the Combine problem is considered easier than the Change problem, which is easier than the Compare problem (Lean, Clements, & Del Campo, 1990; Riley et al., 1983). The Combine problem was the prevalent example given in the participants' textbooks. Both the Change and Compare problems were lacking in the textbooks which the children in this sample use.

Four problems, which differ only in the story context, were created for each problem type, and written Filipino translations were made by an assistant for each problem. To check for consistency, problems were also back-translated to English by another assistant proficient in both languages. To control for differences in computational skill, only small numbers were used – all the sums were between 5 and 9. Number triples were selected according to the guidelines put forward by Carpenter and Moser (1984).

The test consisted of two parts, each having six questions. The six questions in each part of the test represented one English and one Filipino version for each of the three problem types. In one part, the test administrator handed out a booklet containing six questions and asked the students to answer at their own pace. In the other part, the test administrator handed out another booklet containing six questions, but read them aloud twice as the children answered the questions one at a time. (The next question was not read out until everyone had finished with the previous question.) Two rooms were used for the test. Children in the first room were given the read-aloud accommodation first (Filipino then English), followed by the test unaccompanied by a read-aloud accommodation (English then Filipino). Children in the second room solved the non-read-aloud version first (English then Filipino), followed by the test with the read-aloud accommodation (Filipino then English). A fully counterbalanced design was not possible because only two classrooms were available for the test. The researcher administered the test in the first room, and an experienced volunteer tutor administered the test in the second room. The tutor was trained to do the procedures and was advised to follow a manual. The children were not assisted by the test administrators.

Each child answered a total of 12 questions, with four questions for each problem type. Questions of the same problem type were superficially different (e.g. using 'coins' instead of 'pencils'). Problems were chosen in each booklet such that each question in Room 1 was given in Room 2 under two different conditions (different language; read aloud if not read aloud in Room 1 and *vice versa*). Each page of the booklet contained only one problem and enough space for solutions. The children were told that they may use the entire space for their work. The children were told to bring pencils to the test venue and no other materials were allowed.

Students who took the test were withdrawn from their mathematics classes. The test was administered in two batches, as determined by the schedule of their mathematics class, and all participants finished the test in less than thirty minutes.

Data analysis procedures

The participants' solutions to each of the 12 problems were scored as follows. One point was given for each correct answer, and zero was given for an incorrect answer or non-response. The proportion of correct answers for the three items for each condition was computed for each student. Mean scores for high-achievers and for low-achievers were then computed based on this proportion, and used for statistical

analysis. A mixed-factorial ANOVA was employed to test for the influence of each of the three factors and the resulting interactions. Error patterns, based on categories defined by Bernardo (1999) and Cummins et al. (1988), were analysed using chi-squared values. A qualitative analysis of the responses to each problem type was also performed.

Results

Solution accuracy

The mean scores for all high-achieving students (Grade 2 Section 1 and Grade 3 Section 1) and for all low-achieving students (Grade 3 Sections 3, 4, and 6) were then obtained and are shown in Table 1. Note that obtaining the mean score across the three different problem types (Combine, Change, Compare) is justifiable because both empirical data and information processing theorists (e.g. Briars & Larkin, 1984) show that these three problems are unidimensional with respect to difficulty. Children who have developed schemata to solve Compare problems should be able to answer the Change problem. Likewise, children who can solve Change would be expected to be capable of answering the Combine problem. The data were pooled over the tests performed in Room 1 and in Room 2 because initial analysis showed that this factor was not significant, $t(73) = -0.457$, $p = 0.649$ (two-tailed), $d = -0.045$.

The results of the $2 \times 2 \times 2$ ANOVA with achievement (high or low) as a between-groups factor, and language (English or Filipino) and test mode (written, or written and read aloud) as within-groups factors are shown in Table 2.

The only significant interaction was that between language and achievement. Although the use of Filipino appears to help both high- and low-achieving students, the first language advantage was greater for low-achieving students (Figure 1).

Although a read-aloud accommodation facilitated problem solution in previous studies, our results failed to show evidence that reading problems aloud in either language would help Filipino children solve addition and subtraction word

Table 1. Means (± standard deviation) of participants' scores.

	Written		Written and read-aloud	
	English	Filipino	English	Filipino
High-achievers ($n = 35$)	0.705 (± 0.311)	0.752 (± 0.272)	0.667 (± 0.343)	0.733 (± 0.289)
Low-achievers ($n = 40$)	0.417 (± 0.259)	0.625 (± 0.274)	0.400 (± 0.264)	0.592 (± 0.267)

Table 2. Analysis of variance for student scores.

Source	*SS*	*df*	*MS*	*F*	*p*	Partial η^2
Achievement (A)	3.17	1	3.17	14.2	0.000	0.163
Language (L)	1.23	1	1.23	31.1	0.000	0.299
Test mode (T)	0.054	1	0.054	1.68	0.200	0.022
A × L	0.381	1	0.381	9.60	0.003	0.116
A × T	0.000	1	0.000	0.007	0.931	<0.001
L × T	0.000	1	0.000	0.001	0.976	<0.001
A × L × T	0.006	1	0.006	0.201	0.655	0.003

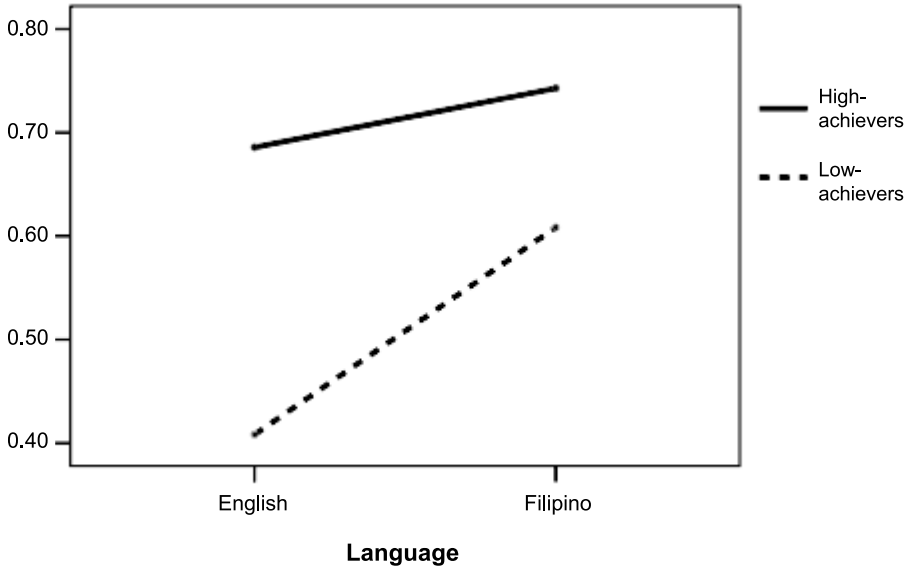


Figure 1. Mean scores of high- and low-achievers.

problems. The three-way interaction and all two-way interactions involving the test mode were insignificant. The main effect of test mode was also insignificant. In fact, an examination of Table 1 shows that students scored somewhat higher if problems were not read aloud. Since this initial analysis revealed that test mode was insignificant, the data will subsequently be pooled for the written and written-and-read-aloud conditions.

There was evidence for main effects of language. Students scored higher when problems were given in Filipino ($M = 0.671$, $SD = 0.263$) than when these were given in English ($M = 0.538$, $SD = 0.291$). There was also a significant main effect for achievement. Not surprisingly, high-achieving students scored higher ($M = 0.604$, $SD = 0.269$) than low-achieving students ($M = 0.508$, $SD = 0.203$).

Error analysis of solutions

To provide a richer account for the explanation of the results, student errors were analysed for each language. The classification of solutions was guided by solution codes used by Cummins and colleagues (1988), and Bernardo (1999). Errors are classified according to the following: first given number errors (FGNE) – the answer produced was the first given number in the problem; second given number errors (SGNE) – the answer produced was the second given number; operation errors (OE) – an incorrect operation was used to solve the problem; and miscellaneous errors which could not be classified in any of the previous categories.

Each problem type was answered four times by each student – twice in English, and twice in Filipino. Because the sample consisted of 75 students, a total of 450 problems were answered in each language. The frequency of solution classifications for English and Filipino problems for high- and low-achieving students is shown in Table 3.

Table 3. Solution classifications for each language, by achievement level.

Language	Correct	FGNE	SGNE	OE	Misc.
High-achievers					
English	144	14	5	38	9
Filipino	156	32	2	14	6
Low-achievers					
English	98	25	21	84	12
Filipino	146	54	14	15	11

Note: FGNE, first given number errors; SGNE, second given number errors; OE, operation errors; Misc., miscellaneous errors; see text for an explanation of error classifications.

Consistent with the results of Bernardo (1999), most errors were of the FGNE and OE type, together accounting for 77.5% of all incorrect solutions. The distribution of solution classifications appears to be different for English and for Filipino problems. Chi-squared analysis revealed that there was a significant difference between these two proportions, and that this difference was evident for both high-achievers (χ^2 = 20.5, $p < 0.001$) and low-achievers ($\chi^2 = 69.6$, $p < 0.001$).

By computing the standardised residuals (cf. Haberman, 1973; Sheskin, 2007) and comparing these with the two-tailed 0.05 value ($z_{.05} = 1.96$) it was found that the following solution strategies for Filipino problems made significant contributions to the obtained chi-squared values: (a) OE for high-achievers, $z_{res} = -2.4$; (b) OE for low-achievers, $z_{res} = -4.9$; (c) correct answers for low-achievers, $z_{res} = 2.2$; and (d) FGNE for low-achievers, $z_{res} = 2.3$. This result means that the use of Filipino in place of English led to fewer OEs for both high- and low-achievers, and an increase in correct answers and FGNEs for low-achievers. Comparing the difference in the proportion of OEs between Filipino and English problems, the difference is more prominent for low-achievers (35.0% vs. 6.3%) than for high-achievers (18.1% vs. 6.7%).

Chi-squared values obtained from the frequencies displayed in Table 3 also indicate that high- and low-achievers had a different distribution of solution strategies for English problems ($\chi^2 = 37.6$, $p < 0.001$) and for Filipino problems ($\chi^2 = 14.5$, $p = 0.006$). An analysis of standardised residuals further revealed that high-achievers produced more correct answers ($z_{res} = 2.9$), less OEs ($z_{res} = -2.5$), and less SGNEs ($z_{res} = -2.0$) in English problems, as compared with low-achievers. There was less difference in the proportion of solutions between high- and low-achievers for Filipino problems. The only cell contribution which led to a significant difference in proportion of solution classifications was the SGNE ($z_{res} = -2.0$), which occurred less frequently among high-achievers than among low-achievers.

Analysis by problem type

To explain the results further, the problem solution success for each problem type was examined for high- and low-achievers. The means and standard deviations of the participants' scores for each problem type are displayed in Table 4. Combine problems were easily solved in both languages. The advantage of using Filipino was most prominent for the Change problem. Finally, Compare problems were difficult even when given in Filipino.

Table 4. Means (± standard deviation) of scores for high- and low-achievers by problem type.

Problem	English	Filipino
High-achievers		
Combine	0.857 (± 0.352)	0.928 (± 0.259)
Change	0.771 (± 0.423)	0.843 (± 0.367)
Compare	0.429 (± 0.498)	0.457 (± 0.502)
Low-achievers		
Combine	0.813 (± 0.393)	0.938 (± 0.244)
Change	0.313 (± 0.466)	0.650 (± 0.480)
Compare	0.100 (± 0.302)	0.238 (± 0.428)

Error analysis by problem type

Since most children were able to solve the Combine problems in either language, there was no striking difference in the error patterns with respect to language or achievement. We thus limit our analysis to the Change and Compare problems.

Change problem

Figure 2 shows the proportion of solution classifications for the Change problem. The interaction between language and achievement is evident not only in solution accuracy but also in the distribution of errors. For low achievers, the use of Filipino led to fewer OEs (41% vs. 6%) and more correct answers (31% vs. 65%). Of the 23 low-achieving students who made an OE in the English Change problem, 18 gave a correct solution

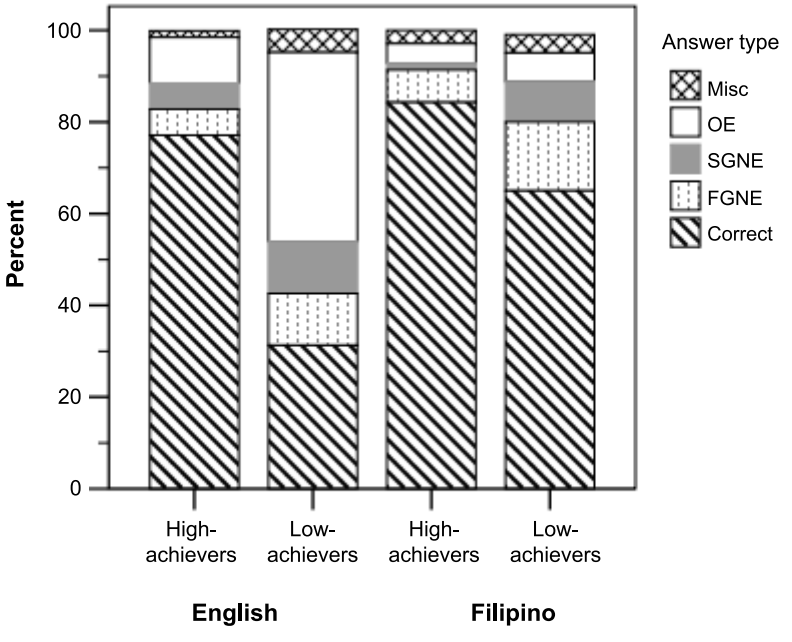


Figure 2. Distribution of solutions for the Change problem.

to the Filipino Compare problem. The proportion of the other types of errors was quite similar for each language. Unlike low-achievers, high-achievers solved the Change problem in both languages.

Compare problem

There was a different pattern for the Compare problem, as shown in Figure 3. For both high- and low-achievers, there is less success in solving the Compare problem than in the Change problem. Both high- and low-achievers produced more OEs in the English Compare problem than in the English Change problem. Although high-achievers were more successful than low-achievers in solving the Compare problem, less than half of the solutions were correct. There was a decrease in OEs for both cohorts of students when Compare problems were presented in Filipino. However, unlike the Change problems where there were fewer OEs and more correct answers among low-achievers when the problem was presented in Filipino, the decrease in OEs in Compare problems was accompanied by almost the same increase in FGNEs. This may explain why the number of correct answers for Compare problems was almost the same in either language. Of the 47 students who produced an OE in the English Compare problem, 28 produced an FGNE and 11 gave a correct solution in the Filipino Compare problem. Finally, although SGNEs were rare, this type of error was more common for low-achievers than for high-achievers.

Discussion

Filipino children find word problems difficult, and the difficulties are often attributed to the inability to comprehend text written in English. This study was conducted to

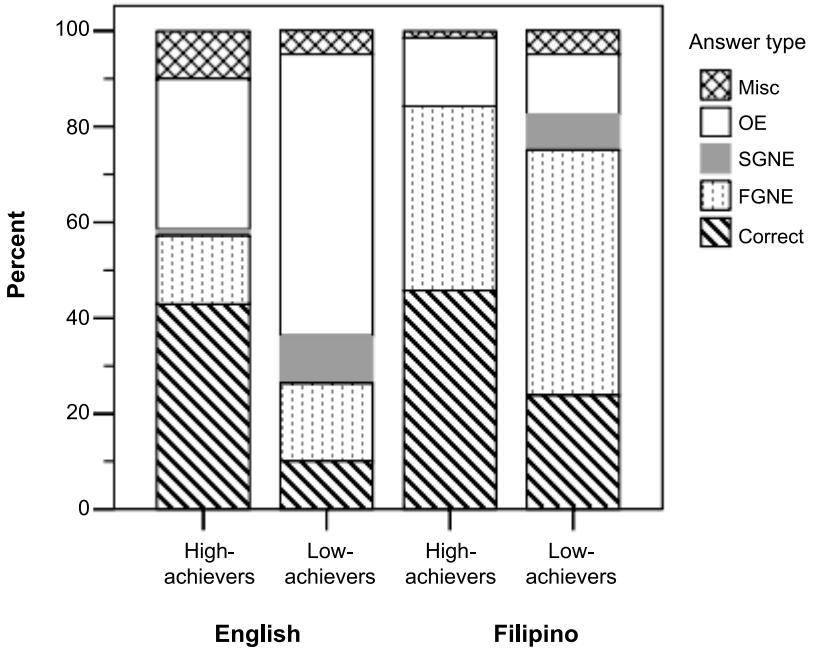


Figure 3. Distribution of solutions for the Compare problem.

gain an understanding of how Filipino children solve English addition and subtraction word problems given in a written test format. This test provided some insight into the difficulties students face and whether these difficulties can be overcome by having word problems read aloud to them, or by giving problems in Filipino.

The analysis revealed that word problem performance improved when problems were presented in Filipino. An important finding from this study was that the use of Filipino led to more correct answers and less OEs. Children who commit an OE may have just done some sort of 'number grabbing'. Bernardo (1999) argues that an OE is 'most likely a result of a gross inability to parse the meaning of the problem text' (p. 155). In fact, two children from this study used multiplication to answer all English, but not Filipino, problems. Apparently, these children totally failed to comprehend the English text, or completely ignored it, forcing them to resort to multiplication (which was the topic they were learning in class at the time of the test). Their lack of understanding may have compelled them to select the most familiar arithmetic operation, or the operation most recently discussed in class (Sowder, 1998; cited by Reed, 1999).

Whereas it was initially predicted, based on the literature, that reading word problems aloud would improve children's performance, this does not appear to have been the case in this sample. Although no time limit was imposed on the children, the procedure where they had to solve problems one at a time, as a group, may have placed pressure on students to give an answer immediately. Reading the problem aloud would then have had no effect on their performance.

Discussion of each problem type

The decrease in OEs on changing from English to Filipino was evident across all three problem types, and was accompanied by quite the same increase in correct solutions, except for Compare. For the Compare problem, performance did not improve substantially, but the distribution of errors was significantly altered by the use of Filipino. In English, close to half of the answers produced was the sum of the two numbers, an OE; in Filipino, the most common answer was an FGNE. Possibly, the use of Filipino made the text more accessible to students. Whereas English text may have been completely incomprehensible, the use of Filipino could have allowed students to realise that adding the two numbers cannot possibly be the correct answer to a question such as 'How many more crabs did Tino catch than Nato?' Still, children failed to give a correct answer, and produced an FGNE instead. One likely explanation is that, although children do not find the text totally senseless, they still lacked the linguistic knowledge necessary to answer Compare problems correctly (Briars & Larkin, 1984; Cummins et al., 1988).

Another explanation for the difficulty in the Compare problems is the children's lack of understanding of additive structures (Vergnaud, 1998; Nunes & Bryant, 1996). This lack of understanding may be due to the lack of exposure to diverse problem types, specifically the Compare problem. Textbooks present limited addition or subtraction situations. The lack of exposure to diverse problem types restricts understanding of other additive situations involving comparisons or missing addends. Addition and subtraction may be viewed as two separate processes, and the inverse relationship between them may not be emphasised.

Among the three problem types in this study, the first language advantage was most evident for the Change problem. The children in this sample may have had limited, or even no exposure to word problems of this type since their textbooks

present addition and subtraction only as joining or taking away operations. Using English for an unfamiliar problem may have caused the text to be meaningless to students. As in the English Compare problem, they may just have combined the two numbers in an attempt to give an answer. However, when problems were given in Filipino, some students were able to understand both the problem text and mathematical structure, allowing them to give a correct answer to the problem. There was less difference in the performance between Combine problems in English and Combine problems in Filipino. Since children had extensive exposure to Combine problems, they could solve this problem regardless of the language used.

Performance of high- and low-achievers

Another important finding was the poor performance of the low-achieving students. The mean score of low-achievers was less than 40% for both the Change and Compare problems. This occurred in spite of giving the written test to Grade 3 students who had already covered more advanced concepts and word problems involving multiplication and division in school. The difficulties are not likely to be the result of computational deficiencies, for two reasons. First, all problems involved only combination sums less than 10. Second, the data showed that most of the errors were either given number errors or errors due to the use of an inappropriate arithmetic operation.

The extent of the poor performance of the children in this study may also be understood in light of previous studies where Grade 3 students were given a written test. For instance, the children in the large-scale study of Lean, Clements and Del Campo (1990) in Victoria, Australia were better in solving word problems than the children in this study. In the Victoria sample, 85% were able to solve the Change problem, and 78% the Compare problem. The results from this study were more comparable with those of Bernardo (1999). In his sample of Grade 3 Filipino children, the mean score for low-achievers was 42% for Filipino problems and 33% for English problems.

The significant language by achievement interaction revealed that the advantage of solving problems in Filipino was greater for low-achievers than for high-achievers. Note that the students in this study were considered as high- or low-achieving based on their performance not just in mathematics but across all subjects, including English, during the previous school year. In Bernardo's (1999) sample of Filipino public school children, high-achievers were better able to understand word problems, whether these were given in English or Filipino. Similarly, the results from this study show that giving problems in Filipino improves the performance of low-achievers more than it does for high-achievers. It may be surmised that high-achieving students have better proficiency in English, allowing them to answer competently problems given in English. This result provides further support for previous findings which show that students with high English proficiency outperform their low English proficiency counterparts on addition and subtraction word problems (Abedi & Lord, 2001; Morales, 1994). It also raises the question of whether tests given in English provide accurate measurements of the mathematical abilities of ELLs (Abedi, 2002).

Limitations and conclusion

The nature of the sample limits the potential generalisability of these results. Our sample was not a random sample and was restricted to one school. In cases such as these, it has been suggested that 'proximal similarity' (cf. Trochim, 2001, p. 42), or

the extent to which a study may be applicable to other situations on the basis of similar contexts, be considered. We described the participants to allow readers to determine whether this study is comparable with their own context. Further, the degree of generalisability is strengthened by the fact that our findings replicate those of Bernardo (1999), which were based on a different school in Quezon City 10 years ago.

Another limitation is the possible difference in the linguistic complexities of the test items. Some problems had greater sentence length which may have contributed to difficulties in problem solution (Martiniello, 2008). The problems also differed in context (e.g. saving coins in a piggy bank, giving mangoes to a friend) and this may have affected children's comprehension of the problem context. Our study was also limited to investigating three possible factors in word problem solution. We considered student achievement as measured by the children's performance across all academic subjects. However, data about students' achievement in particular subdomains such as English, Filipino, or reading may have provided us a fuller understanding of word problem solving performance, especially since it has been well-established that mathematics, language, and reading have related cognitive demands (Jordan & Hanich, 2000; Jordan et al., 1995). Finally, in choosing a pencil-and-paper test for our study, we were not able to collect rich data on children's solutions that we would have collected had we employed individual task-based interviews. Although this study yielded some insights, it also raised several questions: Why are there so many OEs and FGNEs? Why does presenting problems in Filipino facilitate problem solution only in particular problem types? For which problem types does the use of Filipino facilitate solution? Further research is needed to shed light on how Filipino children solve arithmetic word problems in English.

Overall, the results in this study show that the use of Filipino leads to better comprehension of word problem texts, as evidenced by the decrease in OEs. However, to assume that failure in solving word problems is due to the inability to comprehend what was read is highly simplistic. The results from this study indicate that there is more to the solution process than comprehending the problem text because giving problems in Filipino or reading them aloud does not guarantee solution accuracy. An understanding of the problem's mathematical structure is also necessary for success in solving word problems. Still, the use of Filipino led to a decrease in OEs, indicating that the language does help children make some sense of the problem text. As Adetula (1990) argues, 'word problems are reasonable to children, only when the structure of the language in which the problems are presented is understood by the children' (p. 353), and only then can the mathematical structure of the problem be grasped.

The influence of language on learning points to an important consideration for adopting policies which strengthen the use of English as the medium of instruction in Philippine schools. The objective of these policies is for students to develop competence in the English language and be better prepared for job opportunities. This intention operates on the assumption that using English as the medium of instruction leads to better proficiency in the language. Although examining the reliability of this assumption is beyond the scope of this study, our results demonstrate that a lack of English proficiency interferes with mathematics achievement. After two and a half years of formal instruction, the children in this study produced a large number of OEs and FGNEs, more so if the problems were presented in English. The low performance is alarming given that the sample consisted mostly of Grade 3 students who are already expected to solve basic addition and subtraction word problems. Even without showing competent understanding of these foundational concepts, they are expected

to solve problems involving multiplication, division, and rational numbers by the end of the school year. The implications of not being able to solve simple word problems are enormous. For example, Irwin and Irwin (2005) found that children who failed to solve addition problems were unlikely to solve multiplication tasks as well. Thus, this study points to a need to provide support for Filipino students who may not be able to solve basic addition and subtraction problems expected of their grade level.

Guided by the results of this study, the researcher plans to conduct one-on-one interviews on a sample of Filipino children who took the written test. The interviews will consist of word problem tasks, including problem types not presented in this study. One-on-one intervention procedures are also being planned.

References

Abedi, J. (2002). Standardized achievement tests and English language learners: Psychometrics issues. *Educational Assessment, 8,* 231–257.

Abedi, J., Hofstetter, C., & Lord, C. (2004). Assessment accommodations for English-language learners: Implications for policy-based empirical research. *Review of Educational Research, 74,* 1–28.

Abedi, J., & Lord, C. (2001). The language factor in mathematics tests. *Applied Measurement in Education, 14,* 219–234.

Adetula, L.O. (1990). Language factor: Does it affect children's performance on word problems? *Educational Studies in Mathematics, 21,* 351–365.

Bernardo, A.B.I. (1999). Overcoming obstacles to understanding and solving word problems in mathematics. *Educational Psychology, 19,* 149–163.

Bernardo, A.B.I. (2002). Language and mathematical problem-solving among bilinguals. *The Journal of Psychology, 136,* 283–297.

Brawner, F.G., Golla, E.F., Ibe, M.D., De Guzman, F.S., Ogena, E.B., Talisayon, V.M., et al. (1999). *TIMSS-repeat Philippine report* (Vol. 2), *Mathematics.* Manila, Philippines: Department of Education, Culture, & Sports, Department of Science and Technology – Science Education Institute, & University of the Philippines National Institute of Science and Mathematics Education.

Briars, D.J., & Larkin, J.H. (1984). An integrated model of skill in solving elementary word problems. *Cognition and Instruction, 1,* 245–296.

Carpenter, T.P., & Moser, J.M. (1984). The acquisition of addition and subtraction concepts in grades one through three. *Journal for Research in Mathematics Education, 15,* 179–202.

Carraher, T.N., Carraher, D.W., & Schliemann, A.D. (1987). Written and oral mathematics. *Journal for Research in Mathematics Education, 18,* 83–97.

Carteciano, J.A. (2005). *NRCP/DOST addresses dire state of S&T education in the regions.* Retrieved December 23, 2008, from: http://nrcp.dost.gov.ph/index.php?option=com_content&task=view&id=31&Itemid=106

Clarkson, P.C. (2007). Australian Vietnamese students learning mathematics: High ability bilinguals and their use of their languages. *Educational Studies in Mathematics, 64,* 191–215.

Clements, M.A., & Ellerton, N.F. (1995). Assessing the effectiveness of pencil-and-paper tests for school mathematics. In B. Atweh & S. Flavel (Eds.), *Galtha. Proceedings of the 18th annual conference of the Mathematics Education Research Group of Australasia* (pp. 184–188). Darwin, Australia: MERGA.

Cummins, D.D., Kintsch, W., Reusser, K., & Weimer, R. (1988). The role of understanding in solving word problems. *Cognitive Psychology, 20,* 405–438.

De Corte, E., & Verschaffel, L. (April, 1986). *Eye-movement data as access to solution processes of elementary addition and subtraction problems.* Paper presented at the Annual Meeting of the American Educational Research Association, San Francisco, CA. (ERIC Document Reproduction Service No. ED273450.)

De Corte, E., & Verschaffel, L. (1987). The effect of semantic structure on first graders' strategies for solving addition and subtraction word problems. *Journal for Research in Mathematics Education, 18,* 363–381.

De Corte, E., & Verschaffel, L. (1991). Some factors influencing the solution of addition and subtraction word problems. In K. Durkin & B. Shire (Eds.), *Language in mathematical education* (pp. 117–130). Buckingham, UK: Open University Press.

De Corte, E., Verschaffel, L., & De Win, L. (1985). Influence of rewording verbal problems on children's problem representations and solutions. *Journal of Educational Psychology, 77,* 460–470.

Department of Education Bureau of Elementary Education. (2003). *Basic education curriculum (Philippine learning competencies): Mathematics.* Pasig City, Philippines: Department of Education Bureau of Elementary Education.

Fuson, K.C. (1992). Research on learning and teaching addition and subtraction of whole numbers. In G. Leinhardt, R. Putnam & R.A. Hattrup (Eds.), *Analysis of arithmetic for mathematics teaching* (pp. 53–187). Hillsdale, NJ: Lawrence Erlbaum.

Greer, B., Verschaffel, L., & De Corte, E. (2002). The answer is really 4.5: Beliefs about word problems. In G. C. Leder, E. Pehkonen & G. Törner (Eds.), *Beliefs: A hidden variable in mathematics education?* (pp. 271–292). Dordrecht, The Netherlands: Kluwer Academic Publishers.

Haberman, S.J. (1973). The analysis of residuals in cross-classified tables. *Biometrics, 29,* 205–220.

Hughes, M. (1986). *Children and number.* New York: Basil Blackwell.

Irwin, K.C., & Irwin, R.J. (2005). Assessing development in numeracy of students from different socio-economic areas: A Rasch analysis of three fundamental tasks. *Educational Studies in Mathematics, 58,* 289–298.

Jordan, N.C., & Hanich, L.B. (2000). Mathematical thinking in second-grade children with different forms of LD. *Journal of Learning Disabilities, 33,* 567–578.

Jordan, N.C., Levine, S.C., & Huttenlocher, J. (1995). Calculation abilities in young children with different patterns of cognitive functioning. *Journal of Learning Disabilities, 28,* 53–64.

Koedinger, K.R., & Nathan, M.J. (2004). The real story behind story problems: Effects of representations on quantitative reasoning. *Journal of the Learning Sciences, 13,* 129–164.

Latu, V.F. (2005). Language factors that affect mathematics teaching and learning of Pasifika students. In P.C. Clarkson, A. Downton, D. Gronn, M. Horne, A. McDonough, R. Pierce & A. Roche (Eds.), *Building connections: Research, theory and practice. Proceedings of the 28th Conference of the Mathematics Education Research Group of Australasia* (pp. 483–490). Sydney: MERGA.

Lean, G.A., Clements, M.A., & Del Campo, G. (1990). Linguistic and pedagogical factors affecting children's understanding of arithmetic word problems: A comparative study. *Educational Studies in Mathematics, 21,* 165–191.

Martiniello, M. (2008). Language and the performance of English-language learners in math word problems. *Harvard Educational Review, 78,* 333–368.

Mayer, R.E., & Hegarty, M. (1996). The process of understanding mathematical problems. In R.J. Sternberg & T. Ben-Zeev (Eds.), *The nature of mathematical thinking* (pp. 29–53). Mahwah, NJ: Lawrence Erlbaum.

McNeil, N.M., Uttal, D.H., Jarvin, L., & Sternberg, R.J. (2009). Should you show me the money? Concrete objects both hurt and help performance on mathematics problems. *Learning and Instruction, 19,* 171–184.

Meloy, L.L., Deville, C., & Frisbie, D.A. (2002). The effect of a read aloud accommodation on test scores of students with and without a learning disability in reading. *Remedial and Special Education, 23,* 248–255.

Morales, R.V. (1994). Comprehension and solution patterns of simple addition and subtraction math word problems among Mexican American bilingual elementary school students (Doctoral dissertation, University of California, Santa Barbara, 1994).

Moyer, J.C., Sowder, L., Threadgill-Sowder, J., & Moyer, M.B. (1984). Story problem formats: Drawn versus verbal versus telegraphic. *Journal for Research in Mathematics Education, 15,* 342–351.

Mulligan, J.T., & Mitchelmore, M.C. (1997). Young children's intuitive models of multiplication and division. *Journal for Research in Mathematics Education, 28,* 309–331.

National Statistical Coordination Board. (2007). *Students' scores in achievement tests deteriorating; CARAGA and Eastern Visayas rank highest.* Retrieved August 7, 2008, from http://www.nscb.gov.ph/factsheet/pdf07/FS-200705-SS2-01.asp

Nunes, T., & Bryant, P. (1996). *Children doing mathematics.* Cambridge, MA: Blackwell.
Oviedo, G.C.B. (2005). Comprehending algebra word problems in the first and second languages. In J. Cohen, K.T. McAlister, K. Rolstad & J. MacSwan (Eds.), *Proceedings of the 4th International Symposium on Bilingualism* (pp. 267–295). Somerville, MA: Cascadilla Press.
Reed, S.K. (1999). *Word problems: Research and curriculum reform.* Mahwah, NJ: Lawrence Erlbaum.
Riley, M.S., Greeno, J.G., & Heller, J.I. (1983). Development of children's problem-solving ability in arithmetic. In H.P. Ginsburg (Ed.), *The development of mathematical thinking* (pp. 153–196). Orlando, FL: Academic Press.
Sheskin, D.J. (2007). *Handbook of parametric and nonparametric statistical procedures* (4th ed.). Boca Raton, FL: Chapman & Hall/CRC.
Stern, E. (1993). What makes certain arithmetic word problems involving the comparison of sets so difficult for children? *Journal of Educational Psychology, 85,* 7–23.
Trochim, W.M.K. (2001). *Research methods knowledge base* (2nd. ed.). Cincinnati, OH: Atomic Dog Publishing.
Vergnaud, G. (1998). A comprehensive theory of representation for mathematics education. *The Journal of Mathematical Behavior, 17,* 167–181.
Verschaffel, L., & De Corte, E. (1997). Word problems: A vehicle for promoting authentic mathematical understanding and problem solving in the primary school? In T. Nunes & P. Bryant (Eds.), *Learning and teaching mathematics: An international perspective* (pp. 69–97). East Sussex, UK: Psychology Press.
Verschaffel, L., Greer, B., & De Corte, E. (2000). *Making sense of word problems.* Lisse, The Netherlands: Swets & Zeitlinger.
Verschaffel, L., Greer, B., & De Corte, E. (2007). Whole number concepts and operations. In F.K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 557–628). Charlotte, NC: Information Age Publishing.
Whang, W.-H. (1996). The influence of English-Korean bilingualism in solving mathematics word problems. *Educational Studies in Mathematics, 30,* 289–312.
Young, C. (2002). First language first: Literacy education for the future in a multilingual Philippine society. *International Journal of Bilingual Education and Bilingualism, 5,* 221–232.
Zhou, Z., & Lin, J. (2001). Developing mathematical thinking in Chinese kindergarten children: The case of addition and subtraction. *International Journal of Educational Policy, Research and Practice, 2,* 141–155.

Copyright of Educational Psychology is the property of Routledge and its content may not be copied or emailed to multiple sites or posted to a listserv without the copyright holder's express written permission. However, users may print, download, or email articles for individual use.

Ateneo de Manila University

Archīum Ateneo

Mathematics Faculty Publications | Mathematics Department

3-2024

GeoGebra Applets for Fostering Conceptual Understanding in Algebra

Ma. Louise Antonette N. De las Penas

Mark Anthony C. Tolentino

Maria Alva Q. Aberin

Agnes D. Garciano

Juan Carlo F. Mallari

See next page for additional authors

Follow this and additional works at: https://archium.ateneo.edu/mathematics-faculty-pubs

 Part of the Algebra Commons, and the Education Commons

Authors

Ma. Louise Antonette N. De las Penas, Mark Anthony C. Tolentino, Maria Alva Q. Aberin, Agnes D. Garciano, Juan Carlo F. Mallari, Jumela F. Sarmiento, and Debbie Marie B. Verzosa

GEOGEBRA APPLETS FOR FOSTERING CONCEPTUAL UNDERSTANDING IN ALGEBRA

Ma. Louise Antonette N. De Las Peñas[1], Mark Anthony C. Tolentino[1], Maria Alva Q. Aberin[1], Agnes D. Garciano[1], Juan Carlo F. Mallari[1], Jumela F. Sarmiento[†,1] and Debbie Marie B. Verzosa[2]
[1]*Department of Mathematics, Ateneo De Manila University, Philippines*
[2]*Department of Mathematics and Statistics, University of Southern Mindanao, Philippines*

ABSTRACT

This paper discusses two *GeoGebra* applets, *Radical* and *Parabola*, that are designed to strengthen the conceptual understanding of specific topics in algebra. The design and pedagogical basis of the applets are presented. The integration of the applets in teaching Grade 9 mathematics in a partner high school in the Philippines is then discussed. Finally, we report feedback gathered from students and teachers during this integration. Their feedback indicates the potential of these applets for improving students' learning of algebra.

KEYWORDS

Mobile Technology in Teaching Mathematics, Mathematical Applets, *Geogebra* Applets, Algebra Applets

1. INTRODUCTION

In the time of the COVID-19 pandemic in the Philippines, the Department of Education (DepEd) has called for the blended learning modality in schools (DepEd Memo 012, 2020). This introduced several challenges in the teaching and learning of mathematics. Due to the limited time for DepEd to prepare printed materials in lieu of face-to-face classes, there was a need to prepare more instructional materials to help teachers address the Most Essential Learning Competencies (MELCs) prescribed by DepEd's curriculum (DepEd 2020). Under projects funded by the Philippine government (De Las Peñas et al, 2023, De Las Peñas et al 2022), the authors collaborated with particular DepEd School Division Offices to design and create mathematical applications (apps) accompanied with teaching guides and instructional videos to provide teachers with alternative means to teach mathematics. These apps are also being recommended for use in the new normal under the DepEd's *Basic Education Learning Recovery Plan* in which schools are directed towards addressing learning gaps brought about by the pandemic (DepEd Memo 664, 2022).

This short paper contributes to the literature on mathematical learning and the role of technology during the time of COVID-19 and the new normal. In particular, we present two mathematical applets, namely *Radical* and *Parabola*, that were created under the aforementioned government projects and that were designed for Grade 9 students in algebra. We focus on these two applets that give students opportunities to interact with mathematical concepts in a dynamic geometry environment (DGE). These were created using the open-access software *GeoGebra* (https://www.geogebra.org/), making these easily accessible through contemporary technologies such as tablets and mobile phones.

The applets highlight different instructional design features that can be provided with digital tools to support the teaching and learning of mathematics. The *Radical* applet employs the immediate feedback feature that supports practice, where students get the opportunity to strengthen content knowledge previously acquired by practicing at their own pace, and repeatedly, as often as needed (Hillmayr et al., 2020). The *Parabola* applet, on the other hand, makes use of the dragging mode of a DGE to arrive at dynamic families of graphs and to observe relationships among various elements such as the graphs and their corresponding equations. This makes the applet a potential tool for students to engage in conjecture making (Baccaglini-Frank & Marrioti, 2011). Through these features, the two applets aim to add to the large repository of applets that are available online and demonstrate how educators can enhance existing applets with additional instructional design features. For example, while the GeoGebra repository contains many applets on radicals, few if any offer

prompts or hints. Similarly, it includes many applets on parabolas, but there appears to be no applet that simultaneously displays the vertex and standard form of a parabola together with its discriminant.

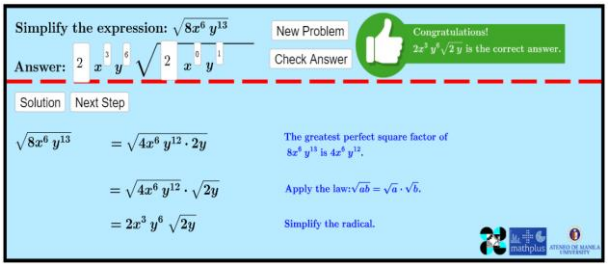
The interested reader can access the applets discussed in this paper and the other mathematical apps of the aforementioned projects from the website https://mathplusresources.wordpress.com/.

2. DESCRIPTION OF THE APPLETS

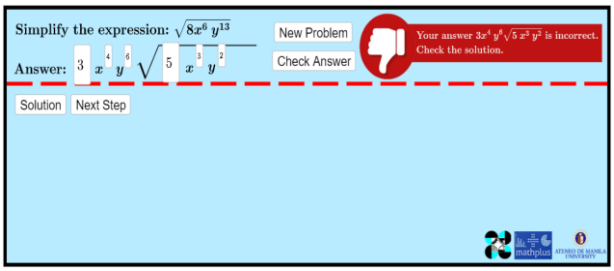
Two of the mathematical apps that we have designed are applets created with the algebra/geometry software *GeoGebra*. Existing studies around the world point towards the affordances in using *GeoGebra* in mathematics education (Antonini & Lisarelli, 2021; Azizah et al., 2021; Tong et al, 2021; Mudaly et al., 2019; Wassie & Zergaw, 2019; Shadaan et al., 2013). In our case, we found that *GeoGebra* applets are appropriate for the Philippine setting as they can provide wider accessibility for teachers and students since these can run on mobile technologies (both Android and iOS). Moreover, *GeoGebra* allows for the creation of content that can be exported as interactive and dynamic worksheets published as applets. This made it easy for us to create instructional materials on topics that were needed immediately based on the requests of teachers in our partner schools during the time of COVID-19. The applets are straightforward to use via *GeoGebra*'s tools that are intuitive and easy to manipulate. Advantageous to teachers was *GeoGebra*'s "construction protocol bar," through which they gathered insights on the processes used to create the applets. This allowed them to create their own applets for other topics and was helpful given their limited time for teaching preparations.

2.1 The *Radicals* Applet

The *Radicals* applet is designed to address one of the learning competencies for Grade 9 mathematics under the content strand algebra, which is "simplifying radical expressions using the law of radicals" (DepEd, 2016). The objective of the app is to strengthen the student's conceptual understanding of simplifying radicals, particularly breaking down the radicand into perfect and non-perfect *n*th powers and applying the property $\sqrt[n]{ab} = \sqrt[n]{a}\,\sqrt[n]{b}$. The students are guided to solve each question provided by the applet (Figure 1(a)). Students can verify their answer using the applet and get appropriate feedback (Figure 1(a)-(b)). Afterwards, the students proceed to the next question. Note that the applet randomizes the questions, so the questions vary per student.



(a)



(b)

Figure 1. Screenshots from the *Radical* app showing: (a) step-by-step solution and feedback on a correct answer; and (b) feedback on a wrong answer

2.2 The *Parabola* Applet

The *Parabola* applet addresses the following Grade 9 learning competencies: graphs a quadratic function of the form $y = a(x - h)^2 + k$ (domain, range, intercepts, axis of symmetry, vertex, direction of opening of the parabola). The applet allows the student to analyze the effects of changing the values of *a*, *h*, and *k* in the quadratic function $y = a(x - h)^2 + k$ on its graph. Sliders are provided so students can experiment on the values of *a*, *h*, *k* (See Fig. 2). It also gives the value of the discriminant, so students can explore its connection to other properties of its graph, such as its *x*-intercepts.

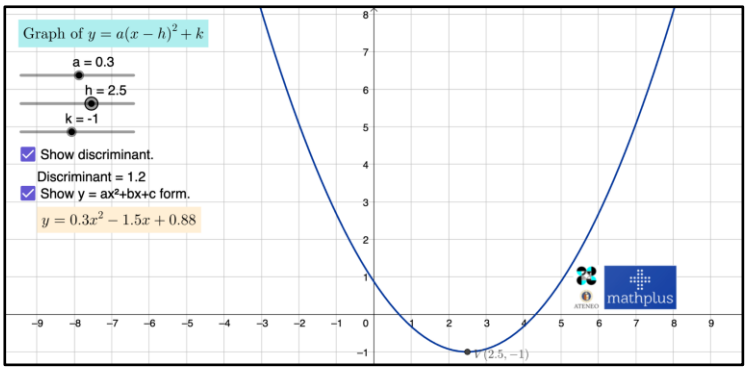


Figure 2. A screenshot of the *Parabola* applet showing the sliders on the values *a*, *h*, and *k*

3. PEDAGOGICAL BASIS OF THE APPLETS

The *Radicals* applet is an example of the use of technology to support "practicing," where repeated use of technology deepens and develops skills that foster understanding of mathematical concepts and strengthens content knowledge (Thurm & Barzel, 2021; Hillmayr et al., 2020). The repetitive feature of the applet allows for deliberate practice, which can modify and enhance previously learned skills and build advanced skills in addition to pre-existing ones (Ericsson, 2016; Garciano et al., 2023). The design of incorporating a feedback mechanism in the applet is crucial so that students engage in a process of practice when working independently without the aid of the teacher (Bokhove, 2011).

The *Parabola* applet is anchored on the framework that technology helps in the teaching and learning of geometry through dynamic interactive graphical representations (Laborde et al., 2006). For example, exploring the graphs of $y = a(x - h)^2 + k$ and using sliders for *a*, *h*, and *k* gives students the opportunity to test, conjecture, and generalize effects of the changing values for *a*, *h*, and *k*. The sliders serve as a dragging mechanism. The dragging feature in a dynamic geometry environment (DGE) illustrates the transformation technology can bring in the representations of mathematical objects, the family of graphs in this case, representing invariant relationships with each of *a*, *h*, *k* (Laborde & Laborde, 2014). The graphs are no longer static objects and are now dynamic objects.

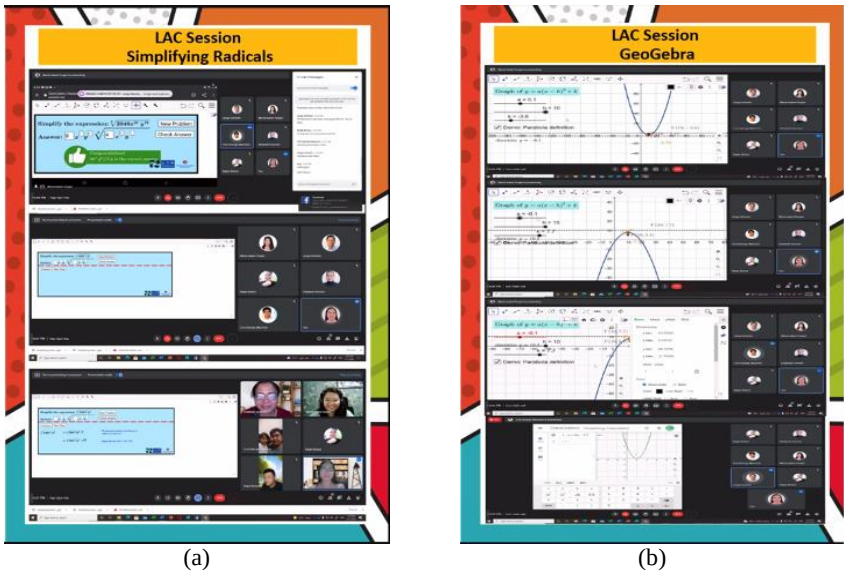
4. INTEGRATION AND USE OF THE APPLETS

In this section we discuss how the *Radical* and *Parabola* applets were used in the teaching and learning of Grade 9 mathematics in a high school in Quezon City, Philippines in school years 2021-2022 and 2022-2023.

For 2021-2022, DepEd implemented a blended-modality form of instruction, so there were synchronous and asynchronous classes. Modules and instructional materials, prescribed by DepEd for the students, were distributed by the school administrators to the parents. During this time, teachers met their students at least once a week through Google Classroom. Written works were required from the students on a regular basis, which were to be done asynchronously. In the synchronous classes, the applets were used to introduce the lesson (Figure 3(a)-(b)). Follow-through sessions were done asynchronously by asking the students to use the applets for exploration and to answer exercises (Figure 4(a)-(b)).

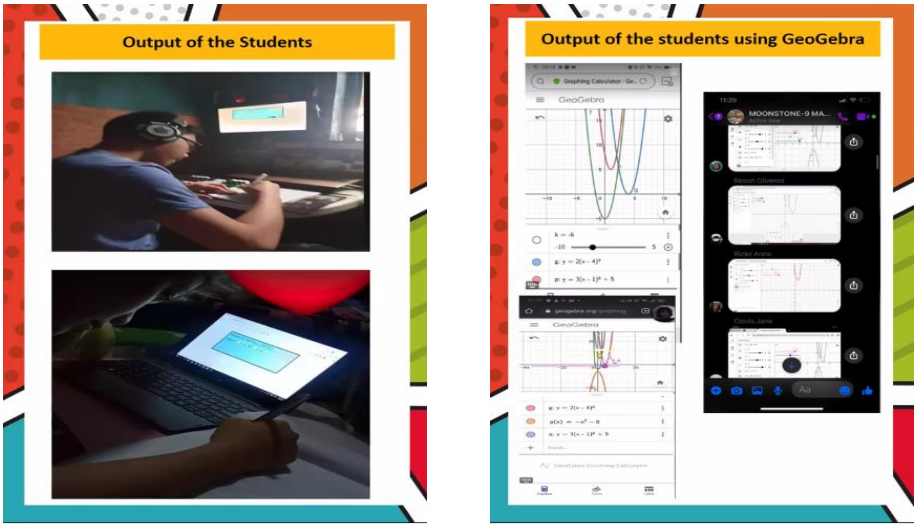
For 2022-2023, DepEd resumed face-to-face classes in schools. The applets were used in the classroom for the lessons, and for assigned homework.

We were able to gather feedback from teachers and students based on the integration of the two applets in school years 2021-2022 and 2022-2023. We discuss these feedback below.



(a) (b)

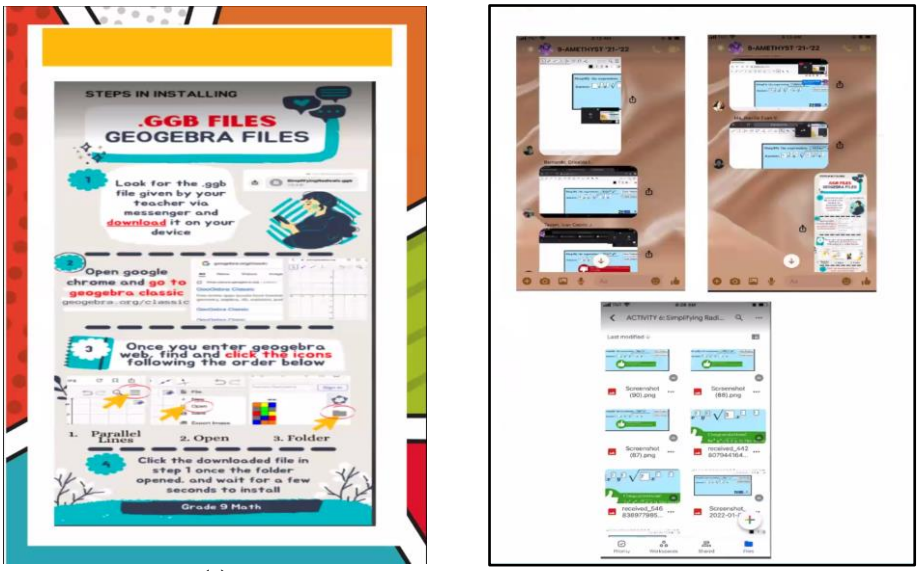
Figure 3. Screenshots of the use of the a) *Radical* applet and b) *Parabola* applet in Google classroom



(a) (b)

Figure 4. Screenshots of the work of the students carried out asynchronously

a) *Radical* applet: We were able to get qualitative feedback from teachers and students on the use of the applet during the blended modality of instruction in 2021-2022. First, specific instructions were given by the teachers to the students on how to use the *GeoGebra* applets, including downloading instructions (Figure 5(a)). The applet was used in the synchronous sessions for teaching the concept. Then students were asked to answer exercises given by the applet asynchronously. For each radical expression, they were first asked to record their solutions in their notebooks. Then afterwards, they were asked to check their solutions versus the detailed solution provided by the applet. They submitted snapshots of their work through Messenger to their teachers (Figure 5(b)).



(a) (b)

Figure 5. (a) Instructions on the *GeoGebra* files; (b) screenshots of sample solutions on simplifying radicals sent through messenger app to the algebra teacher

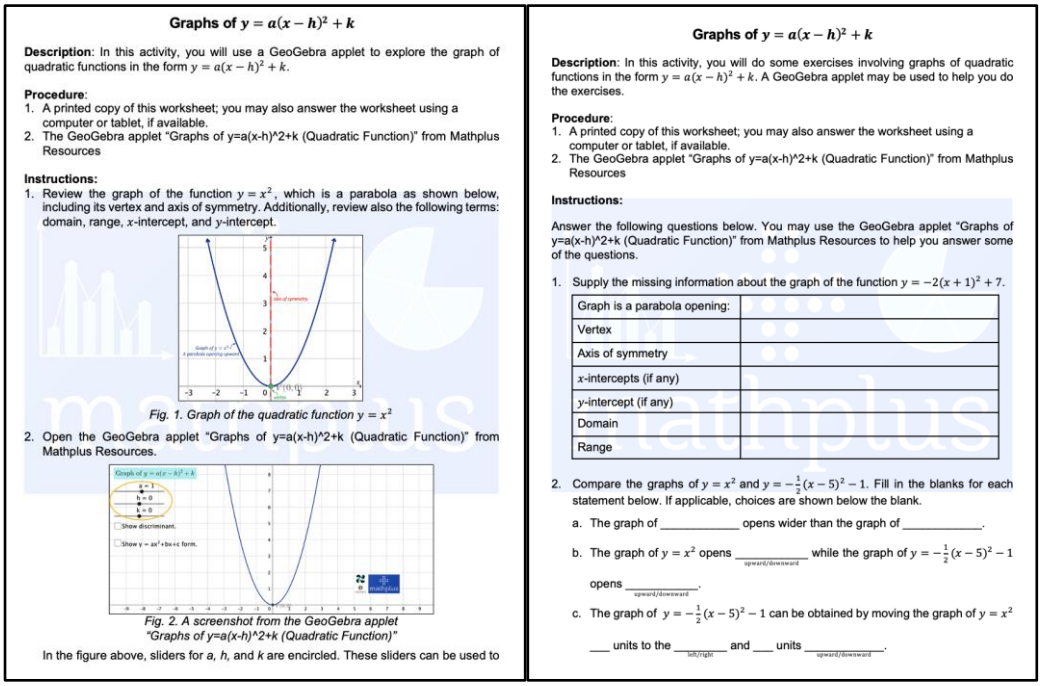
During an online Program Implementation Review (PIR) a few months after the use of the *Radical* applet, the teachers reported the experiences in their classes. Parents also gave comments with regards to the experiences of their children. In general, the applet helped the students understand the simplification of radical expressions through constant practice. It was useful that the applet was able to generate various types of expressions, and the students did not only rely on the exercises given by the teacher or those provided in the modules. This was also useful for the teachers, because of the limited opportunities they had to create as many exercises as possible for the students. The teachers shared the following during the PIR: The students remarked that being able to see the detailed solutions with the reasoning of each step from the applet was helpful. They liked that they were able to identify their mistakes from the solution provided by the applet since they were not always with their teachers. Working with the applet kept them interested in doing the exercises.

b) *Parabola* applet: In 2022-2023, during the first year of implementation of face-to face classes after the pandemic, we were able to obtain an initial quantitative assessment on the use of the applet. While teaching the lesson on quadratic functions, four Grade 9 sections were randomly selected to form two pairs of experimental and control classes in such a way that each pair (experimental and control) of classes are taught by the same teacher; thus, two teachers are involved. Pretests and posttests were administered to the four classes.

The experimental classes used the *Parabola* applet. Each student had a tablet where the applet was installed. The worksheets (Figure. 6(a)-(b)) were answered by the students individually on different days. One worksheet (Figure. 6(a)) dealt with exploring the graph $y = a(x - h)^2 + k$ and using sliders for a, h, and k. The students were first asked to explore the effects of changing the values of a, h, and k on the graphs and write their observations. Then they were asked to make conjectures on properties of the graphs such as the coordinates of the vertex, equation of the axis of symmetry, opening of the graph, and domain and range of $f(x) = a(x - h)^2 + k$. For the second worksheet (Figure. 6(b)), one type of questioning involved giving specific equations (e.g., $y = -2(x + 1)^2 + 7$), and students were asked to provide the vertex, x- and y-intercepts, domain, and range. Another type of question involved giving properties of the graph of the parabola and the student was asked to give a possible equation for the parabola that satisfied the properties.

The control sections employed the traditional approach, where the teachers taught the lesson without the *Parabola* applet and the worksheets.

Based on the data shown in the table below, for one pair of experimental and control classes, the experimental class showed a higher level of improvement from their pretest to posttest scores when compared to the control. On the other hand, the other pair of classes only exhibited little difference in their levels of improvement. Thus, these results indicate that the use of the *Parabola* applet and other related resources (teaching guide, student worksheets) may have the potential to affect students' learning positively.



Graphs of $y = a(x-h)^2 + k$

Description: In this activity, you will use a GeoGebra applet to explore the graph of quadratic functions in the form $y = a(x-h)^2 + k$.

Procedure:
1. A printed copy of this worksheet; you may also answer the worksheet using a computer or tablet, if available.
2. The GeoGebra applet "Graphs of y=a(x-h)^2+k (Quadratic Function)" from Mathplus Resources

Instructions:
1. Review the graph of the function $y = x^2$, which is a parabola as shown below, including its vertex and axis of symmetry. Additionally, review also the following terms: domain, range, x-intercept, and y-intercept.

Fig. 1. Graph of the quadratic function $y = x^2$

2. Open the GeoGebra applet "Graphs of y=a(x-h)^2+k (Quadratic Function)" from Mathplus Resources.

Fig. 2. A screenshot from the GeoGebra applet "Graphs of y=a(x-h)^2+k (Quadratic Function)"

In the figure above, sliders for *a*, *h*, and *k* are encircled. These sliders can be used to

(a)

Graphs of $y = a(x-h)^2 + k$

Description: In this activity, you will do some exercises involving graphs of quadratic functions in the form $y = a(x-h)^2 + k$. A GeoGebra applet may be used to help you do the exercises.

Procedure:
1. A printed copy of this worksheet; you may also answer the worksheet using a computer or tablet, if available.
2. The GeoGebra applet "Graphs of y=a(x-h)^2+k (Quadratic Function)" from Mathplus Resources

Instructions:

Answer the following questions below. You may use the GeoGebra applet "Graphs of y=a(x-h)^2+k (Quadratic Function)" from Mathplus Resources to help you answer some of the questions.

1. Supply the missing information about the graph of the function $y = -2(x+1)^2 + 7$.

Graph is a parabola opening:	
Vertex	
Axis of symmetry	
x-intercepts (if any)	
y-intercept (if any)	
Domain	
Range	

2. Compare the graphs of $y = x^2$ and $y = -\frac{1}{2}(x-5)^2 - 1$. Fill in the blanks for each statement below. If applicable, choices are shown below the blank.
   a. The graph of _____ opens wider than the graph of _____.
   b. The graph of $y = x^2$ opens _____ (upward/downward) while the graph of $y = -\frac{1}{2}(x-5)^2 - 1$ opens _____ (upward/downward).
   c. The graph of $y = -\frac{1}{2}(x-5)^2 - 1$ can be obtained by moving the graph of $y = x^2$ ___ units to the _____ (left/right) and ___ units _____ (upward/downward).

(b)

Figure 6. Worksheets for the experimental class

Table 1. Mean pretest, gain, and posttest scores for the two pairs of experimental and control classes

	Teacher A		Teacher B	
	Experimental A	Control A	Experimental B	Control B
Number of Students	36	28	31	31
Mean Pretest Scores	20.24%	22.70%	22.58%	22.81%
Mean Gain Scores	31.15%	11.73%	13.36%	15.21%
Mean Posttest Scores	51.39%	34.44%	35.94%	38.02%

However, assessing the effectiveness of the applet and complementary resources more accurately will require further studies. As evident in Table 1, all four classes involved in the study had relatively similar mean pretest scores. The mean gain scores then indicate that all four classes actually received low mean posttest scores, with only Experimental A receiving a mean posttest score (i.e., 51.39%) above 50%. The performance of these classes indicates that there has been a *learning gap* in mathematics that might have transpired even before (aside from during) the integration of the resources. Considering this, further studies on assessing the effectiveness of the aforementioned resources will need to carefully consider the possible presence of such learning gaps.

6. CONCLUSION AND OUTLOOK

In this paper, we discussed two *GeoGebra* applets *Radicals* and *Parabola* that were designed and developed to support the teaching and learning of algebra, especially in the context of the COVID-19 pandemic and the new normal in the Philippines. Capitalizing on sound pedagogy and the affordances of mobile technology, the two applets demonstrate how educators can enhance existing apps with additional instructional design features. These two applets were integrated into Grade 9 classes in one of our partner high schools in the Philippines. Based on feedback gathered from teachers and students during this integration, the applets, as well as some complementary resources, can ease teachers' work and have the potential to improve students' learning of algebra topics.

However, various factors such as learning gaps in mathematics or given the possibly extraordinary education situation brought about by the pandemic, further studies are needed to accurately assess the effectiveness of these applets. In the near future, additional experimental studies employing a pretest-posttest design will be conducted. This will involve the inclusion of more classes, thereby expanding the overall sample size for a comprehensive analysis. Given their potential, however, it is also important to continue the development and improvement of such resources.

ACKNOWLEDGEMENT

This paper is one of the outputs of the Ateneo Mathplus Resources Laboratory housed at the Department of Mathematics, School of Science and Engineering, Ateneo de Manila University. The authors thank the Department of Science and Technology-Philippine Council for Industry, Energy, and Emerging Technology Research and Development (DOST-PCIEERD) and the University Research Council (URC), Ateneo de Manila University for the support of the development of the mathematical resources for Grades 1 to 11 Mathematics. Much appreciation is extended to our DepEd partners particularly DepEd School Division Quezon City, Dr Joel Feliciano, Ms. Gina Acosta and Ms. Nora Villalobos and the other teachers and administrators of our partner schools.

REFERENCES

Antonini, S. and Lisarelli, G. (2021). Designing tasks for introducing functions and graphs within dynamic interactive environments. *Mathematics*, Vol. 9, No. 5, 572.

Azizah, A. N. et al. (2021). The effectiveness of GeoGebra to improve visual representation ability. *Journal of Physics: Conference Series*, Vol. 1808, 012059.

Baccaglini-Frank, A. and Mariotti, M. A. (2011). Conjecture-generation through dragging and abduction in dynamic geometry. In Méndez-Vila, A., (Ed.), *Education in a Technological World: Communicating Current and Emerging Research and Technological Efforts* (pp. 100-107).

Bokhove, C.,9 (2011). *Use of ICT for acquiring, practicing and assessing algebraic expertise*, Dissertation, Utrecht University.

De Las Peñas, M. L. A. N. et al. (2022). *Technology Interventions for Mathematical Reasoning, Statistical Thinking and Assessment, DOST-PCIEERD Terminal Report*.

De Las Peñas, M. L. A. N. et al. (2023). *Mathematical Resources for Distance Learning Utilizing Community LTE Networks and Television Frequencies DOST-PCIEERD Terminal Report.*

Department of Education (DepEd), (2016). *K to 12 Curriculum Guide: Mathematics*. Retrieved September 17, 2023, from https://www.deped.gov.ph/wp-content/uploads/2019/01/Math-CG_with-tagged-math-equipment.pdf

Department of Education (DepEd), (2020). *K to 12 Most Essential Learning Competencies with Corresponding CG Codes*. Retrieved September 17, 2023, from https://commons.deped.gov.ph/K-to-12-MELCS-with-CG-Codes.pdf

Department of Education (DepEd), *DepEd Order No. O12, s.(2020): Adoption of the Basic Education Learning Continuity Plan for SY 2020-2021 in light of the COVID 19 Public Health Emergency.* Retrieved September 19, 2023, from https://authdocs.deped.gov.ph/deped-order/do_s2020_012-adoption-of-the-be-lcp-sy-2020-2021/

Department of Education (DepEd) Region I, (2022). *Regional Memorandum No. 664, s. 2022: Basic Education Learning Recovery Plan in light of the COVID 19 Pandemic*. Retrieved September 19, 2023, from https://depedro1.com/wp-content/uploads/2022/06/ rm0664s2022.pdf

Ericsson, K.A. (2016). Summing up hours of any type of practice versus identifying optimal practice activities: commentary on Macnamara, Moreau, & Hambric. *Perspectives on Psychological Science*, Vol. 11, No. 3, pp. 351-354.

Garciano, A.D. et al. (2023). Practice through play using mobile technology. In P. Kommers, I. Arnedillo Sánchez, & P. Isaías (Eds.), *Proceedings of the International Conferences on E-society 2023 and Mobile Learning 2023* (pp. 351-358). International Association for the Development of the Information Society.

Hillmayr, D. et al. (2020). The potential of digital tools to enhance mathematics and science learning in secondary schools: a context-specific meta-analysis. *Computers & Education*, Vol. 153, 103897.

Laborde, C. and Laborde, J.M. (2014). Dynamic and tangible representations in mathematics education, In S. Rezat, M. Hattermann, & A. Peter-Koop (Eds.), *Transformation – A Fundamental Idea of Mathematics Education: A New Approach* (pp. 187-202).

Laborde, C. et al. (2006). Teaching and learning geometry with technology, In A. Gutiérrez & P. Boero (Eds.), *Handbook of Research on the Psychology of Mathematics Education: Past, Present and Future* (pp. 275-304).

Mudaly, V. and Fletcher, T. (2019). The effectiveness of GeoGebra when teaching linear functions using the iPad. *Problems of Education in the 21st Century*, Vol. 77, No. 1, pp. 55-81.

Shadaan, P. and Leong, K. E. (2013). Effectiveness of using GeoGebra on students' understanding in learning circles. *Online Journal of Educational Technology*, Vol. 1, No. 4, pp. 1-11.

Thurm, D. and Barzel, B. (2022). Teaching mathematics with technology: a multidimensional analysis of teacher beliefs. *Educational Studies in Mathematics*, Vol. 109, pp. 41-63.

Tong, D. H. et al. (2021). The effectiveness of using GeoGebra software in mathematics classrooms: a case study of teaching continuous functions in high schools. *Journal of Hunan University*, Vol. 48, No. 9, pp. 256-268.

Wassie, Y. S. and Zergaw, G. A. (2019). Some of the potential affordances, challenges and limitations of using GeoGebra in mathematics education. *EURASIA Journal of Mathematics, Science and Technology Education*, Vol. 15, No. 8, em1734.

Interactive Mobile Applications for Teaching and Learning Mathematics

Ma. Louise Antonette N. De Las Peñas*, Ma. Alva Q. Aberin*, Evangeline P. Bautista*, Flordeliza F. Francisco*, Len Patrick Dominic M. Garces*, Winfer C. Tabares*, Debbie Marie B. Verzosa†

*Department of Mathematics, Ateneo de Manila University (ADMU)

†Mathematics and Statistics Department, University of Southern Mindanao

Abstract

The presentation tackles mobile applications on some math topics developed as part of the DOST-PCIEERD project "Development of Interactive Software and Teaching Guides for Grade 7 to 10 Mathematics" headed by De Las Peñas [4]. The project is also involved in the construction of comprehensive resources (e.g. teaching guides, GeoGebra files) for teachers in order to help them find ways to best incorporate the apps into their respective curricula.

While the applications are primarily for Grade 7 to 10 mathematics, they can also be used to supplement discussions in college algebra and mathematical proof writing. The project features the apps named AlgeTiles [5], DrawLine [6], GeoMatch [1], and ProveIt [2], which are geared towards promoting a more interactive approach in learning and teaching basic algebraic operations, equations of a line, and proving basic geometric results on triangles and quadrilaterals. Other applications on statistics and probability created in the project are also anticipated to play a significant role in the instruction of Math in the Modern World, an upcoming math GE course.

This project recognizes the fact that mobile technology is poised as a ubiquitous way of imparting and honing mathematical knowledge. Related literature studies show the novelty of these kinds of mobile apps in mathematics.

Overview of the Research Project

Technology has become an indispensable tool in the mathematics classroom, not only because it supplements learning, but because competence in the use of technological tools is essential in today's society. The K-to-12 mathematics framework in the Philippines recognizes that appropriate tools are necessary in mathematics instruction–tools which include computers, smart phones and tablet PCs, and the Internet. However, the use of technology in aiding mathematical instruction depends on the availability of such technological resources and, just as important, the teachers' ability and disposition to use these devices.

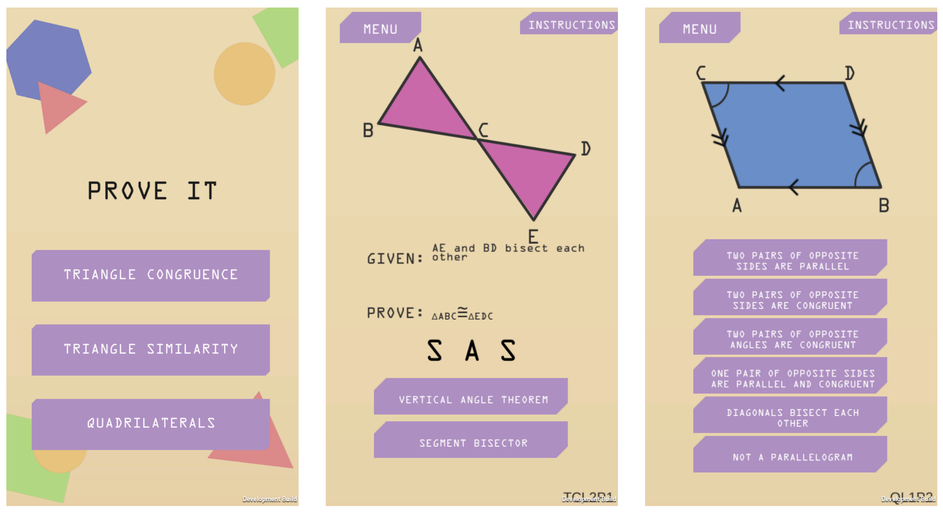
This project hopes to contribute to the current wealth of ICT resources for mathematics education by creating software and fortify teachers' competency in delivering instruction through the provision of teaching guides that are tied to the use and optimization of the developed software. The software developed are not only usable in the computer laboratory setting, but also in classrooms where there are limited technological resources available.

The topics covered by the software and the teaching guides address those that are cited in the Department of Education's Least Learned Competencies for Grade 6 and 4th Year High School based on the National Achievement Test (NAT) for S.Y. 2014-2015. This list is supplemented by the inclusion of other major themes in high school mathematics, such as the discipline of *proving* mathematical results.

Interactive Software Developed

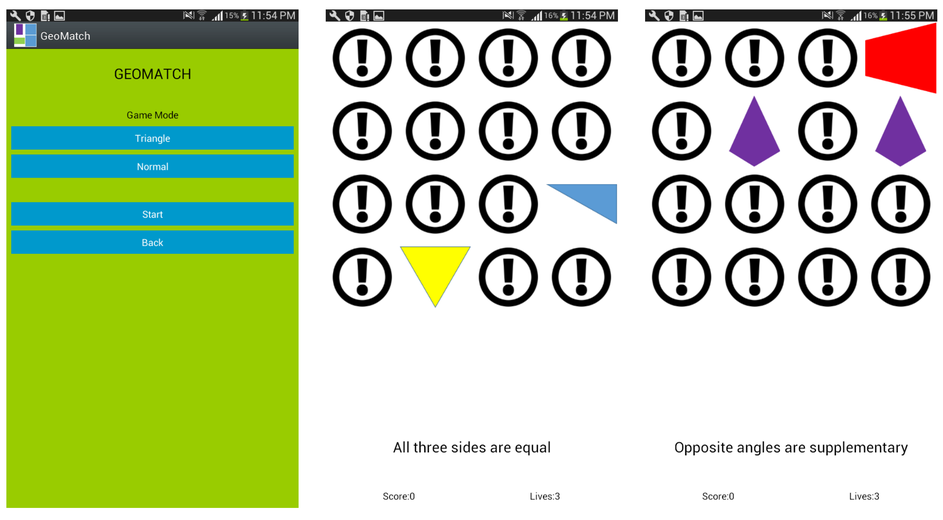
ProveIt

ProveIt is an Android app that contains exercises on proving triangle congruence, triangle similarity, congruence of quadrilaterals, and conditions for quadrilaterals to be classified as parallelograms. The app allows the students to prove geometric results by selecting corresponding parts under the guidance of pre-specified theorems [2]. An iOS version of this app is also currently in development.



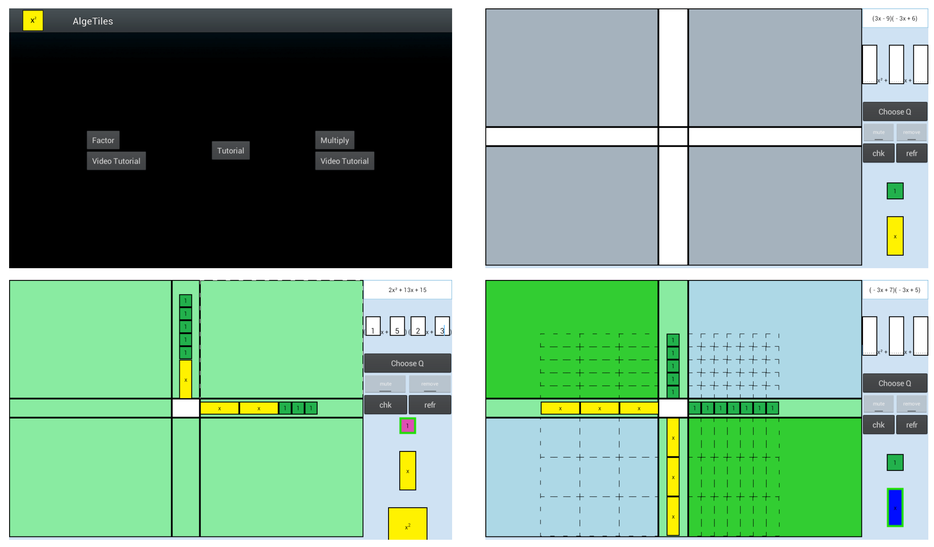
GeoMatch

GeoMatch is an Android-based game that aims to boost the students competencies on classifications of triangles and classifications of quadrilaterals. Adaptability to one's learning capacity is complemented by the gamified design and the customizable game speed [1].



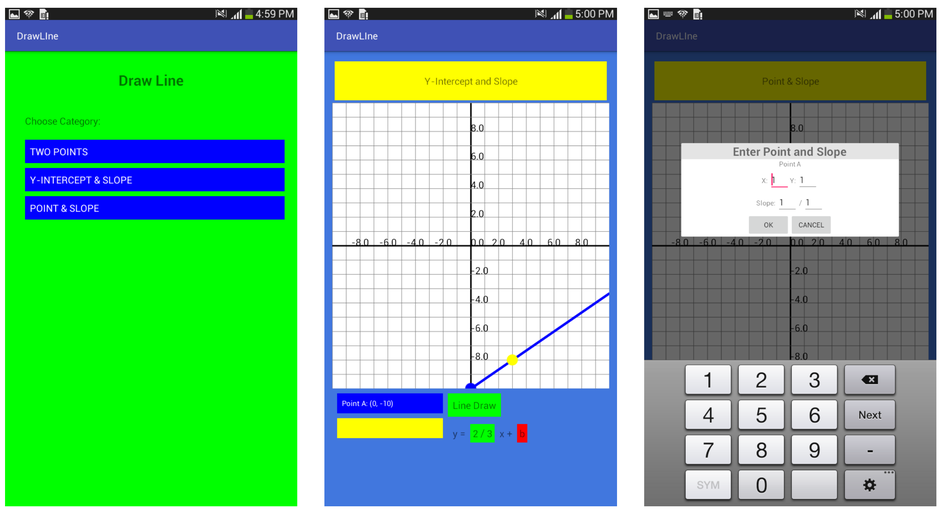
AlgeTiles

AlgeTiles is an Android app that facilitates the learning of factoring quadratic trinomials and multipling binomials in one or two variables through the use of algebra tiles. Students and teachers are also allowed customize the given problem so that the app can be used in conjunction with classroom instruction [5].



DrawLine

DrawLine is an Android app covering the following topics: equations of a given line and graphing a line given its equation. Students are free to choose what equation form they will use and to customize the given information in the problem [6].



Conclusions and Future Outlook

The project is currently in the final stages of developing the mobile and GeoGebra applications, in addition to the finalization of the teaching guides for web uploading. Android applications on topics in counting, probability, and statistics conceptualized by E. P. Bautista and M. A. C. Tolentino are currently in development [3]. All outputs–teaching guides, GeoGebra apps, and Android/iOS software–resulting from the project will be uploaded to an online platform and will be branded as the **MathPlus** online repository.

The finalized apps will then be field tested in a selected private school and a selected public school at the start of S.Y. 2017-2018. Teacher training sessions on how to use the teaching guides and the interactive software will be hosted in August 2017. Finally, the team expects to publish the results of auxiliary studies on the use of the software and teaching guides toward the end of 2017.

While the need addressed dominantly by the project is the dearth of technological resources for mathematics education at the Junior High School level, the said resources can also be used to supplement tertiary education in algebra and analytic geometry at the basic or remedial level. The discipline of proof-writing honed by apps such as ProveIt can also be used to complement instruction in the upcoming GE course in Mathematics.

References

[1] Aberin, M. A. Q. & M. L. A. N. De Las Peñas. *GeoMatch*. Mobile Application.

[2] Aberin, M. A. Q., M. L. A. N. De Las Peñas & D. M. B. Verzosa. (2017). *ProveIt*. Mobile Application.

[3] Bautista, E. P. & M. A. C. Tolentino. (2017). *Counting, Statistics, and Probability*. Mobile Application.

[4] De Las Peñas, M. L. A. N., M. A. Q. Aberin, E. P. Bautista, F. F. Francisco, L. P. D. M. Garces, D. M. B. Verzosa & W. C. Tabares. (2017). *Development of Interactive Software and Teaching Guides for Grade 7 to 10 Mathematics*. DOST-PCIEERD Research Project.

[5] Francisco, F. F. (2017). *AlgeTiles*. Mobile Application.

[6] Francisco, F. F. (2017). *DrawLine*. Mobile Application.

Acknowledgements

The research team would like to express its immense gratitude to the Department of Science and Technology (DOST) - Philippine Council for Industry, Energy, and Emerging Technology Research and Development (PCIEERD). The team would also like to thank the ADMU School of Science and Engineering and ADMU Department of Mathematics for their unending support in all the team's research endeavors. We would also like to acknowledge the team's partners in software development: SparkSoft Solutions, Inc. and the Ateneo Java Wireless Competency Center (AJWCC). This presentation was also made possible through the support of the ADMU Loyola Schools.

Layer Groups of Batak Weaves

Ma. Louise Antonette De Las Peñas[a], Agnes Garciano[a],
Debbie Marie Verzosa[b] and Mark Tomenes[a]

[a]Department of Mathematics, Ateneo de Manila University, Loyola Heights, Quezon City, Metro Manila, 1108, Philippines
[b]Department of Mathematics and Statistics, University of Southern Mindanao, Kabacan, Cotabato, 9407, Philippines

Abstract. This paper discusses the symmetry structures of the decorative weave patterns in baskets and other household items of the Batak, an indigenous community in the Philippines. The study confirms the realization of 15 layer groups that occur as symmetry groups of the Batak weaves. Layers groups are crystallographic space groups that have translational symmetries in two dimensions.

Keywords: Baskets; Batak basketry; symmetry groups; layer groups

I. Introduction

Basketry has a long cultural history, spanning thousands of years (Gilhus, 2020). Baskets from various cultures show unique patterns and designs, making them intriguing subjects for mathematical symmetry investigations.

Within the Philippines, diverse cultures developed distinct basket weaving traditions, including the *apugan* of the Kabihug in Camarines Norte (Rubio, 2016), the *bakat* of the Sugbuanon in Cebu and Bohol (Inocian, *et al.*, 2019), and the *Tingkep* of the Pala'wan in Palawan (Dressler, 2019). There is also the long basket weaving tradition of the Batak. This community is spread out in settlements in the island of Palawan (City Government of Puerto Princesa, 2009). They create baskets which, aside from being functional, are also pieces of art, containing elaborate black and white patterns that demonstrate the weavers' technical expertise and understanding of algebraic and geometric principles (De las Peñas, *et al.*, 2021). There are detailed records of Batak woven patterns (Calderon, 1986; Novellino, 2009) and ethnographic accounts of Batak basket-weaving knowledge (Novellino, 2009).

In this paper, we investigate the presence of layer group symmetries in the weave patterns woven by the Batak, which appear predominantly in baskets. In particular, we determine which of the 80 layer group structures known in Crystallography (Kopský & Litvin, 2002) are present in the Batak weave patterns. This work continues from a previous study (De Las Peñas *et al*., 2021) where a detailed discussion of the weaving techniques of the Batak was presented, including how the decorative repeating patterns in the Batak weaves were created from these techniques.

The Batak patterns analyzed in this paper were culled from existing records (Calderon, 1986; Novellino, 2009), museums, and exhibits (the Batak Visitor Center, the National Commission for Culture

and the Arts, the Cotabato museum). Basket designs were also obtained from social enterprises focusing on Batak baskets, and from a two-day visit to a Batak community in Palawan.

This article contributes to existing works on symmetries of culture, particularly on the applications of crystallographic groups to study symmetries of artwork of indigenous communities. See for example the works by Washburn and Crowe and references therein (Washburn & Crowe, 1988, 2004), and other related studies on symmetries of fabrics (De Las Peñas & Amores, 2016; De Las Peñas *et al.*, 2018; Pebryani, 2018; Chudasri & Sukantamala, 2023). Moreover, symmetry analyses have been performed on basket designs of the Yurok, Karok and Hupa indigenous people of California (Washburn, 1986), the Tlingit people of Alaska and Canada (Mukhopadhyay, 2009), and those found in the United States Southwest (Washburn & Webster, 2006). The work described in the present paper, which investigates the presence of layer group structures in Batak baskets, adds to the aforementioned existing studies on symmetries of baskets. Layer groups have been discussed as symmetries of woven fabrics in the literature (Roth, 1993; De Las Peñas *et al.*, 2023), but not in the context of cultural artwork such as baskets. This paper also intends to contribute to studies in the mathematics of baskets. The ethno-mathematician Gerdes discussed the mathematics that emerges in the basketry of Mozambique and of the Peruvian and Brazilian Amazon (Gerdes, 2004, 2010, 2011).

The organization of this paper is as follows. Section 2 presents the Batak weaving process. Section 3 gives the geometric setting with regards to weaves, and explains how to carry out the analysis of their symmetry structures. Section 4 presents the layer groups found in the Batak weaves. Finally, Section 5 gives the concluding remarks and future direction of the work.

II. Weaving Process

Batak baskets are made from a variety of bamboo that grows abundantly in the mountainous villages of Puerto Princesa, where Batak communities are found. Two sets of thin bamboo strips are used in basket-making to arrive at black and white patterns: a set of bamboo strips in their natural color ("white-like"); and a set of one-sided black bamboo strips from a blackened pole, obtained by holding the bamboo pole over a flame of a lighted resin, until soot covers its outer portion. Details of the preparation of these white and black bamboo strips as well as the weaving process itself which will be described below, may be found in the literature (Calderon 1986; Novellino, 2009).

Weaving is predominantly done by women. A Batak weaver begins by weaving the base of a basket. First, a set of thin, white bamboo strips are laid out vertically on the floor. The weaver then inserts white strips over and under the vertical ones, interlacing them perpendicularly so that these do not fall apart. The plaited white strands will form the base of the basket. To weave the sides of the basket, the weaver uses a finished woven basket to serve as a model or template for the new one (Fig. 1(a)). The model basket is turned upside down and the white plaited strands are then put on top of the base of the model basket and tied to it temporarily. Then four sticks are secured and tied on each corner of the model basket to form the frame of the sides of the new basket (Novelino, 2009).

In order to start weaving the sides of the new basket, the strands hanging from the base are bent vertically towards the sides of the finished basket (Fig.1(b)). Thus, from a two-dimensional planar weave, the basket weaver now transitions into working out a three-dimensional craft. All the strands used in making the base of a basket now become the vertical strands of the sides of the basket. To make a basket with a black and white decorative pattern that appears on the basket's sides, one-sided black strands are inserted as horizontal strands, with the black part of the strands facing front. In weaving, the vertical and horizontal

strands are called *warp* and *weft* strands, respectively. The weft strands are woven continuously around the sides of the basket until a basket of a desired height is achieved. Finally, the four sticks are removed and rattan strands are entwined using a cross stitch pattern around a space, approximately one inch below the rim as well as around the rim of the basket itself in order to secure the basket's opening.

A characteristic of Batak baskets is that the base is partitioned into quadrants; hence the base consists of an even number of vertical and horizontal strands. Two perpendicular lines (quadrantal axes) resulting from the weaving process, are apparent at the base (Fig. 1 (c)). When the base strands are folded up to form the sides of a basket, these quadrantal axes extend and transform into vertical lines through the middle on each of the four sides of the basket. Because of the nature of the Batak weaving technique, warp strands are always colored white while the weft strands are always black. Furthermore, at any step of the weaving exercise, the warp and weft strands are perpendicular, thus either a black portion of a strand covers a white strand or vice versa. The weaver precisely determines the number of strands in each quadrant of the base, based on the specific black and white pattern she intends to create. (De Las Peñas *et al.*, 2020). For example, to arrive at the *Piyakdan* pattern in the basket shown in Fig. 1(d), the weaver uses 16×16 strands per quadrant (Fig.1(c)).

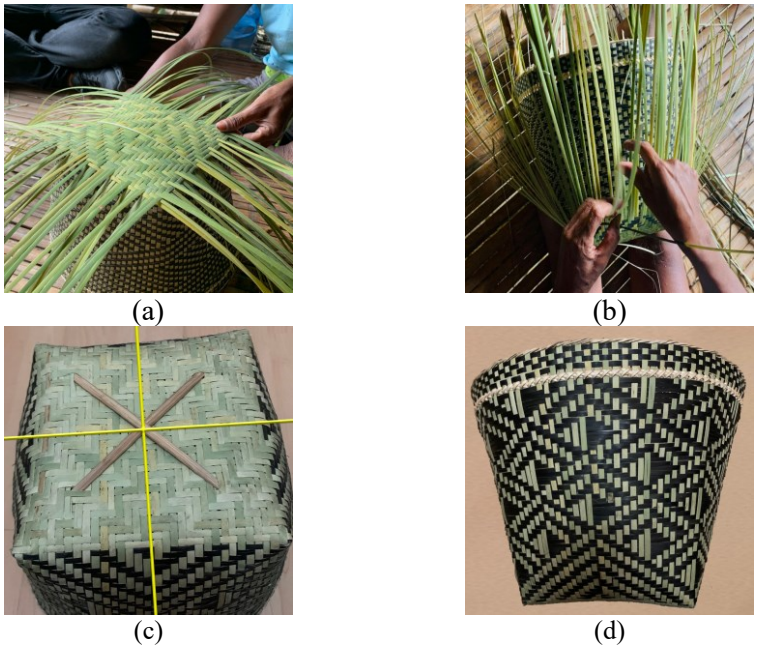


Figure 1: (a) Plaited white strands temporarily tied on the base of the model basket; (b)strands in the base being bent towards the sides of the model basket then black horizontal strands inserted to create the black and white pattern; (c) base of the basket showing quadrantal axes; and (d) basket showing the *Piyakdan* pattern.

Although the Batak are known for their baskets, the Batak also weave non-baskets such as trays, and mats. For the non-baskets, the Batak use one-sided black strips. On one side, all strands of the same direction will have the same color. For example, all warp stands at the front and back of the non-basket will be black, and white, respectively. Then all weft strands at the front and back will be white and black, respectively. This way, the same decorative patterns appear at the front and back (see for example, Figs. 2(a)-(b)).

One of the weaving techniques used by the Batak to arrive at the repeating black and white patterns in non-baskets, can be likened to the concept of *thick stripping* (Roth, 1995) where a fixed number x of

adjacent weft strands alternate colors black and white; and the same goes for the warp strands. Consider for example two variants of the *Binalang* pattern shown in Figs. 2(c)-(d). In these trays, the number x is changed to show different variants of a pattern, x takes on the values 3 (Fig. 2(c)) and 6 (Fig. 2(d)) respectively. The underlying weave (without colors) is a twill – in particular, a 2-over and 2-under weave of warp over weft (and vice versa).

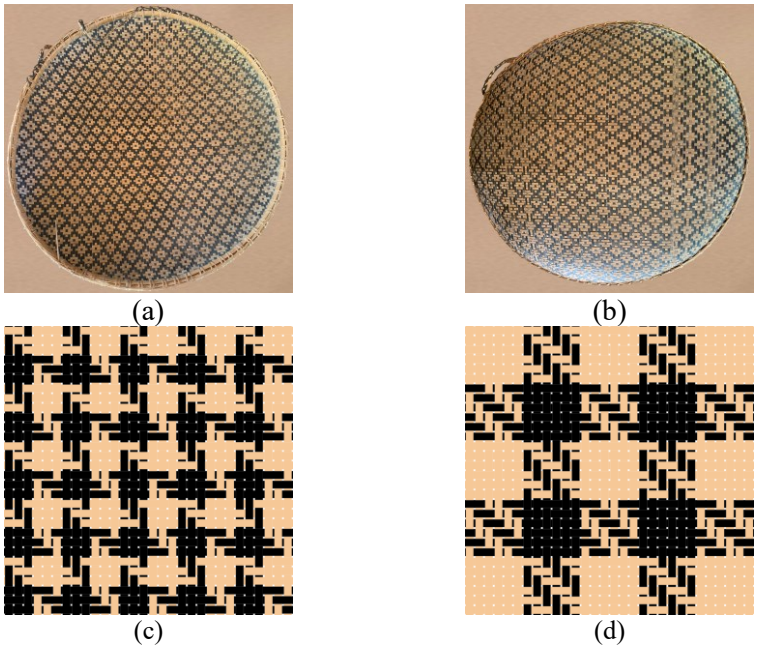


Figure 2: (a)-(b) Tray showing the same decorative pattern in front and back; kind of twill using thick stripping of (c) three white and three black strands; and (d) six white and six black strands alternating in the warp and the weft of two Batak mats.

III. Mathematical Considerations

There are several weaving techniques employed in the creation of baskets such as the 2-way, 2-fold weave; the 3-way, 3-fold weave; and the *kagome* (hexagon-triangle-hexagon) pattern (Tarnai, 2006). The Batak use the 2-way and 2-fold weave in constructing their baskets. In a *2-way weave*, the strands have two directions: warp (horizontal) and weft (vertical); and are at right angles to each other. A *2-fold weave* means there are no more than two strands crossing each other (Hoskins & Thomas, 1991).

One way to understand the geometric properties of a 2-way 2-fold weave W is to represent it by means of its *design*, $D(W)$. Assuming that the weave repeats indefinitely in the plane via two linearly independent translations, and is periodic, we use the regular tiling of the plane by unit squares. Each square is the intersection of a row of squares (a warp strand) and a column of squares (a weft strand). The square is given a particular color (in this paper, grey) if the weft strand passes over the warp strand at the given intersection, and another color (white) if the warp strand passes over the weft strand. Any weave that we assume to be periodic, therefore, can be represented by a 2-coloring of the regular tiling by squares. In this paper, we will assume that the Batak weaves under consideration are periodic.

Illustration 3.1. To illustrate the concept of the design of a weave, let us consider the weave pattern W_1 appearing at the front side of the Batak basket given in Fig. 3(a), and assume it is periodic. Its design $D(W_1)$ as a 2-coloring of the regular tiling by squares is shown in Fig. 3(b). □



(a) (b)

Figure 3: (a) Batak basket showing a particular weave W_1; and (b) the design $D(W_1)$ corresponding to W_1.

To determine the group of symmetries of W mathematically, we will employ the setting of analyzing a 2-way 2-fold fabric given in Roth (1993) as follows. We fix $D(W)$ on a plane P, described by $z = 0$ in three-dimensional space. The weave W is then considered to lie above and below P at a certain uniform distance, the warp strands "lying parallel" to the rows of $D(W)$ and the weft strands also "lying parallel" to the columns of $D(W)$.

A *symmetry* of W is an isometry, a distance preserving transformation of the Euclidean space $\mathbb{E}^3$, that sends the weave W to itself. The *symmetry group* G of W consists of all the symmetries of W. This is a three-dimensional group of isometries leaving the plane P invariant. G is a *layer group*, which is a group of isometries of $\mathbb{E}^3$ that has translations in two directions. From Roth (1993), G is embedded in a group $U = \bar{G} \times \{e, \tau\}$ where e is the identity isometry of $\mathbb{E}^3$, τ is a reflection about P and $\bar{G}$ is the maximal group of symmetries of $D(W)$, that is, $\bar{G} \cong p4mm$, a plane crystallographic group. The symmetry group G that leaves W or $D(W)$ invariant is a subgroup of U. An element of G is either of the form (g, e) or (g, τ) where $g \in \bar{G}$. Since τ is not in G, an element g appears in at most one pair. The set $S = \{g \in \bar{G}: g \text{ in either } (g, e) \text{ or } (g, \tau)\}$ is a subgroup of $\bar{G}$. The set $S_1 = \{g \in \bar{G}: g \text{ in } (g, e)\}$ is a subgroup of index 1 or 2 in S. The layer group G is described by the pair (S, S_1) where $S = S_1 \cup S_2$, $S_2 = \{g \in \bar{G}: g \text{ in } (g, \tau)\}$ (Roth, 1993). If $S = S_1$ the notation used is $(S, -)$. The elements of S_1 correspond to the elements of G that preserve the side of the fabric; those in S_2 correspond to elements of G that reverse the side of the fabric.

Illustration 3.2 Let us assume the weave W_1 presented earlier lies on a plane P. We discuss some symmetries of W_1. W_1 has symmetry (ρ_1, e) where ρ_1 is a reflection with red axis shown in Fig. 4(a). The effect of (ρ_1, e) is that it preserves the side of W_1, that is a side of W_1 is sent to itself. Observe that the highlighted warp strands are sent to warp strands so the symmetry is indeed side preserving. Another symmetry of W_1 is (ρ_2, τ) where ρ_2 is a reflection with blue axis shown in Fig. 4(b) followed by a reflection τ about the plane P. The effect of (ρ_2, τ) is that it sends the front of W_1 to its back and reverses the side of W_1. Observe that the highlighted region in green consisting of warp strands is sent to the highlighted region in yellow consisting of weft strands so this symmetry is indeed side reversing. Other examples of symmetries of W_1: (σ_1, e) (side preserving) where σ_1 is a glide reflection with red axis shown in Fig. 4(c), (σ_2, τ) (side reversing) where σ_2 is a glide reflection with blue axis given in Fig. 4(d) and (μ_2, τ) (side reversing) where μ_2 is a 2-fold rotation with blue center shown in Fig. 4(e). To illustrate other symmetries, we consider another weave W_2. The symmetries present in W_2 are: (μ_1, e) (side preserving) μ_1 a 2-fold rotation with red center in Fig. 4(f), (λ_1, e) (side preserving) where λ_1 is a translation with red vector given in Fig. 4(g) and (λ_2, τ) (side reversing) where λ_2 is a translation with blue vector shown in Fig. 4(h). We highlight regions of the weaves in green and yellow as a guide in Figs. 4(a)-(h). □

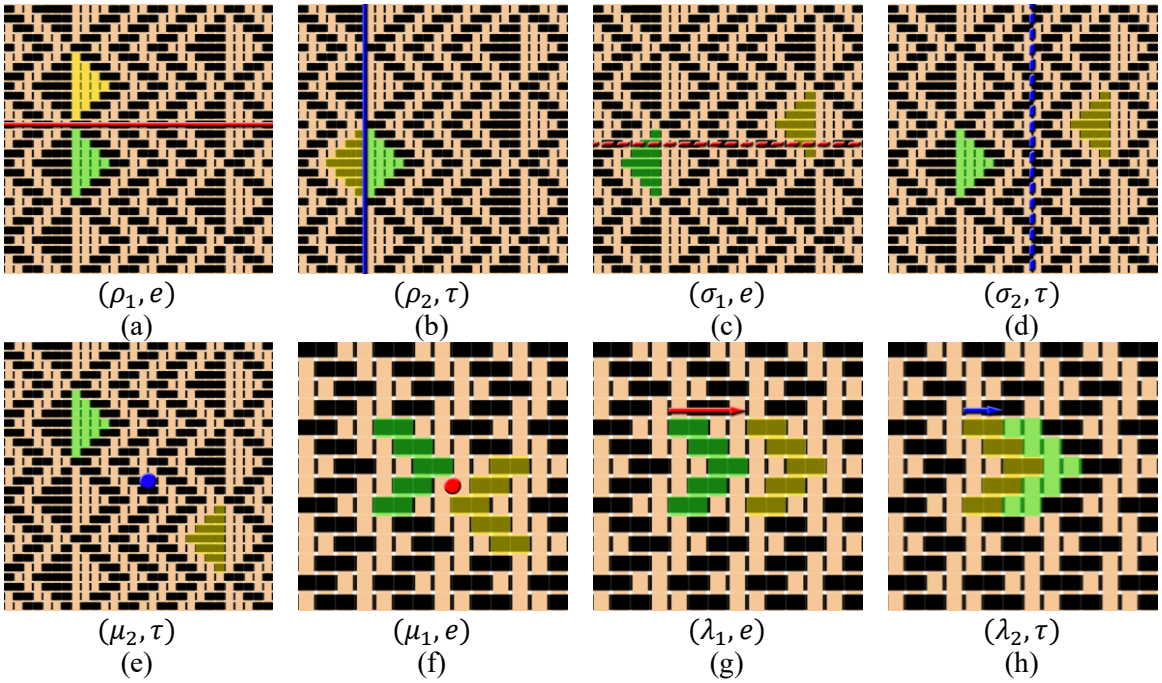


Figure 4: Different kinds of symmetries present in two examples of Batak weaves.

To describe the symmetry group G of a weave W using the pair (S, S_1) where $S = S_1 \cup S_2$ the following result from De Las Peñas *et al*. (2023) is helpful in determining S.

Theorem. Consider a 2-way 2-fold weave W with design $D(W)$. If $D(W)$ has color group G^*, then $G^* = S$, where $S = \{g \in \bar{G}\colon\ g$ in either (g, e) or (g, τ), elements of $G\}$; G is the symmetry group of W, $\bar{G}$ is the symmetry group of the uncolored tiling by squares.

Note that the design $D(W)$ of a weave W is a 2-coloring of the regular tiling by squares so from color symmetry theory we can refer to its color group, which we define below.

Consider the set X of square tiles of a regular tiling $\mathcal{T}$ by squares with symmetry group of the uncolored tiling given by $\bar{G}$. If $C = \{\text{grey}, \text{white}\}$, the set of two colors, a 2-coloring of $\mathcal{T}$ is a surjective function $f\colon \mathcal{T} \to C$ which assigns to each $x \in X$ a color $f(x)$ in C. Let G^* be the subgroup of $\bar{G}$ that effect a permutation of the colors in C called the *color group* of the given coloring of $\mathcal{T}$. Then $g^* \in G^*$ if for every $c_i \in C$ there is an element $c_j \in C$ such that $g^*\left(f^{-1}(c_i)\right) = f^{-1}(c_j)$.

In this paper, the color group $G^* = S$ of the design $D(W)$ will be illustrated geometrically by considering its diagram. The *diagram* of the color group G^* is the union of the set consisting of the centers of rotations of G^* and the set consisting of the axes of reflections and glide reflections of G^*. From the color group $G^* = S$ of $D(W)$, we determine the group S_1 by determining all the elements of S that correspond to the side preserving symmetries of G.

Illustration 3.3. Consider the design of weave W_1 given in Fig. 3(b). Its color group G_1^* consisting of all elements of $\bar{G}_1$ that effect a permutation of the colors is given by the plane crystallographic group $c2mm$. The diagram of $G_1^* \cong c2mm$ superimposed in $D(W_1)$ is illustrated in Fig. 5(a). From the color group G_1^* we determine the index 2 subgroup consisting of elements that correspond to the elements of S_1. In this case we find that $S_1 \cong c1m1$. In the diagram of G_1^*, we color the fixed points (centers) and fixed lines (axes of symmetries) of elements of S_1 or S_2, red or blue respectively. (We will use this coloring scheme

throughout the paper). For instance, S_1 contains the reflections and glide reflections with horizontal axes so these axes are colored red. Those axes corresponding to the reflections and glide reflections with vertical axes, as well as the centers of 2-fold rotations which are elements of S_2, are colored blue (Fig. 5(b)). We only show a portion of the diagram in Fig. 5(c) that corresponds to the unit cell of S_1. The symmetry group of W_1 is described by $G_1 \coloneqq (c2mm, c1m1)$. □

Now given a weave with symmetry group G described by (S, S_1), the next step would be to characterize its layer group structure. This is carried out by determining the three-dimensional isometries corresponding to each of the symmetries given in the diagram of the color group $G^* = S$. Then the appropriate lattice diagram of the layer group is drawn and the type of layer group can be identified from Kopský & Litvin (2002). We use Table 1 which is obtained from De Las Peñas *et al*. (2023) for our analysis based on the symmetries present in the color groups of the designs representing the Batak weaves.

Table 1: Correspondence of elements of S with three dimensional symmetries

Element of S	**Three-dimensional symmetry**
Translation with vector v (element of S_1)	Translation with vector v
Translation with vector v (element of S_2)	Glide reflection with glide component v through the plane P
2-fold rotation with center A (element of S_1)	2-fold rotation with axis normal to P at A
2-fold rotation with center A (element of S_2)	Inversion with center A
Reflection with axis l (element of S_1)	Reflection with plane normal to P at l
Reflection with axis l (element of S_2)	2-fold rotation with axis l
Glide reflection with glide vector v and axis l (element of S_1)	Glide reflection with glide component v through the plane normal to P at l
Glide reflection with glide vector v and axis l (element of S_2)	2-fold screw rotation with screw axis v and axis l

Illustration 3.4. We now describe the symmetry group of the weave W_1 given earlier in Fig. 3(a) as a layer group. Let us consider the design $D(W_1)$ of the weave in Fig. 5(c) with the given diagram of its color group $S = G_1^*$. The 2-dimensional isometries present in the lattice diagram in Fig. 5(c) are analyzed with respect to their 3-dimensional counterparts as stated in Table 1. The 3-dimensional isometries are exhibited in Fig. 5(d) as follows. The reflections with red axes correspond to reflection planes perpendicular to P passing through axes represented by solid black lines. Similarly, the glide reflections with dashed red axes correspond to glide reflection planes perpendicular to P passing through axes represented by dashed black lines. The 2-fold rotations with centers at the blue circles correspond to inversions in P with centers denoted by hollow circles. The reflections with vertical blue axes correspond to 2-fold rotations in P with axes indicated by lines with arrowheads. Finally, the glide reflections correspond to 2-fold screw rotations in P with axes indicated by lines with half arrowheads. With these correspondences, we are able to construct the lattice diagram of the layer group $c2/m11$ (Fig. 5(d)) corresponding to $G_1^* \cong c2mm$ (Fig. 5(c)). □

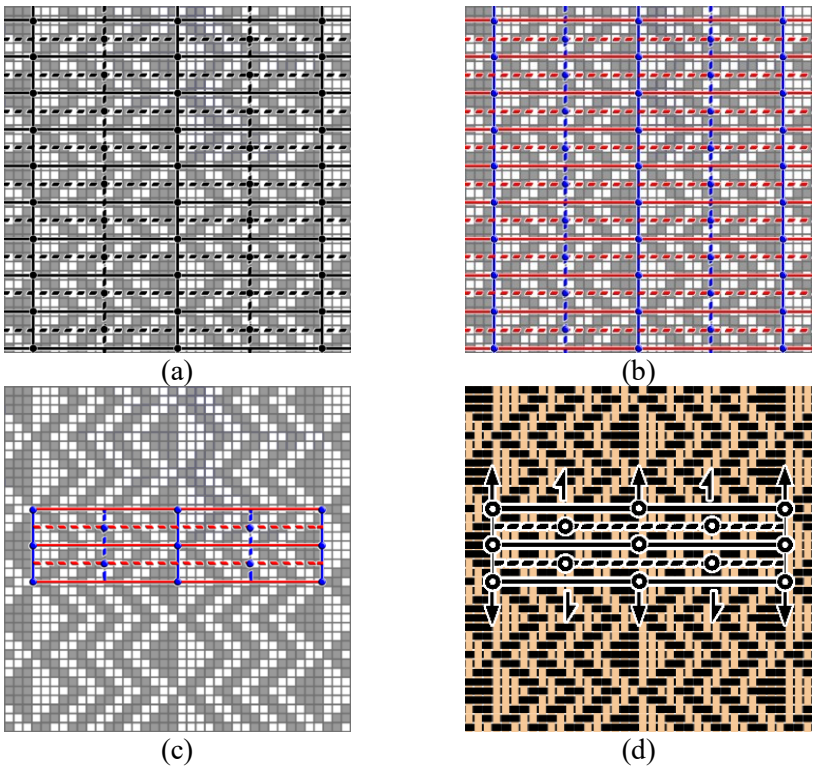


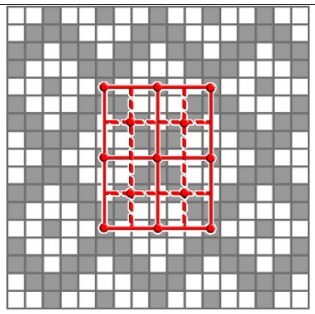
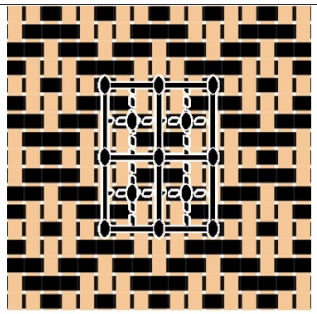
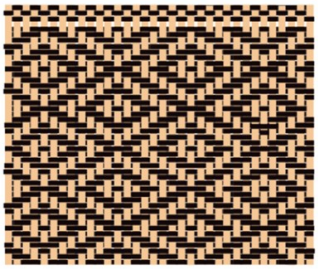
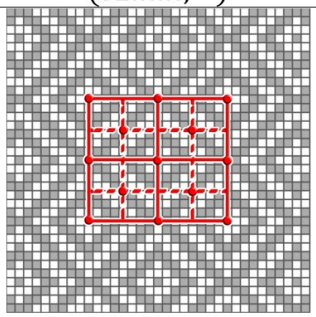
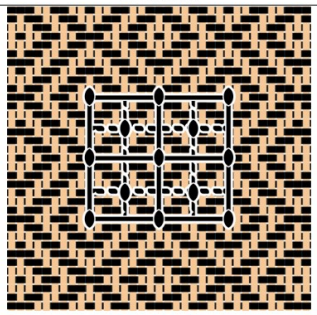
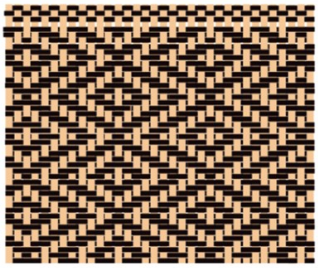
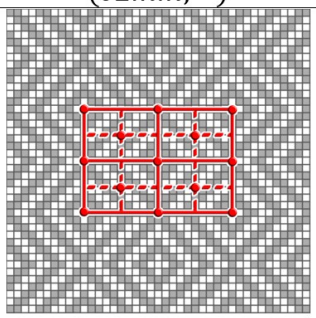
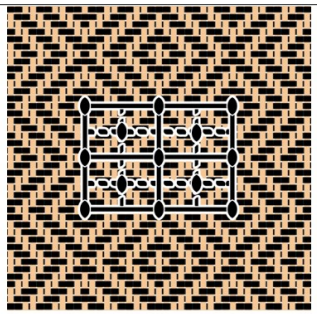
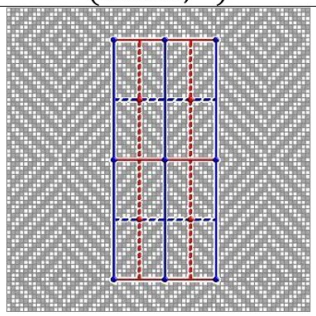
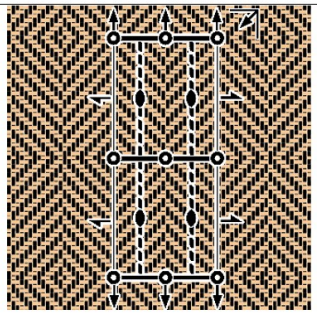
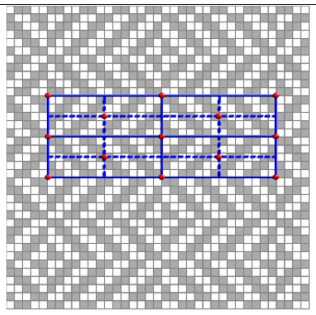
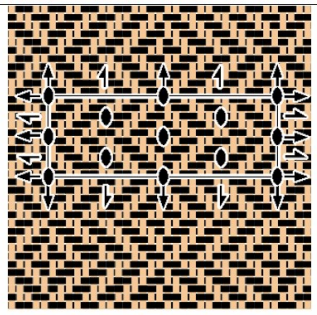
Figure 5: (a)-(b) The fixed points and fixed lines of elements of the color group G_1^*; (c) diagram of $G_1^* \cong c2mm$; and (d) lattice diagram of $c2/m11$.

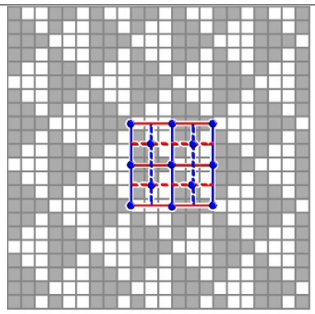
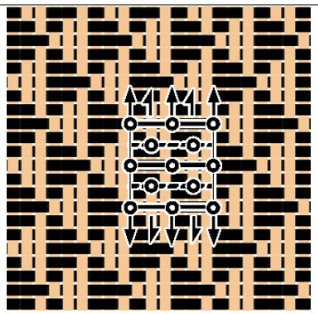
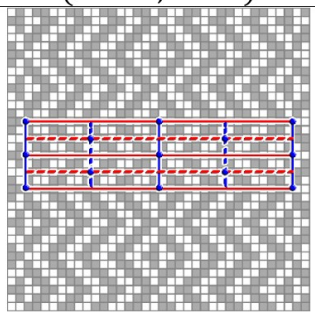
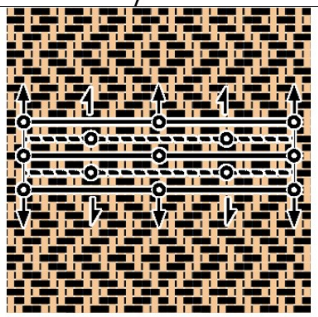
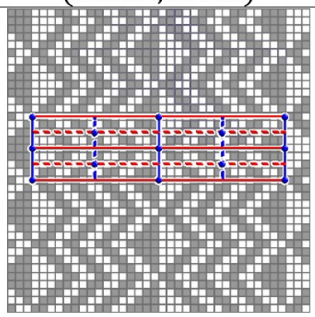
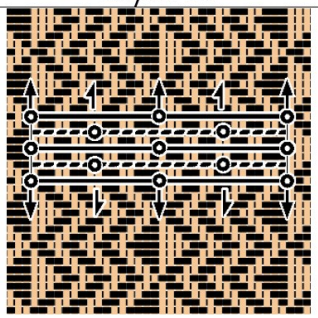
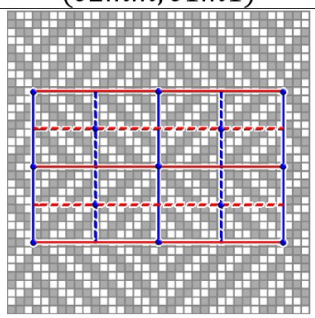
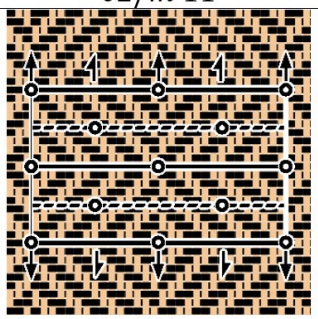
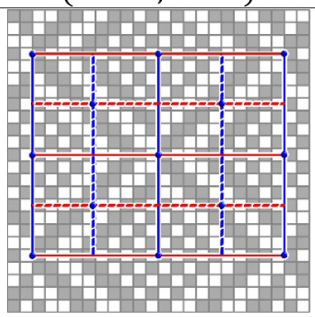
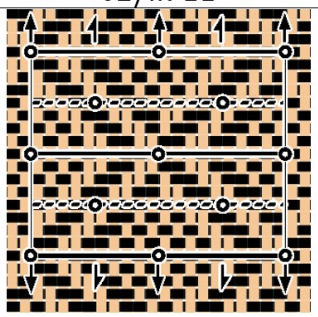
IV. Symmetry Groups of the Batak weaves

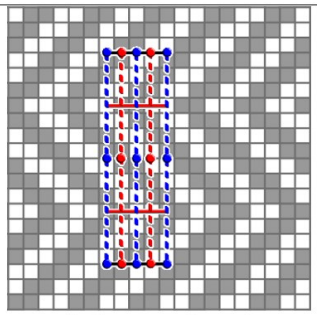
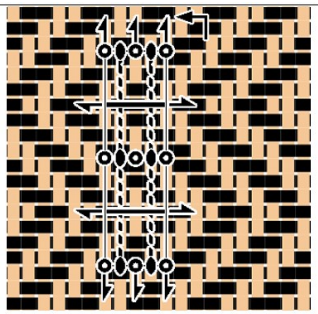
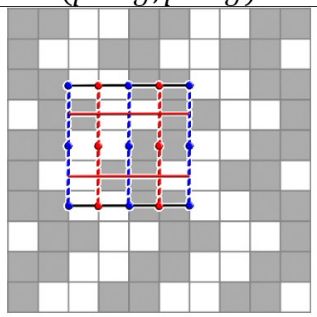
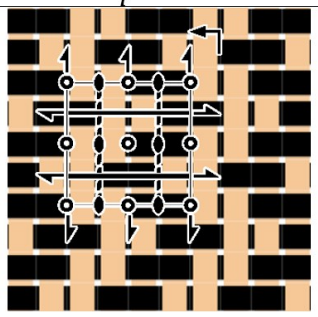
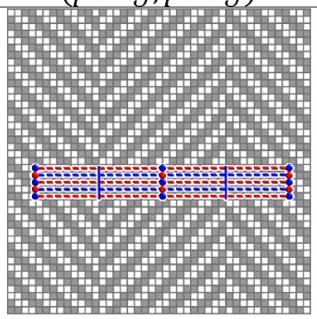
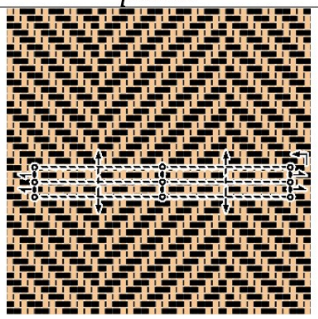
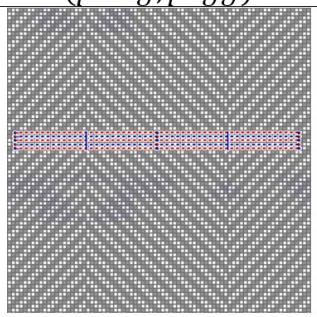
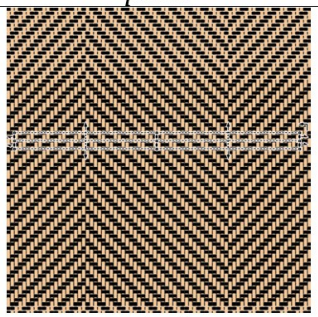
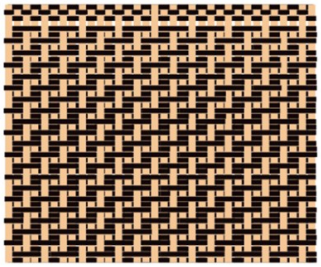
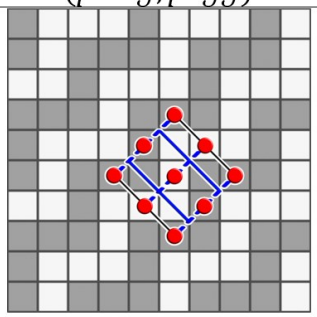
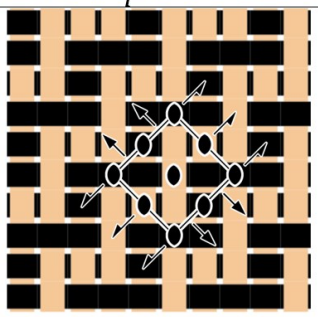
Twenty-two black and white motifs in Batak baskets were catalogued by Novellino where a pattern may include variations (Novellino, 2009). These patterns are based on cultural tradition and have been passed on through several generations (Novellino, 2009). Twenty-nine patterns were recorded by Calderon (1986). In our studies, we found 33 patterns in baskets and 11 in trays and mats with distinct motifs (see also De las Peñas, *et al.*, 2021). To refer to a particular type of pattern or its variant, the Batak weaver usually gives the patterns names. In our research, 22 patterns have been given names by the weavers themselves and Batak advocates.

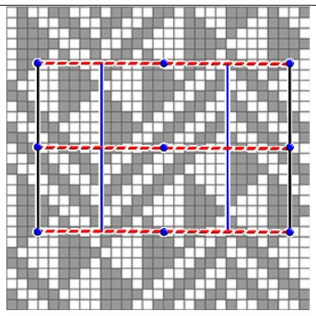
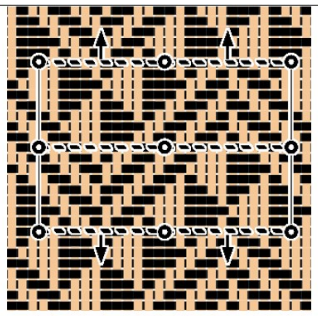
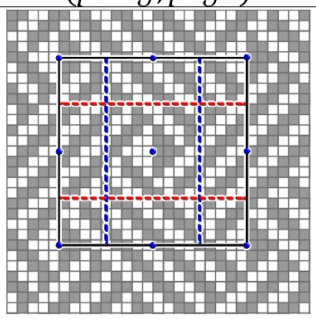
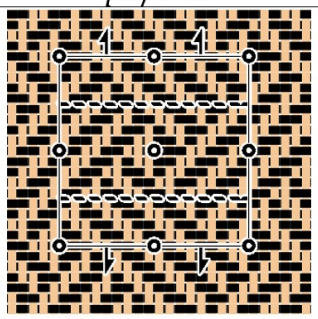
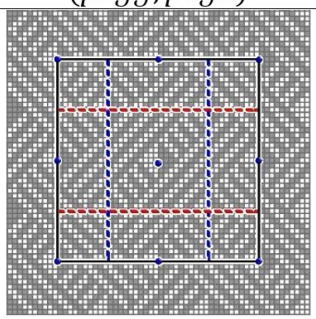
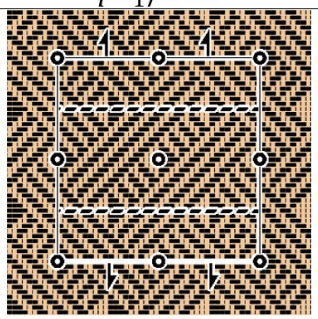
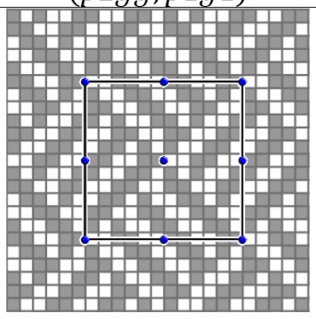
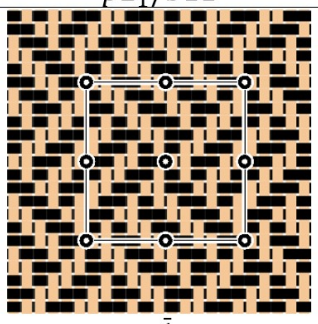
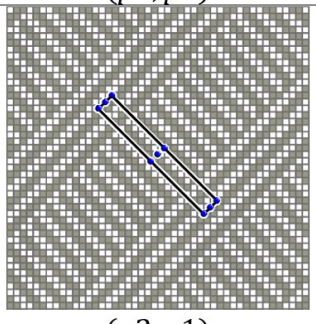
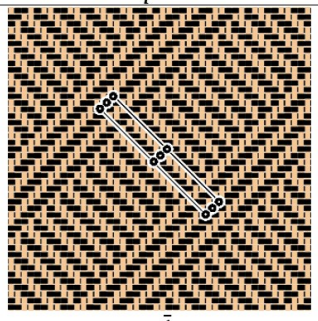
The types of motifs and how these are repeated in the weave contribute to the symmetry structure present in the weave. In De Las Peñas *et al*. (2023) it is known that there are exactly 50 layer groups which are possible symmetry groups structures of 2-way 2-fold weaves. Our results show that 15 of these layer groups occur as symmetry groups of the Batak weaves. We have seen a prevalence of glide reflection symmetries, about 72.72% of all the patterns have symmetry groups with glide reflections.

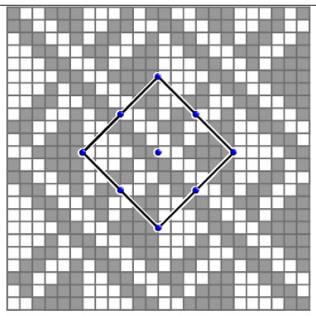
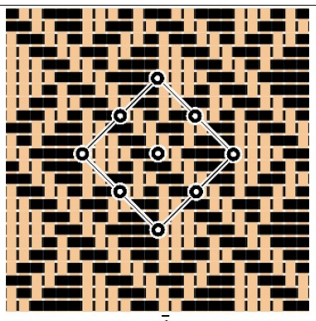
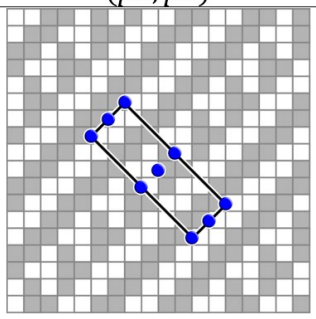
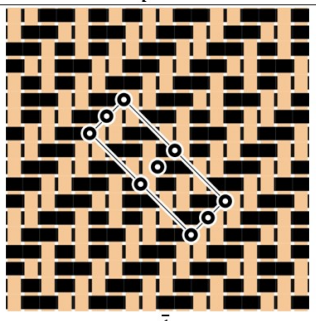
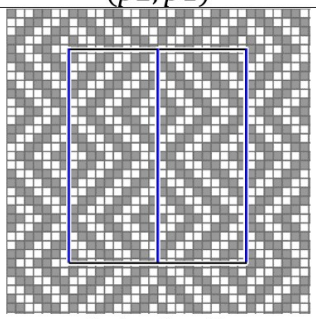
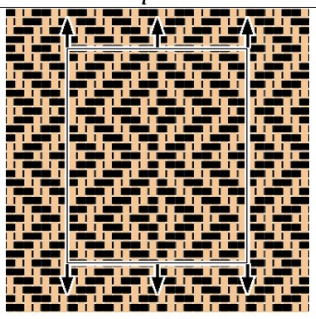
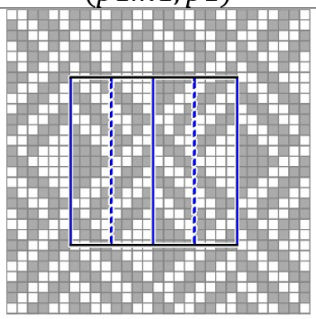
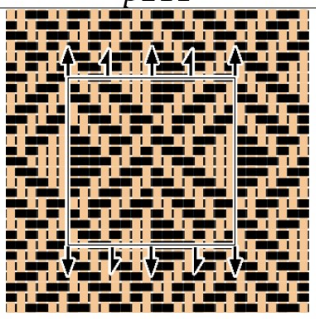
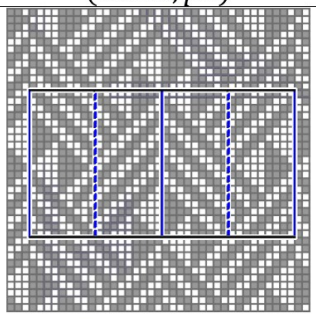
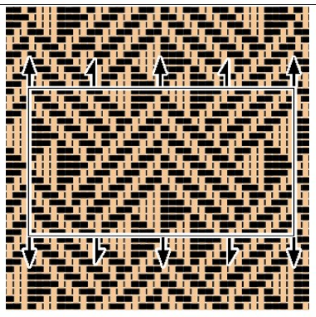
In Fig. 6 we present the results for the basket weaves and in Fig. 7 we present the results for the non-basket weaves. The description of each symmetry group G in terms of the pair (S, S_1) and the layer group structure is given. Most of the geometric elements present in the weaves are triangles and diagonal lines. Diagonal lines are particularly prominent in patterns such as *Piyatwad*, *Libo-libo*, *Tiagudua*, and *Biyanig*. Additionally, some patterns incorporate both triangles and diagonal lines, such as *Enasang*, *Peitak-pietak*, *Piyakdan*, *Tiningkulob*, and *Pianpo*. In most cases triangular motifs come in pairs of black and white strands to form a diamond motif.

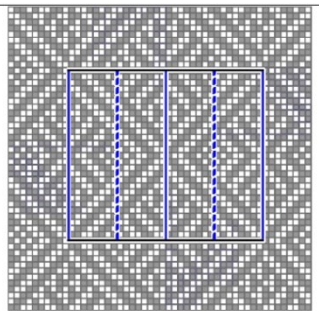
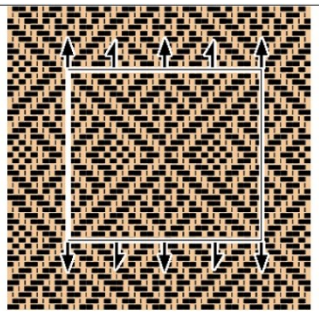
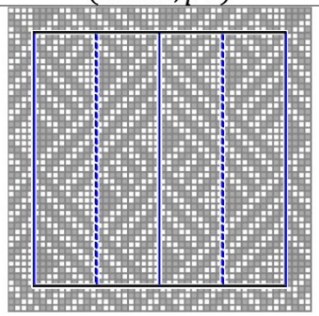
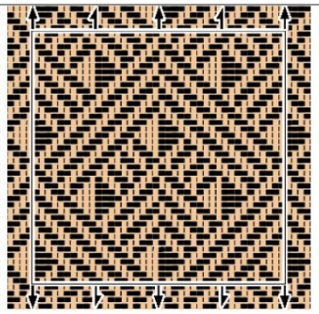
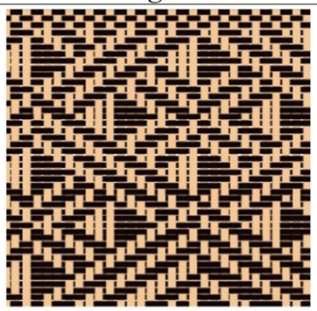
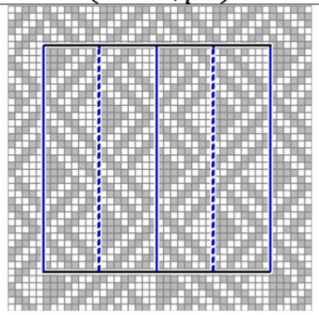
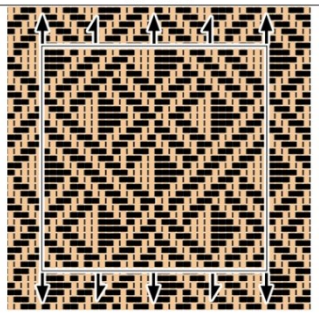
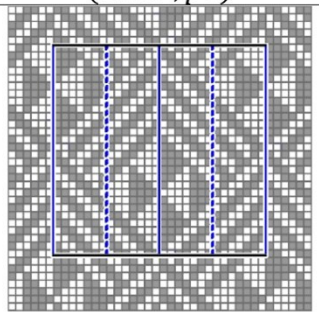
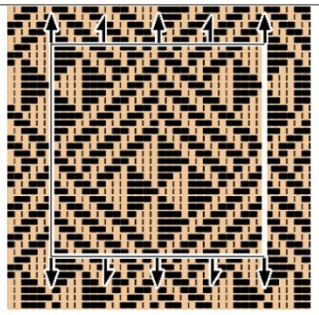
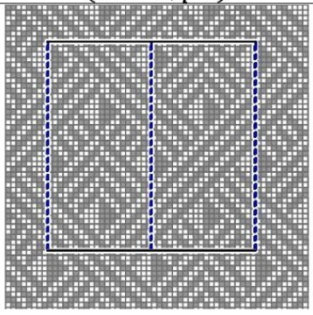
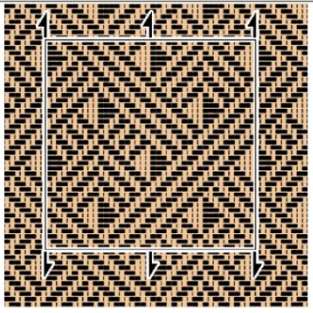
	Batak Weave W	Lattice Diagram of S on $D(W)$	Lattice Diagram of Layer Group on the Idealized Weave of W
1		 $(c2mm, -)$	 $cmm2$
2	 *Kiyubo*	 $(c2mm, -)$	 $cmm2$
3	 *Piyaglipusan*	 $(c2mm, -)$	 $cmm2$
4	 *Piyatwad/ Giyanggangan bialingan/*	 $(c2mm, p2mg)$	 $pman$
5	 *Timograk*	 $(c2mm, p2)$	 $c222$

6		 $(c2mm, c1m1)$	 $c2/m\ 11$
7		 $(c2mm, c1m1)$	 $c2/m\ 11$
8		 $(c2mm, c1m1)$	 $c2/m\ 11$
9	 *Giyanggangan*	 $(c2mm, c1m1)$	 $c2/m\ 11$
10	 *Giyanggangan piyangapaulan*	 $(c2mm, c1m1)$	 $c2/m\ 11$

11	 *Liangob-liangob*	 $(p2mg, p2mg)$	 *pmab*
12	 *Liolo*	 $(p2mg, p2mg)$	 *pmab*
13	 *Libo-libo*	 $(p2mg, p2gg)$	 *pbab*
14	 *Tiagudua*	 $(p2mg, p2gg)$	 *pbab*
15	 *Koyukoy*	 $(p2mg, p211)$	 $p2_122$

16	 *Enasang*	 $(p2mg, p1g1)$	 $p2/b\ 11$
17		 $(p2gg, p1g1)$	 $p2_1/b\ 11$
18	 *Pietak pietak*	 $(p2gg, p1g1)$	 $p2_1/b11$
19	 *Natagainpun*	 $(p2, p1)$	 $p\bar{1}$
20	 *Biyanig*	 $(p2, p1)$	 $p\bar{1}$

21		 $(p2,p1)$	 $p\bar{1}$
22		 $(p2,p1)$	 $p\bar{1}$
23		 $(p1m1,p1)$	 $p211$
24	 *Piyakdan/Kiarumata*	 $(c1m1,p1)$	 $c211$
25		 $(c1m1,p1)$	 $c211$

26	 *Piyaglipusan* variant	 $(c1m1, p1)$	 $c211$
27	 *Tiningkulob*	 $(c1m1, p1)$	 $c211$
28	 *Tiningkulob* variant	 $(c1m1, p1)$	 $c211$
29		 $(c1m1, p1)$	 $c211$
30	 *Pianpo*	 $(p1g1, p1)$	 $p2_111$

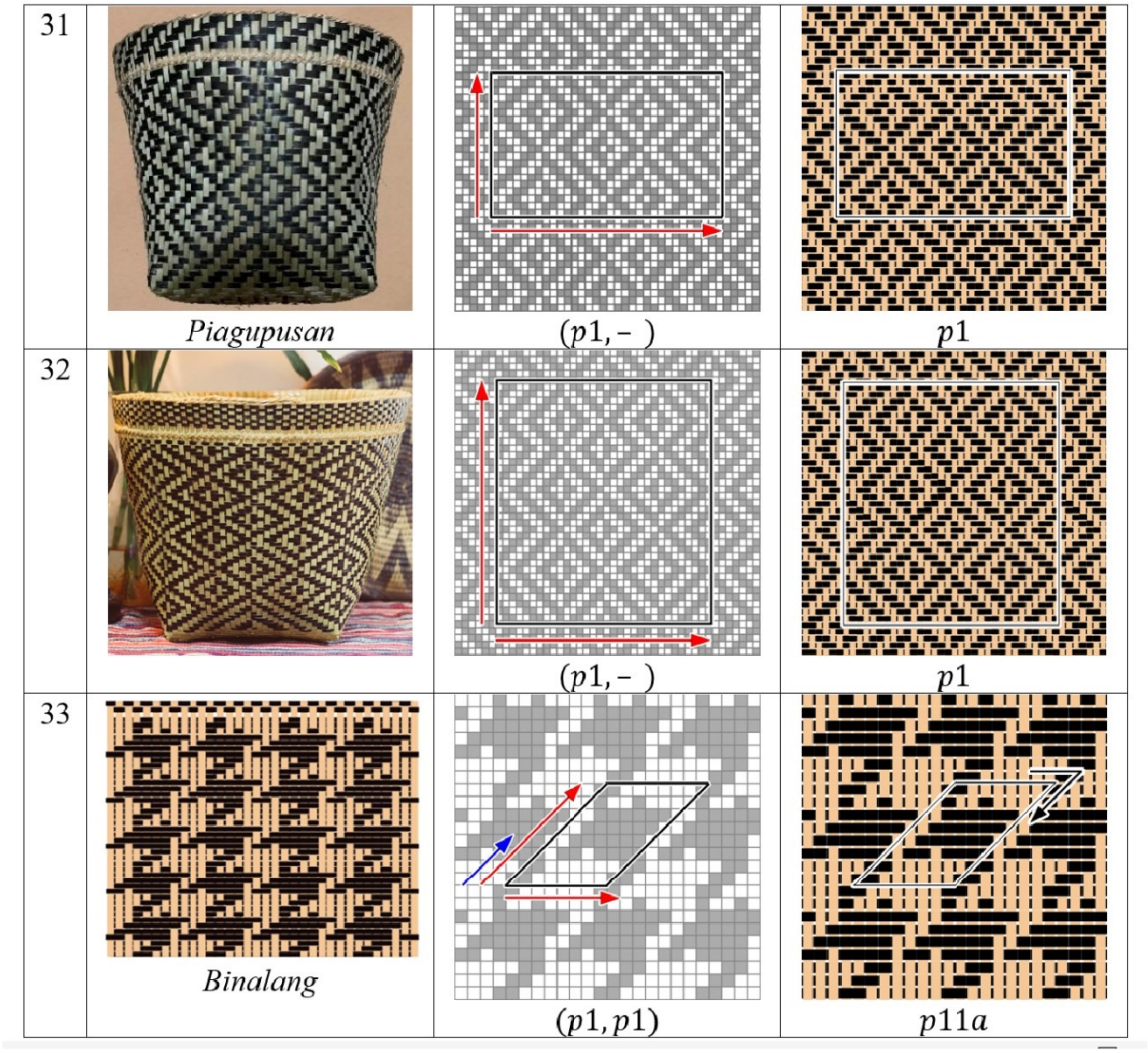
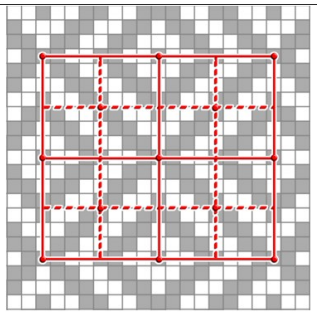
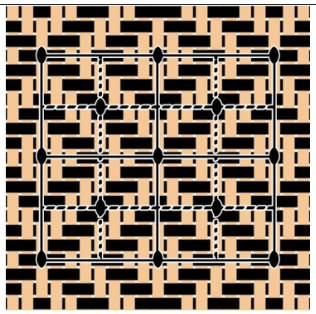
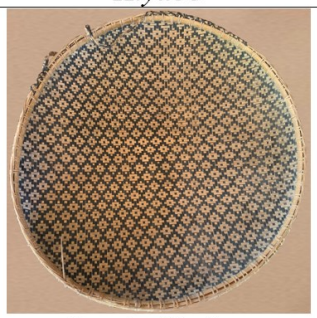
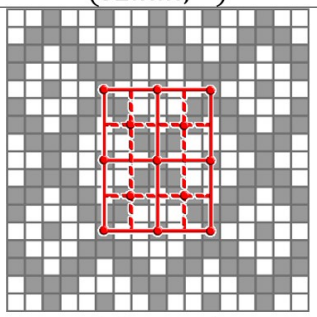
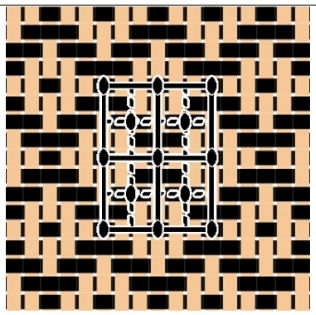
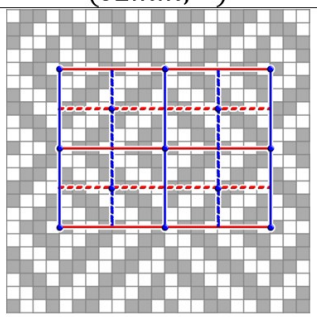
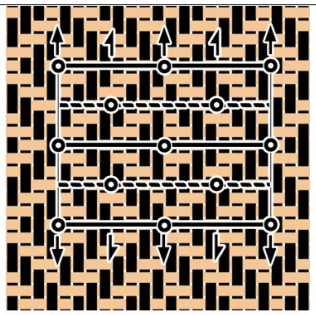
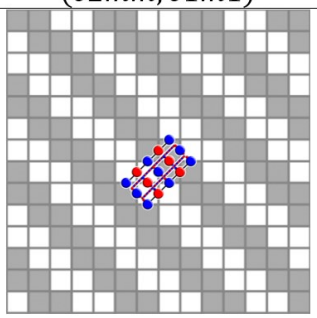
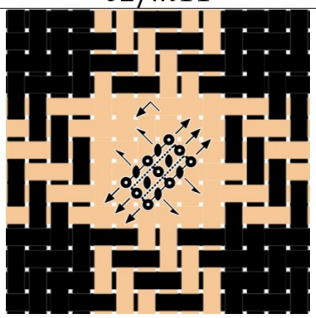
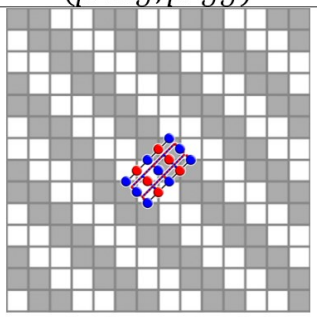
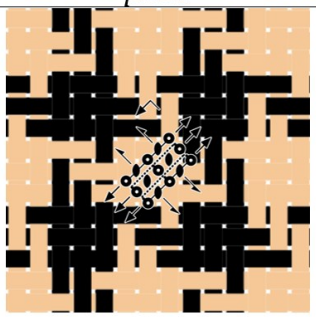
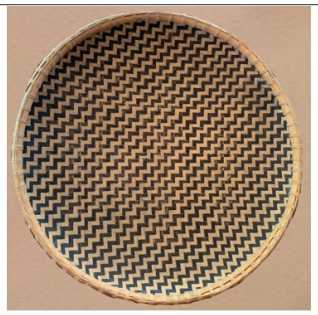
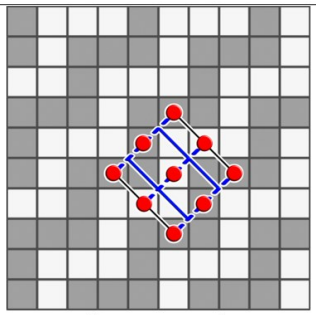
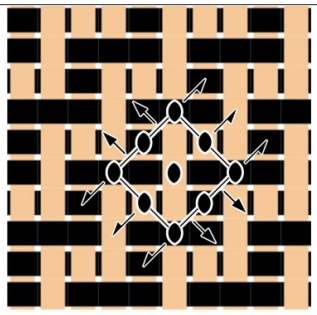
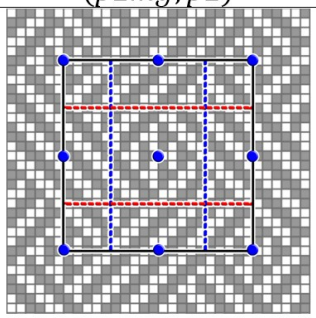
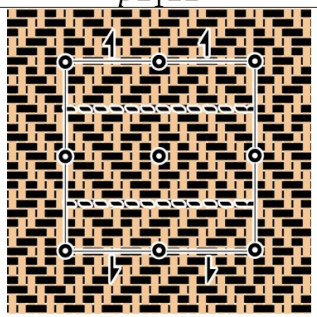
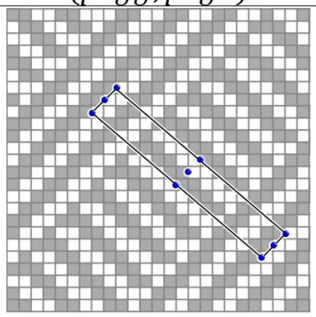
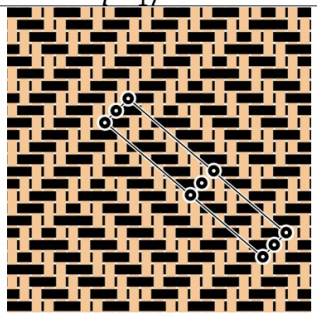
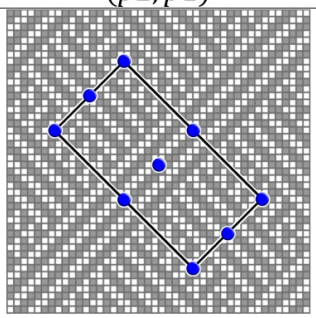
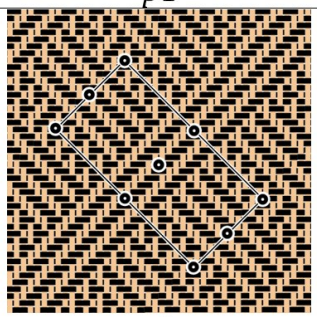
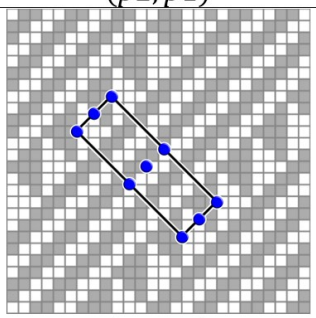
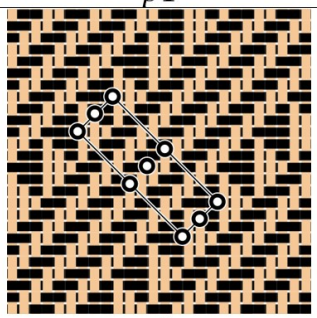


Figure 6: The symmetry groups of Batak weaves present in baskets. The first column shows either a basket exhibiting the weave at the front, or a portion of the weave; the second column gives the lattice diagram of the color group on the design of the weave with the corresponding description of (S, S_1) and the third column gives the lattice diagram of the layer group in the idealized weave with the corresponding layer group structure.

	Batak Weave W	**Lattice Diagram of S on $D(W)$**	**Lattice Diagram of Layer Group on the idealized Weave W**
1	 *Kiyubo*	 $(c2mm, -)$	 $cmm2$
2	 *Kiyubo* Variant	 $(c2mm, -)$	 $cmm2$
3	 *Giyanggangan*	 $(c2mm, c1m1)$	 $c2/m11$
4		 $(p2mg, p2gg)$	 $pbab$
5		 $(p2mg, p2gg)$	 $pbab$

6	 *Koyukoy*	 $(p2mg, p2)$	 $p2_122$
7		 $(p2gg, p1g1)$	 $p2_1/b11$
8	 *Biyanig*	 $(p2, p1)$	 $p\bar{1}$
9		 $(p2, p1)$	 $p\bar{1}$
10		 $(p2, p1)$	 $p\bar{1}$

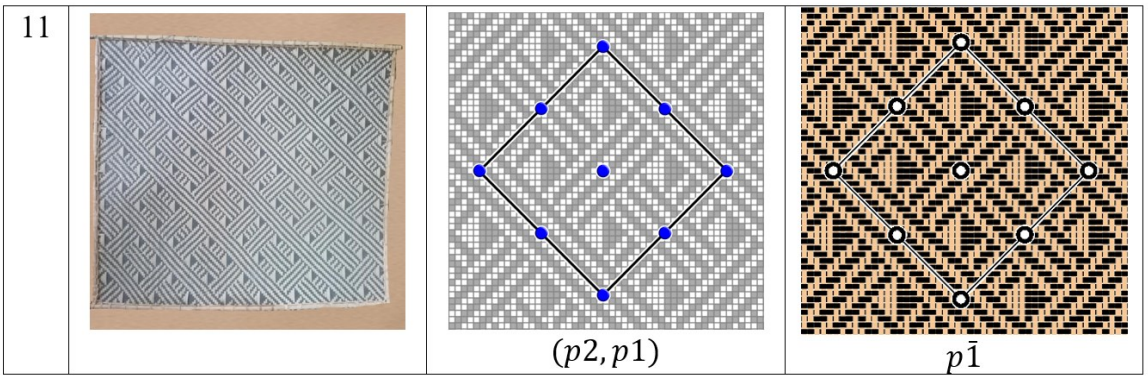


Figure 7: The symmetry groups of Batak weaves present in mats and trays. The first column shows either a mat or a tray exhibiting the weave; the second column gives the lattice diagram of the color group on the design of the weave with the corresponding description of (S, S_1) and the third column gives the lattice diagram of the layer group in the idealized weave with the corresponding layer group structure.

V. Concluding Remarks and Future Direction

This paper discusses the symmetry structures of the repeating patterns present in the baskets and other household items woven by the Batak community in Palawan. In particular, it reports the occurrence of 15 layer group structures that are present in the Batak weaves. We have observed that about 73% of the patterns studied contain glide reflection symmetries.

Several factors contribute to the occurrence of these groups. One is the use of the 2-way 2-fold weaving method and the other is due to Batak weaving techniques. The presence of these groups is also due to the types of repeating patterns present which are influenced by motifs that are culturally significant to the Batak, and have been passed on from generation to generation. For example, from the results in Figs. 6 and 7, it can be verified that the rotational symmetries present are 2-fold rotations. We have not found a 4-fold rotational symmetry in the Batak weaves even if this symmetry can occur in a 2-way 2-fold weave (see an example in Figs. 8(a)-(b)). The Batak motifs and how these are repeated in the patterns using their weaving method, do not allow for the strands to meet at right angles in the same way exhibited in Fig. 8b. The absence of the 4-fold rotational symmetries thus, does not allow for the presence of 11 layer groups as symmetry groups particularly those with four 4-rotational symmetries or 4-fold rotoinversion symmetries.

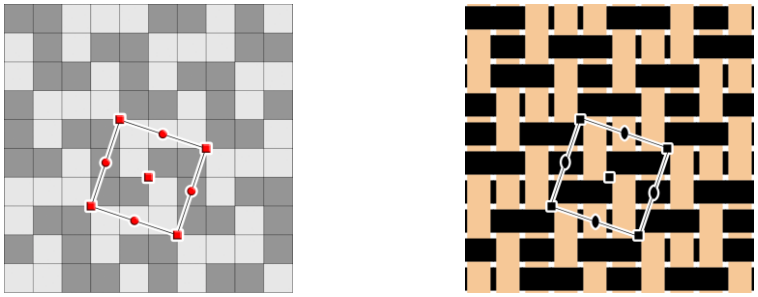


Figure 8: Example of a (a) design; and (b) its corresponding weave with 4-fold rotational symmetry not found in Batak weaves.

It is apparent from the results of the study that the Batak can create symmetric black and white patterns, whether this is repeated in four sides of a basket or whether these appear in the front and side of trays and mats. Very remarkable is not just the counting methods that the Batak employ to arrive at these patterns, but also the calculated technique of inserting one-sided black strips to create the patterns as a form of cultural expression. The patterns gathered in this research could be a starting point to produce a color symmetry framework in weaves using one-sided colored strands and add to existing literature on colored fabrics (e.g. Roth, 1995; Thomas, 2013).

This study shows the realization of crystallographic groups in cultural artifacts. A possible next step is to continue studies of symmetries in culture through the analysis of the weave structures that occur in other types of basket weaving traditions, including those that employ a realization of other weaves, such as the three-way three-fold weaves. It would be interesting to see how the symmetries in patterns on baskets from other cultures compare with those found in the Batak weaves.

Acknowledgements. Thanks go to Ms. Lara Frayre and Mr. Renato Estepa Jr. for facilitating the authors' site visit to the Batak weaving community and for sharing their pictures of the Batak baskets.

References

Calderon, M.J. (1986). *Philippine Quarterly of Culture and Society* **14**(2), 128–136.
Chudasri, D. & Sukantamala, N. (2023). *J. Appl. Cryst.* **56**(1), 81-94.
City Government of Puerto Princesa. (2009). Community-based monitoring systems (CBMS) report, Puerto Princesa City: City Government of Puerto Princesa.
De Las Peñas, M.L.A.N., Garciano, A.D. & Verzosa, D.M. (2021). *Philippine Studies: Historical and Ethnographic Viewpoints* **69**(1), 35-70.
De Las Peñas, M.L.A.N., Garciano, A.D., Verzosa, D.M. & Taganap, E. (2018). *J. Appl. Cryst.* **51**, 456-469.
De Las Peñas, M.L.A.N. & Salvador-Amores, A. (2016). *Philippine Journal of Science* **145**(1), 89-103.
De Las Peñas, M.L.A.N., Tomenes, M.D. & Liza, K. (2023). *Acta Cryst.* A, (accepted for publication, to appear).
Dressler, W.H. (2019). *Political Geography* **73**, 123–137.
Gerdes, P. (2004). *Special E-Book issue of the International Journal Visual Mathematics*. http://www.mi.sanu.ac.rs/vismath/gerdbook/gerdcontents/contents. htm.
Gerdes, P. (2010). *Otthava: Making baskets and doing geometry in the Makhuwa culture in the northeast of Mozambique.* Morrisville: Lulu Enterprises.
Gerdes, P. (2011). In Reza Sarhangi and Carlo H. Séquin (Eds.), *Bridges 2011: Mathematics, music, art, architecture, culture*, pp. 9-16. Phoenix: Tessellations Publishing.
Gilhus, I.S. (2020). In D. Johannsen, A. Kirsch, & J. Kreinath (Eds.), *Narrative cultures and the aesthetics of religion*, pp. 249-269. Leiden, the Netherlands: Brill.
Hoskins, J. & Thomas, R. (1991). *Bull. Aust. Math. Soc.* **44**(1), 33-43.
Inocian, R.B., Cuestas, N.J.P., Carin, J.K.L., & Canoy, J.D.E. (2019). *Journal of Cultural Heritage Management and Sustainable Development* **9**(4), 445-467.
Kopský, V. & Litvin, D. B. (2002). *International Tables for Crystallography Volume E.* Dordrecht: Kluwer Academic Publishers.
Mukhopadhyay, S. (2009). *ZDM Mathematics Education* **41**, 117-130.
Novellino, D. (2009). *Indonesia and the Malay World* **37**(108), 203–224.
Pebryani, N. D. (2018). *LEKESAN: Interdiciplinary Journal of Asia Pasific Arts* **1**(2), 59-73.
Roth, R. (1993). *Geometriae Dedicata* **48**, 191–210.

Roth, R. (1995). *Geometriae Dedicata* **56**, 307-326.
Rubio, J.S. (2016). *Malaysian Journal of Mathematical Sciences* **10**, 211–231.
Tarnai, T. (2006). In Proceedings of IASS-APSC 2006 International symposium: New Olympics new shells and spatial structures, IASS and Beijing University of Technology, China, 2006. Paper no. IL09. Madrid, Spain: International Association for Shell and Spatial Structures.
Thomas, R. S. (2013). *Contrib. Discrete Math* **8**, 38–65.
Washburn, D. K. (1986). *Empirical Studies of the Arts* **4**(1), 19-45.
Washburn, D. K. & Crowe, D. W. (1988). *Symmetries of culture, theory and practice of plane pattern analysis.* Seattle: University of Washington Press.
Washburn, D. K. & Crowe, D. W. (2004). *Symmetry Comes of Age: The Role of Pattern in Culture.* University of Washington Press.
Washburn, D. K. & Webster, L. D. (2006). *Kiva: Recent Perishables Research in the U.S. Southwest* **71**(3), 253–264.

Ateneo de Manila University

Archīum Ateneo

Mathematics Faculty Publications | Mathematics Department

1-1-2023

Mathematical Mobile Apps Via Rural Casting

Ma Louise Antonette N. De Las Peñas
Ateneo de Manila University

John Chris T. Kwong
Ateneo de Manila University

Peter Antonio B. Banzon
Advanced Science and Technology Institute

Philip A. Martinez
Ateneo de Manila University

Wilbur Isaac G. Lapada
Advanced Science and Technology Institute

See next page for additional authors

Follow this and additional works at: https://archium.ateneo.edu/mathematics-faculty-pubs

 Part of the Mathematics Commons, and the Science and Mathematics Education Commons

Custom Citation

De Las Peñas, M.L.A.N., Kwong, J.C.T., Banzon, P.A.B., Martinez, P.A., Lapada, W.I.G., Eballa, J.N., Sebastian, J.E.C., Asido, M.D., San Juan, J.D.M., Versoza, D.M.B., Sarmiento, J.F., Garciano, A.D., Tolentino, M.A.C., Aberin, M.A.Q., & Mallari, J.C.F. (2023). Mathematical Mobile Apps Via Rural Casting. In P. Kommers, I.A. Sánches, & P. Isaías (Eds.), International Conferences on e-Society 2023 and Mobile Learning 2023, 343-350.

This Conference Proceeding is brought to you for free and open access by the Mathematics Department at Archīum Ateneo. It has been accepted for inclusion in Mathematics Faculty Publications by an authorized administrator of Archīum Ateneo. For more information, please contact oadrcw.ls@ateneo.edu.

Authors

Ma Louise Antonette N. De Las Peñas, John Chris T. Kwong, Peter Antonio B. Banzon, Philip A. Martinez, Wilbur Isaac G. Lapada, Julian N. Eballa, John Emil C. Sebastian, Miguel D. Asido, Jan Daryl M. San Juan, Debbie Marie B. Verzosa, Jumela F. Sarmiento, Agnes D. Garciano, Mark Anthony C. Tolentino, Maria Alva Q. Aberin, and Juan Carlo F. Mallari

This conference proceeding is available at Archīum Ateneo: https://archium.ateneo.edu/mathematics-faculty-pubs/239

MATHEMATICAL MOBILE APPS VIA RURAL CASTING

Ma. Louise Antonette N. De Las Peñas[1], John Chris T. Kwong[1,2], Peter Antonio B. Banzon[2], Philip A. Martinez[1,2], Wilbur Isaac G. Lapada[2], Julian N. Eballa III[2], John Emil C. Sebastian[2], Miguel D. Asido[2], Jan Daryl M. San Juan[2], Debbie Marie B. Verzosa[3], Jumela F. Sarmiento[1], Agnes D. Garciano[1], Mark Anthony C. Tolentino[1], Maria Alva Q. Aberin[1] and Juan Carlo F. Mallari[1]

[1]*Department of Mathematics, Ateneo De Manila University, Philippines*
[2]*Department of Science and Technology - Advanced Science and Technology Institute, Philippines*
[3]*Department of Mathematics and Statistics, University of Southern Mindanao, Philippines*

ABSTRACT

This paper discusses the distribution, through a digital datacasting framework, of mathematical resources for Grades 1 to 7 to two schools in a community in the Philippines. Among the mathematical resources made available, are mathematical applications (apps), which run on mobile technologies, that have been created to help in the mathematical learning of students in a remote setting. The distribution is facilitated by the RuralCasting set-top box, a developed custom set-top box capable of receiving digital TV broadcasts and providing local content access through its Wi-Fi network. This paper presents the performance of the set-top box in distributing the mobile mathematical apps, and a short discussion on the mathematical applications deployed.

KEYWORDS

Mobile Technology in Teaching Mathematics, Datacasting, Mathematical Apps

1. INTRODUCTION

In the Philippines, based on the National ICT Household Survey (Department of Information and Communications Technology, 2019) only about 17.7% of households have access to the internet. As such, the Philippine Department of Education's, blended learning approach during the time of the Covid-19 pandemic, focused largely on non-internet sources such as printed modules, radio and television. There was a need to explore alternative avenues for the delivery of information and resources for teaching and learning. The Department of Science and Technology- Philippine Council for Industry, Energy and Emerging Technology Research and Development (DOST-PCIEERD) and the Department of Science and Technology - Advanced Science Technology Institute (DOST-ASTI) led an initiative aiming to develop technologies that maximize the use of the UHF spectrum (e.g., the use of TV whitespaces for LTE mobile broadband services, use of digital TV datacasting framework) for the deployment of information.

This paper discusses the results of a project under this government initiative, where the datacasting capabilities of the ISDB-T Digital TV standard adopted by the Philippines has been maximized by sending different file formats furthering the cause of distance learning and contributing to the availability of educational resources in areas and communities where internet infrastructure is unreliable or non-existent. In particular, a custom *set-top box* was developed that was able to receive the usual TV programming, along with the attached data content, providing multiple users access to the content thru Wi-Fi, and return information back using alternative transmission methodologies.

One of the resources that were distributed through the set top box were mathematical applications (apps) that run on mobile technologies. Even before the Covid-19 pandemic, there has always been the need to provide support for the mathematics education of Filipino learners. The challenges of achieving the twin goals of mathematics education, as explicated by the Department of Education (2016) are well-documented (Bernardo & Limjap, 2012; Verzosa & Vistro-Yu, 2019). Although the twin goals of mathematics education are critical thinking and problem solving, mathematical learning in the Philippines is still characterized by rule-based

methods and rote memorization (Verzosa, 2020). Further, a large number of Filipino students perform poorly in mathematics assessments that demand higher levels of cognitive thinking (Schleicher, 2019). The problem was exacerbated by the Covid-19 pandemic where students were not with their teachers, and remote learning was mostly done through modules, especially in the areas without internet. In this connection, mathematical applications were developed by the math group of the project team. These were interactive, engaging, easy to use with limited supervision, and were designed to address learning competencies prescribed by the Department of Education. One venue of distribution for these mathematical apps is through the set-top box and can be easily downloaded by the parents and students.

The use of datacasting technology for educational delivery to under-resourced communities has already been trialed elsewhere. Recently, in the United States, due in part to the disruption of onsite classes brought about by the COVID-19 pandemic, SCETV, South Carolina state's public educational broadcasting network (South Carolina ETV, 2020), and Pennsylvania PBS, a public television organization (Pennsylvania PBS, 2020), have pioneered a pilot project to explore how the datacasting technology can address the digital divide by providing learning content to students without home internet access. Although datacasting is not meant to take the place of broadband internet, its potential to provide an equitable solution using a well-established technological infrastructure to access online educational materials without home internet connection is boundless.

2. CONCEPTUAL FRAMEWORK

2.1 RuralCasting

The RuralCasting system mainly utilizes Wi-Fi technology to distribute the educational resources that are normally unavailable to users without the internet. As shown in Figure 1, the set-top box can be accessed using any Wi-Fi device or using the set-top box itself as a stand-alone unit. When accessed through the Wi-Fi network, multiple users will be able to download, view, and stream the content made available offline. The content can also be updated periodically using the datacasting transmission.

The contents, namely, the applications, videos, text files, images will be locally stored in the set-top box. A lightweight learning management system like Moodle and Canvas but specifically for the RuralCasting set-top box, named *Edukastv* was created. In *Edukastv*, the content creators or teachers can upload their subject outline, reading modules, quizzes, etc. for the students to access.

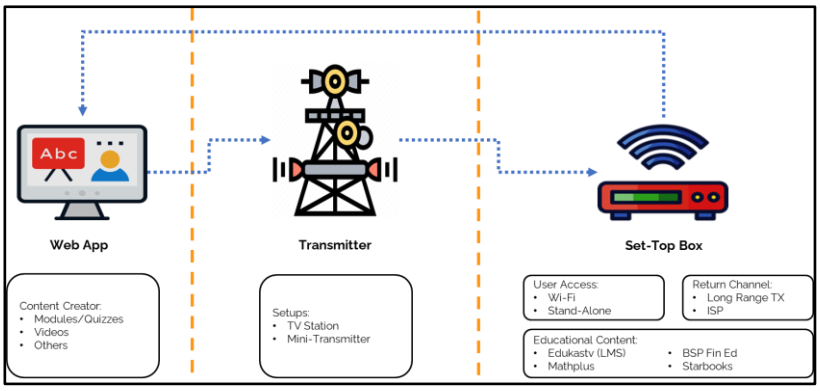


Figure 1. RuralCasting system overview

Users connect to the RuralCasting's Set-Top Box Wi-Fi network and will be able to access the RuralCasting Portal (Figure 2a). Once connected, the user can then navigate through the services hosted by the Set-Top Box, such as Edukastv, Math Applications, and videos on Financial Education produced by the Bangko Sentral ng Pilipinas (BSP). The Set-Top box also hosts an instance of DOST's Science and Technology Academic and Research-Based Openly Operated Kiosk (STARBOOKS) (Figure 2b).

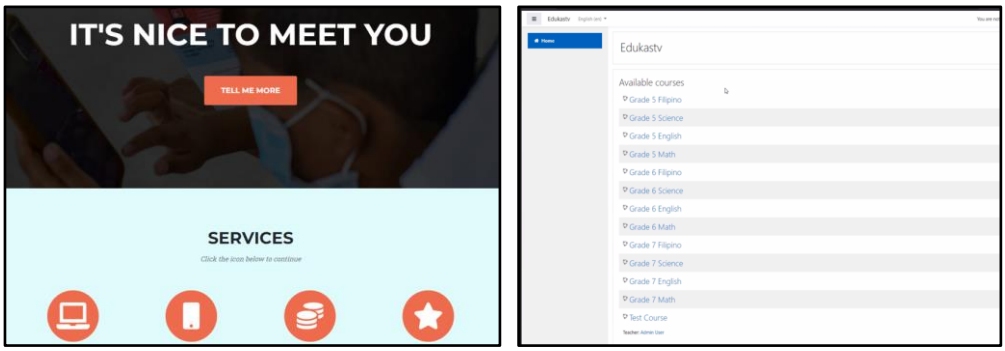


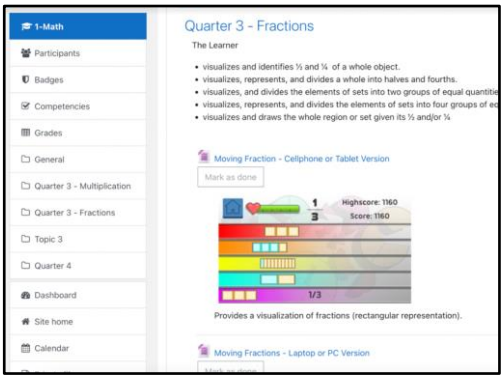
Figure 2. (a) RuralCasting Portal; (b) Edukastv page

For the users to access the mathematical mobile applications, they will need to be connected to the set-top box's Wi-Fi, go to the rcast.local page, and navigate the portal to the Math Applications page. The page will display all the available mathematical mobile apps available for download. An instructional page is also available for the users shown in Figure 3.

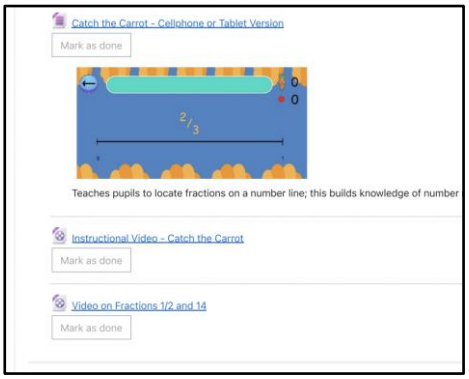


Figure 3. Instructional guide in installing the mobile app

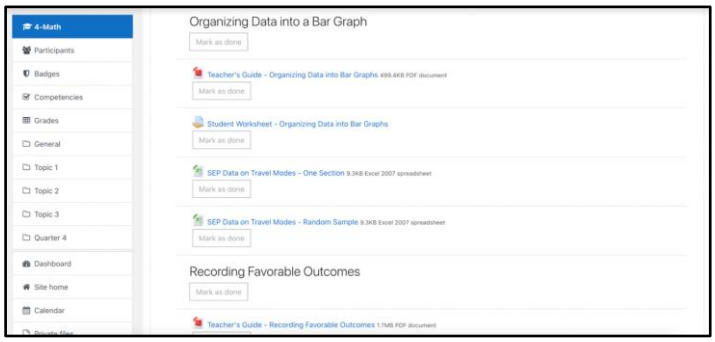
The teachers can incorporate the apps into their learning modules in Edukastv as shown in Figure 4(a). Since the mobile apps have been curated for the learners' grade level, the teachers will have an easier time in assigning the appropriate mathematical concept for their learners. Accompanying the mobile apps are other related resources such as teaching guides, student worksheets, or instructional videos that have been developed through the project. Examples or models for different grade levels on how the apps and these resources can be integrated and organized in Edukastv are also provided to the teachers as shown in Figure 4(b-c).



(a)



(b)



(c)

Figure 4. (a) Incorporating the mobile app in Edukastv; (b-c) Organizing mobile apps and other educational resources (e.g., videos, teacher's guide, student worksheet) in Edukastv

2.2 Mathematical Applications

The mobile mathematical applications have been designed and developed for Grades 1-6 but can also be used for Grade 7. The applications focus on place value, fraction number sense, and number sense strategies. All these concepts are tied to the understanding of number magnitude, which provides a unifying framework for mathematical understanding (Siegler et al., 2011). Examples of apps that have been developed by the project are *Catch the Carrot* (Verzosa et al., 2021a) and *Grid Game* (Verzosa et al., 2021b).

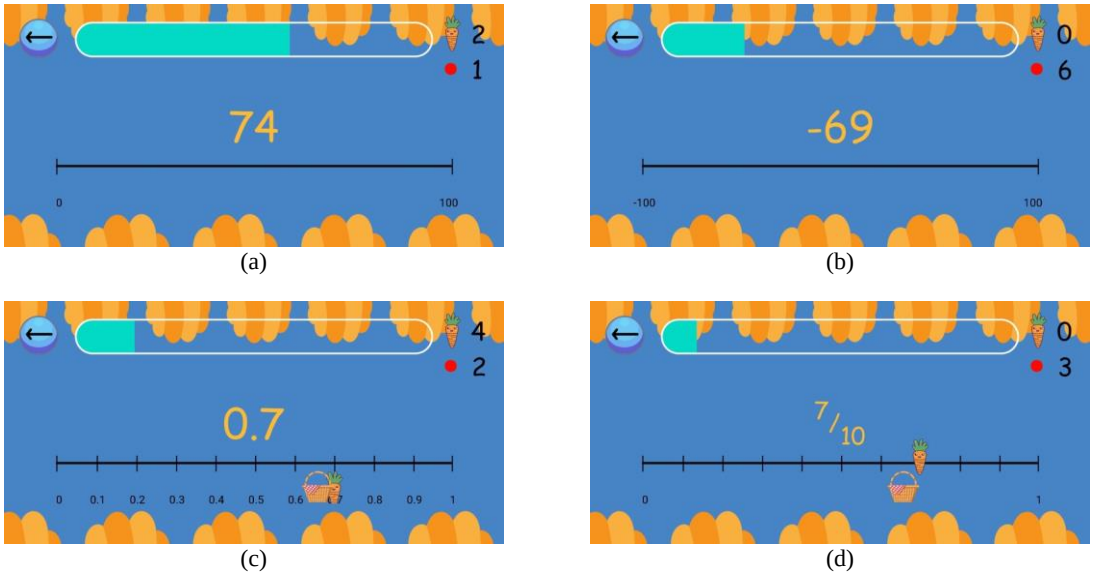
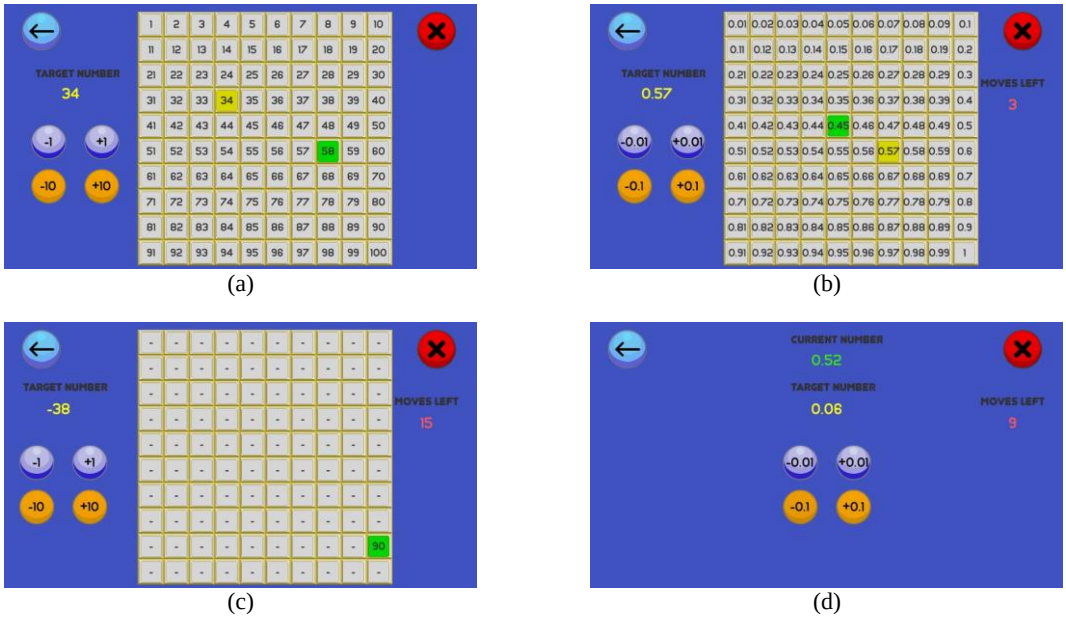


Figure 5. Screenshots of *Catch the Carrot* consisting of different levels such as (a) whole numbers, (b) integers, (c) decimals, and (d) fractions

Catch the Carrot is a number line mobile app designed to develop students' knowledge of whole number and fraction magnitude, which are important predictors of later mathematical achievement (Siegler, 2016). In the app, a segment of a number line is shown, and students need to estimate the location of a given number on the segment. For example, in Figure 5(a), the segment is from 0 to 100, and the given number is 74. The student then needs to indicate the location of 74 on the number line. After the student enters a response, by tapping or clicking on the number line, a carrot appears and falls into a basket if the student's estimate is close enough. Different levels and challenges are present in the app. Students may choose to play with integers (Figure 5(b)), decimals (Figure 5(c)) or fractions (Figure 5(d)).

On the other hand, *Grid Game* is an app aimed to develop conceptual place value (Ellemor-Collins & Wright, 2011). The app shows a starting number and a target number (Figure 6(a)). The student needs to click on the buttons so that the starting number reaches the target number. In higher levels, the student must perform the task in the minimum number of moves. There is also an option to work with higher levels of abstraction through a blank grid (i.e., a grid with no numbers) as in Figure 6(c) or through no grid at all (i.e., only the starting and target numbers are shown) as in Figure 6(d).

As further detailed by Verzosa et al. (2021a, 2021b), both *Catch the Carrot* and *Grid Game* are based on research in mathematical learning and are aimed to promote mathematical reasoning and visual thinking so that children can learn to think independently. Both apps also contain various levels to address official learning competencies across the elementary mathematics curriculum. Further, both apps have been designed with careful integration of educational game design principles. This has been intentionally done to maximize the apps' potential not only in the classroom or in online synchronous classes but also for students' independent and regular use. Specifically, *Catch the Carrot* has been designed following the Game-based Learning Design Model (Shi & Shih, 2015) where 11 interrelated game-design factors are considered. Meanwhile, the game design of *Grid Game* conforms to the Educational Games Design Model (Ibrahim & Jaafar, 2009), which focuses on game design, pedagogy, and learning content modeling.



(a) (b) (c) (d)

Figure 6. Screenshots of the *Grid Game* consisting of different levels such as (a) whole numbers, (b) decimals, (c) integers with a blank grid, and (d) decimals with no grid (for increased abstraction)

The full set of mathematical apps developed by the project can be found on the website https://mathplusresources.wordpress.com/. As is the case with *Catch the Carrot* and *Grid Game*, all the mathematical apps found therein have been developed based on research in mathematics education and with due consideration to their alignment to the official curricula of the Philippines' Department of Education (DepEd, 2020). The design and development of apps that are game-based or gamified also follow educational gaming principles that have been corroborated in literature on technology in mathematics education. All the grade school level apps found in the aforementioned website are included in deployments via the RuralCasting system.

3. METHODOLOGIES

The mathematical applications were deployed by the team to two elementary schools: San Andres Elementary School and Madilaydilay Elementary School in the province of Rizal, Philippines. Location was a crucial

component in choosing the schools; the team had to identify schools with low interference in television frequencies for RuralCasting. The deployment had the full support of the Department of Education School Division Office of Rizal, which facilitated the cooperation of school administrators and teachers. Community officials also provided assistance. Meetings were held between the project team and the officials of the school division office, and the school administrators and teachers for the onboarding process. The math project team met with mathematics teachers of both schools and gave them webinars on the use of the mathematical applications.

During deployment, the team devised instructions and scenarios where the participants will be able to access the services, and simultaneously browse and download the applications. The team then tested the maximum number of participants able to smoothly access, stream, and download the content from the RuralCasting set-top box. Also, after downloading the mathematical applications, feedback on the applications (Figures 5 and 6) were gathered. Improvements were done on the mathematical applications based on the feedback. For example, an opportunity for improvement shown in Figure 7 is the integration of audio feedback, which was later on incorporated into the app. Further, the lack of instructions was identified as a weakness. Thus, some video demonstrations of the application were made to accompany the application.



Figure 7. Sample SWOT analyses

4. RESULTS AND DISCUSSIONS

In Table 1, it is shown that issues arose when there were more than 15 devices connected and utilizing the Wi-Fi network simultaneously. The mathematical applications that were given for transmission testing in RuralCasting worked perfectly and were compatible with the set-top boxes (Table 2).

In Table 2, it is shown how the set-top box performs in terms of distributing the mobile mathematical applications with respect to the increase in participants. As shown, it is observed that the participants have no problems in downloading, installing, and using the application. But issues such as delays in downloading and awaiting the pop-up for the device were observed once more than 15 participants simultaneously accessed the custom set-top box. Fortunately, once fully downloaded, the application was successfully installed and was fully utilized.

Table 1. Maximum users, activities, and remarks per set-top box

No. of Participants	Wi-Fi Connected	Browsing	Streaming	Downloading	Remarks
5	5	5	5	5	all were able to connect, browse, stream, and download the available content
10	10	10	10	10	all were able to connect, browse, stream, and download the available content
15	15	10	10	10	all were able to connect, browse, stream, and download the available content
18	16	14	13	15	some were not able to connect, some had issues loading the webpage, some had difficulty in downloading the app

Table 2. Example of app performance with respect to participants and experience

MathPlus App	No. of Participants	Remarks
Catch the Carrot	10	app successfully downloaded from the set-top box, installed in an Android device, and used the app
	15	app successfully downloaded from the set-top box, installed in an Android device, and used the app
	18	not everyone successfully downloaded the app from the set-top box, others had a delay in the download, but once successfully downloaded the app was successfully installed and utilized
Grid Game	10	app successfully downloaded from the set-top box, installed in an Android device, and used the app
	15	app successfully downloaded from the set-top box, installed in an Android device, and used the app
	18	not everyone successfully downloaded the app from the set-top box, others had a delay in the download, but once successfully downloaded the app was successfully installed and utilized

5. CONCLUSION AND OUTLOOK

This paper discusses the deployment of a customized set-top box capable of distributing offline content thru Wi-Fi. The resources offered from the set-top box range from images and texts to videos and Android applications. The users have been able to connect their personal Wi-Fi devices successfully and to navigate through the RuralCasting Portal. The contents have been successfully downloaded, installed, and utilized simultaneously by up to 15 participants during the technical demonstrations in the targeted sites in Rizal, Philippines.

The deployment continues in the current school year, where more schools in remote areas in the Philippines are identified to have access to set-top boxes with more educational content. A point of consideration and improvement in the set-top box is the testing with the participants of the return channel feature that allows information to be returned to the teachers (e.g., grades, scores, feedback from explorations with apps). This will provide valuable information to the teachers, and thus offer a mechanism for an alternative communication system to these areas without internet access.

This school year as well, the Philippines' Department of Education is implementing a "back-to-basics" program in reading and mathematics as part of a learning recovery endeavor to address the serious learning gaps among school children who have been locked down and only learned remotely during the two years of the pandemic. The mathematical apps that have been developed in this project are envisioned to contribute towards this purpose. Part of the outlook for the school year is to conduct studies in the areas of deployment with respect to the effectiveness of the apps in addressing the learning of the students.

ACKNOWLEDGEMENT

The authors would like to acknowledge the support of the Department of Science and Technology- Philippine Council for Industry, Energy and Emerging Technology Research and Development (DOST-PCIEERD) and the Department of Science and Technology - Advanced Science Technology Institute (DOST-ASTI). They are also grateful to the Ateneo de Manila University University Research Council, the School of Science and

Engineering, the Department of Education - Rizal, and all the participants during the initial testing of the set-top box and the deployment/distribution of the mobile mathematical applications. Thanks are also extended to the app developers: Victor Ortega, Nigel Cargo, and Theo Lacson; and to the project managers and deployment team, Pinky Manio, Maria Aya Lei Banzuela, Julius Urbina, and Grace Datiles.

REFERENCES

Bernardo A.B.I. and Limjap A., (2012). Investigating the Influence of Teachers' Pedagogical Beliefs and Reported Practices on Student Achievement in Basic Mathematics. 12th International Congress on Mathematical Education. Seoul, Korea.

Department of Education (DepEd), (2016). K to 12 Curriculum Guide: Mathematics. https://www.deped.gov.ph/wp-content/uploads/2019/01/Math-CG_with-tagged-math-equipment.pdf

Department of Education (DepEd), (2020). K to 12 Most Essential Learning Competencies with Corresponding CG Codes. https://commons.deped.gov.ph/K-to-12-MELCS-with-CG-Codes.pdf

Department of Information and Communications Technology (National ICT Household Survey 2019). https://dict.gov.ph/ictstatistics/nicths2019/

Eballa J. N. et al., (2022). RuralSync: Providing Digital Content to Remote Communities in the Philippines through Opportunistic Spectrum Access. 2022 IEEE Global Humanitarian Technology Conference (GHTC). California, USA, pp. 125-128.

Ellemor-Collins D. and Wright R., (2011). Developing Conceptual Place Value: Instructional Design for Intensive Intervention. Australian Journal of Learning Difficulties, Vol. 16, No. 1, pp. 41-63.

Ibrahim R. and Jaafar A., (2009). Educational Games (EG) Design Framework: Combination of Game Design, Pedagogy and Content Modeling. Proceedings of the 2009 International Conference on Electrical Engineering and Informatics (ICEEI 2009), Bangi, Malaysia, pp. 293-298.

National Telecommunications Commission (NTC), (2016). Memorandum Circular No. 02-03-206, ISDB-T Standards. https://ntc.gov.ph/wp-content/uploads/2016/MC/MC-No-02-03-2016-Attachment.pdf

Pennsylvania PBS, (2020). Datacasting. Retrieved 10/30/2020 from https://pennsylvaniapbs.org/datacasting/

Rusciano A., 2021. ATSC Datacasting Paves Way for Flexibility of ATSC 3.0 in Distance Education - ATSC: Nextgen TV. https://www.atsc.org/news/atsc-datacasting-paves-way-for-flexibility-of-atsc-3-0-in-distance-education/

Schleicher A., (2019). PISA 2018: Insights and Interpretations. OECD, Paris, France. https://www.oecd.org/pisa/PISA%202018%20Insights%20and%20Interpretations%20FINAL%20PDF.pdf

Shi Y.-R. and Shih J.-L., (2015). Game Factors and Game-based Learning Design Model. International Journal of Computer Games Technology, Vol. 2015, pp. 1-11.

Siegler R.S., (2016). Magnitude Knowledge: The Common Core of Numerical Development. Developmental Science, Vol. 19, No. 3, pp. 341-361.

Siegler R.S. et al., (2011). An Integrated Theory of Whole Number and Fractions Development. Cognitive Psychology, Vol. 62, No. 4, pp. 273-96.

South Carolina ETV, (2020). Datacasting. Retrieved 10/30/2020 from https://www.scetv.org/datacasting

Verzosa D.M.B., (2020). Pre-service Elementary Teachers' Reasoning in Introducing Multidigit Subtraction. International Journal of Mathematical Education in Science and Technology, Vol. 51, pp. 876-892.

Verzosa D.M.B. et al., (2021a). Development of a Gamified Number Line App for Teaching Estimation and Number Sense in Grades 1 to 7. Proceedings of the 29th International Conference on Computers in Education. Taoyuan, Taiwan, pp. 648-656.

Verzosa D.M.B. et al., (2021b). Using Mobile Technology to Promote Higher-Order Thinking Skills in Elementary Mathematics. Proceedings of the International Conferences Mobile Learning and Educational Technologies. Lisbon, Portugal, pp. 19-26.

Verzosa D.M.B. and Vistro-Yu C.P., (2019). Prospects and Challenges in Implementing a New Mathematics Curriculum in the Philippines. School Mathematics Curricula, pp. 207-226.

Ateneo de Manila University

Archīum Ateneo

Mathematics Faculty Publications | Mathematics Department

1-1-2023

Practice Through Play Using Mobile Technology

Agnes D. Garciano
Ateneo de Manila University

Debbie Marie B. Verzosa
University of Southern Mindanao

Ma Louise Antonette N. De Las Peñas
Ateneo de Manila University

Maria Alva Q. Aberin
Ateneo de Manila University

Juan Carlo F. Mallari
Ateneo de Manila University

See next page for additional authors

Follow this and additional works at: https://archium.ateneo.edu/mathematics-faculty-pubs

 Part of the Mathematics Commons, and the Science and Mathematics Education Commons

Custom Citation

Garciano, A.D., Versoza, D.M.B., De Las Peñas, M.L.A.N., Aberin, M.A.Q., Mallari, J.C.F., Sarmiento, J.F., & Tolentino, M.A.C. (2023). Practice Through Play Using Mobile Technology. In P. Kommers, I.A. Sanchez, & P. Isáias. (Eds.), International Conferences on e-Society 2023 and Mobile Learning 2023, 351-358.

This Conference Proceeding is brought to you for free and open access by the Mathematics Department at Archīum Ateneo. It has been accepted for inclusion in Mathematics Faculty Publications by an authorized administrator of Archīum Ateneo. For more information, please contact oadrcw.ls@ateneo.edu.

Authors

Agnes D. Garciano, Debbie Marie B. Verzosa, Ma Louise Antonette N. De Las Peñas, Maria Alva Q. Aberin, Juan Carlo F. Mallari, Jumela F. Sarmiento, and Mark Anthony C. Tolentino

This conference proceeding is available at Archīum Ateneo: https://archium.ateneo.edu/mathematics-faculty-pubs/240

PRACTICE THROUGH PLAY USING MOBILE TECHNOLOGY

Agnes D. Garciano[1], Debbie Marie B. Verzosa[2], Ma. Louise Antonette N. De Las Peñas[1], Maria Alva Q. Aberin[1], Juan Carlo F. Mallari[1], Jumela F. Sarmiento[1] and Mark Anthony C. Tolentino[1]
[1]*Department of Mathematics, Ateneo De Manila University, Philippines*
[2]*Department of Mathematics and Statistics, University of Southern Mindanao, Philippines*

ABSTRACT

This paper discusses the *Just Keep Solving* apps that are designed based on deliberate practice model for developing mathematical skills. Features of deliberate practice include well-defined goals involving areas of weakness as determined by a knowledgeable other such as a teacher. The integration of game design features provides a positive environment wherein the learning goals critical in a deliberate practice model are emphasized. Possible strategies for integrating the apps in a classroom are also discussed using Hughes, Thomas and Scharber's RAT (Replacement, Amplification, Transformation) framework. The games can replace traditional pen-and-paper classroom activities, amplify learning by personalizing a student's experience and providing opportunities for deliberate practice, and transform instruction from being teacher-centered to student-centered.

KEYWORDS

Mobile Technology in Teaching Mathematics, Mathematical Apps, Drill-and-Practice

1. INTRODUCTION

In school year 2022-2023, the Department of Education (DepEd) in the Philippines implemented the gradual transition to the in-person learning modality after purely distance learning due to the Covid 19 pandemic. The DepEd developed the *Basic Education Learning Recovery Plan* to guide schools in addressing learning gaps brought about by the pandemic (DepEd Memo 664, 2022). Under a government-funded project *Mathematical Resources for Distance Learning Utilizing Community LTE Networks and Television Frequencies*, the authors collaborated with local DepEd school divisions to deliver mathematical content that included mathematical applications (apps), teaching guides/instructional videos, and performance tasks or activity sheets that are distributed not just through the internet but also through community LTE networks (De Las Peñas et al., 2022) and local networks powered by datacasting technology (De Las Peñas et al., 2023). The mathematical apps were designed to address the Most Essential Learning Competencies (MELCs) prescribed by DepEd (DepEd, 2020) as well as to help narrow the existing learning gaps on particular competencies determined by the school administrators and teachers.

The apps have also been developed to be compatible with the use of mobile technology, which offer additional advantages. First, given the growing popularity and access to mobile technology (e.g., smartphones, tablets) in the Philippines, majority of the project's apps are compatible with a large range of Android devices and have relatively small storage requirements. Secondly, the use of mobile technology can potentially improve the learning opportunities and experiences of students. The portability and convenience of mobile devices mean that students use them more often (Norris et al., 2011). Consequently, students become capable of personalizing their learning experiences (Shuler, 2009) because their mobile devices are available for use even outside the classroom and/or without the supervision of their teachers.

This paper discusses two particular mathematical apps, namely *Just Keep Solving* and *Just Keep Solving 2*, that focus on linear equations and inequalities, and statistics and probability, respectively. These apps are based on a strategic application of the deliberate practice model for developing mathematical skills, particularly those that have not yet been learned (Lehtinen et al., 2017). The interested reader can access *Just Keep Solving* and *Just Keep Solving 2* as well as all the mathematical apps of the project from the website https://mathplusresources.wordpress.com/.

ISBN: 978-989-8704-47-4 © 2023

2. DRILL-AND-PRACTICE AND DELIBERATE PRACTICE IN MATHEMATICS EDUCATION

The approach of using game-based drill-and-practice activities has been previously used and studied in different fields and has been associated with some positive outcomes. For instance, Foss et al. (2014) assessed the use of an online game that provides medical calculation drills intended for student nurses. For learning vocabulary, Yip & Kwan (2006, p.246) report that their research results indicate, "both quantitatively and qualitatively, that learners playing online vocabulary games tend to learn better and could retain the learnt vocabulary for a longer period and retrieve more words than those who simply attended face-to-face lessons without accessing the vocabulary games." It is important to note that the vocabulary games in Yip and Kwan's study are all drill-and-practice games. As a last example, in mathematics education, Ke (2008) has reported that computer math drill games (particularly ASTRA EAGLE, a series of web-based games consisting of drill-and-practice math activities) have a positive effect on students' attitudes towards learning mathematics.

While drill-and-practice is often associated with low-level or procedural skills, deliberate practice involves the learning of higher-order skills (Lehtinen et al., 2017). Deliberate practice is typically associated with music or sport (Ericsson et al., 2006). Applying this concept to mathematics education, Lehtinen summarized features of deliberate practice based on Ericsson's (2016) work. These include developing skills that have not yet been learned but are known by a knowledgeable other (such as a teacher) who can provide support, and the provision for feedback. Further, the emphasis on well-defined goals involving areas of weakness suggests that the process is not necessarily enjoyable (Lehtinen et al., 2017).

Fuchs et al. (2010) investigated whether deliberate practice can improve low-performing mathematics students' skills in strategic counting, which is a skill that typically developing children usually learn on their own. They found that low-performing students who received deliberate practice performed significantly better on strategic counting tasks than the control group who did not receive any form or instruction, and another group who received instruction but no opportunity to practice. By isolating the effect of deliberate practice on learning, they argued that practice is indeed valuable for students with mathematical difficulties.

In another study, Pachman et al. (2013) investigated the effect of deliberate practice on the learning of geometry. Similar to Ericsson (2016), they considered an emphasis on weakness as a critical feature of deliberate practice. They found that more knowledgeable students derived the most benefits from deliberate practice. For these students, the teacher can impose that they focus solely on their weak areas. By contrast, less knowledgeable students improved more if they practiced in both their weak and strong areas. These results suggest that some level of competence must first be attained before students are compelled to focus solely on their weak areas.

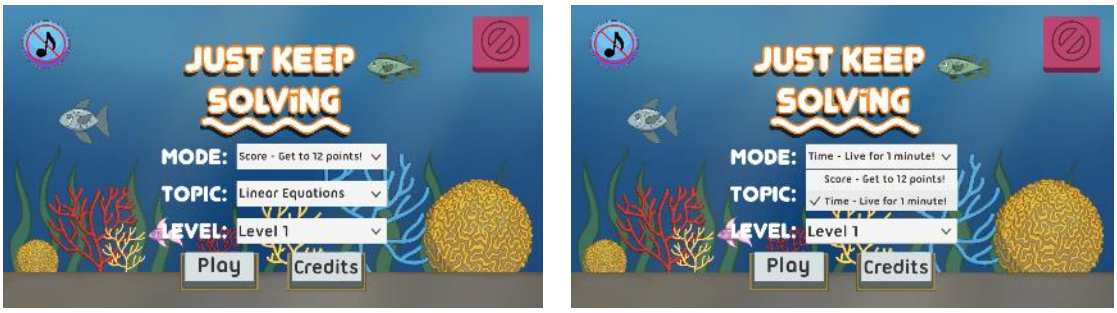
3. THE *JUST KEEP SOLVING* APPS

The *Just Keep Solving* apps are designed in a game-like environment to address particular learning competencies in mathematics, based on a deliberate practice model. Both apps are compatible with and run efficiently in a wide range of Android devices (e.g., smartphones, tablets). Versions for use in Windows computers (laptops or desktops) are also available. Each app focuses on topics, and a topic has several levels. The apps show an underwater environment with bombs that threaten to destroy the ocean corrals. The questions are written on the bombs, and to prevent a bomb from reaching the ocean floor, the student must answer the question corresponding to the bomb correctly. A wrong answer results in some destruction to the corals. After a certain number of wrong answers, the game is terminated.

3.1 *Just Keep Solving*

One of the five content areas in the mathematics curriculum given in the K to 12 Curriculum Guide in Mathematics in the Philippines is that of Patterns and Algebra (DepEd, 2016). In particular, one of the Most Essential Learning Competencies (MELCs) in Grade 7 mathematics under this content area is: solve linear equations and inequalities (DepEd, 2020). The achievement of this competency is a prerequisite to solving quadratic and higher order equations.

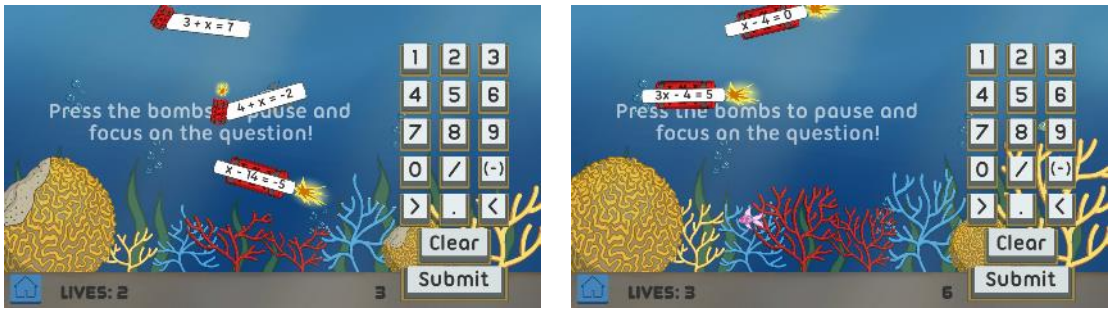
The *Just Keep Solving (JKS)* app (Figure 1(a)) offers students opportunities to practice solving linear equations and inequalities. It is meant to provide repetitive exercises necessary for students to master algebraic skills.



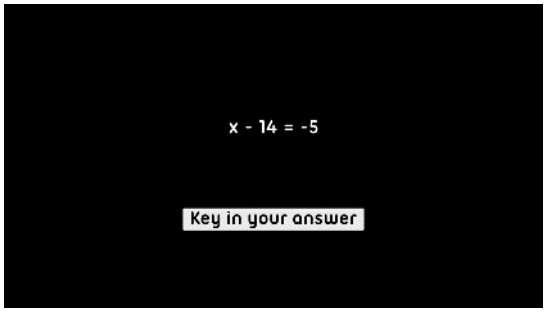
(a) (b)

Figure 1. (a) Home screen of *JKS*; (b) Two modes to play *JKS*

The *JKS* app is divided into two topics: *Linear Equations* and *Linear Inequalities.* Each topic can be played in two modes: one where the student can score up to 12 points before the game ends, and the other, where the student is given one minute to answer as many questions correctly as he can (Figure 1(b)). There are two levels under the topic *Linear Equations*. The first level deals with one-step linear equations (Figure 2(a)) and the second level deals with one and two-step linear equations (Figure 2(b)). When a bomb is selected, a screen containing only the question appears and allows students to focus on solving the question (Figure 2(c)). At this point, the falling of the bombs is paused. The equations are basic and are meant to invite students to play while providing practice needed for procedural fluency in algebra.



(a) (b)



(c)

Figure 2. *JKS* topic *Linear Equations*: (a) Level 1; (b) Level 2; (c) screen displaying a question

Likewise, there are two levels under the topic *Linear Inequalities*. The first level displays linear inequalities which are solvable using addition or subtraction of positive or negative numbers (Figure 3(a)), while the second level deals with linear inequalities which are solvable using multiplication or division of positive or negative numbers (Figure 3(b)). Under this topic, the correct answer requires the student to provide the appropriate inequality symbol (< or >) as well as the correct number. For example, in answering the question $x - 1 > 0$, the inequality ">" and the number "1" is provided.



(a) (b)

Figure 3. *JKS* topic *Linear Inequalities*: (a) Level 1 and (b) Level 2

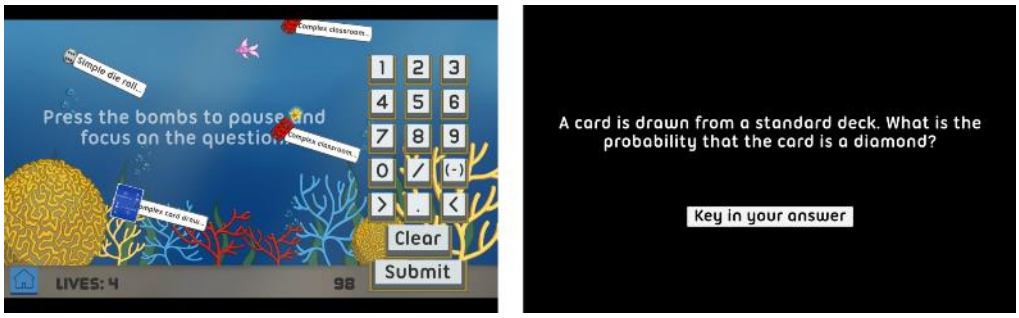
3.2 *Just Keep Solving 2*

Given the potential of *JKS* in developing students' skills on implementing simple computations and applying mathematical formulas, *Just Keep Solving 2* (*JKS* 2) (Figure 4) has been conceptualized as an expansion of *JKS* with topics *Statistics* and *Probability*. The topic *Statistics* has three levels that are intended for Grade 7 students and are focused on the following Most Essential Learning Competencies (MELCs) under Statistics: i) calculates the measures of central tendency of ungrouped data; ii) uses appropriate statistical measures in analyzing and interpreting statistical data; and iii) calculates the measures of variability for ungrouped data (DepEd p.307, 2020). Moreover, there are also two levels on the topic *Probability*. The first level is intended for the following Grade 8 MELCs under Probability: i) finds the probability of a simple event; and ii) solves problems involving probabilities of simple events. (DepEd p.312, 2020) On the other hand, the second level involves MELCs under the Grade 11 content area Random Variables and Probability Distributions. These MELCs are: i) illustrates a probability distribution for a discrete random variable and its properties; ii) computes probabilities corresponding to a given random variable; and iii) solves problems involving mean and variance of probability distributions. (DepEd, 2013; DepEd, 2020).



Figure 4. Home screen of *Just Keep Solving 2*

While *JKS 2* retains most of the features of the first *JKS*, certain changes had to be applied to adapt to the relatively lengthier questions and computations involved in the aforementioned statistics and probability MELCs. The most significant of these changes is that only the first part of each question, or a key phrase to represent the question, is displayed on a falling bomb (Figure 5(a)). The complete question can only be shown by tapping on the bomb itself. The students get access to a full-screen view where the question is displayed (Figure 5(b)). Another change, as shown in Figure 5(a), is that some questions use different icons (e.g., a card or a die) instead of the bomb icon. This is to give a visual cue about the corresponding question's context, when applicable.



(a) (b)

Figure 5. (a) In *Just Keep Solving 2*, only the first parts of the questions are shown beside the falling bombs. (b) Tapping a falling bomb gives access to the full-screen view of the corresponding question.

Each level of *JKS 2* features six to eight types of questions. Specific questions are generated randomly within prescribed parameters. Due to this variety, some game aspects in *JKS 2* have been adjusted to allow for longer game play. Specifically, the modes are now "Score – Get to 20 points!" (previously 12 points) and "Time – Live for 2 minutes!" (previously 1 minute) while the number of game lives has been increased from three lives in *JKS* to five. At the end of each game, regardless of the player succeeding or failing to complete the mode's goal, relevant game records are displayed (Figure 6) for easy recording (e.g., via a screenshot).



(a) (b)

Figure 6. (a) Game screen after a successful completion of a level of the "Time – Live for 2 minutes!" mode; (b) Game screen after failing to complete a level "Score – Get to 20 points!" mode.

4. GAME DESIGN FACTORS

The *Just Keep Solving* apps have been designed to enhance the potential benefits of traditionally pen-and-paper or flash-card-based activities by transforming them into digital game applications. As many students are fascinated with playing games in smartphones, tablets, and/or computers (Castellar et al., 2014), it is envisioned that the *Just Keep Solving* apps can increase the interest and attention of the students. The gamelike environment can positively transform their experience in doing deliberate practice activities which, due to its demands and emphasis on weak areas, can tend to be unenjoyable (Lehtinen et al. 2017).

While some have expressed that drill-and-practice games may lead to some players developing the undesirable behavior of "try[ing] actions with no reflection on outcomes" (Kiili, 2005 p.14), previous studies (Foss et al., 2014; Ke, 2008 Yip & Kwan, 2006) exemplify that well-designed drill-and-practice games may lead to positive outcomes on learning. Thus, the Just Keep Solving apps were designed to promote features of the deliberate practice model through the integration of game-design factors. For this purpose, we employed Shi and Shih's (2015) Game-based Learning (GBL) Design Model, which has been developed as a result of an extensive review of research on game design factors. The GBL Design model identifies 11 game-design factors (game goals, game mechanism, game fantasy, game value, interaction, freedom, narrative, sensation, challenges, sociality, and mystery) which form the environment wherein learning goals, a critical feature of deliberate practice, can be emphasized.

For both *Just Keep Solving* apps, the *game goal* can be identified by a knowledgeable other, such as a teacher, to enable students to work on weak areas in Algebra, Statistics, or Probability. Since it had been shown that purely focusing on weak areas may not be optimal for the weaker students (Pachman et al., 2013), other game-like features were necessary to make the experience more enjoyable and effective. The *game fantasy* is established by using clear and aesthetically appropriate graphics in the background (i.e., ocean floor with fishes and corals) and other game elements (i.e., falling bombs, damaged corals). These graphics assets, together with the playful background music and sound effects (e.g., for correct or wrong answers), form the multimedia presentation of the game's setting and contribute to players' *sensation*. The game's *narrative* is implied both by the multimedia presentation and the game *mechanism*, which as previously described, requires the players to answer questions that are attached to falling bombs so that they do not fall and explode on the ocean floor.

Players *interact* with the game by tapping the bombs to access a full-screen view of the questions and by using a digital number keypad for inputting answers. While the bombs' falling pauses on the full-screen view of questions, the game's *challenge* is that there is only a limited time for each bomb to reach the ocean floor and explode and that bombs continue to appear as the game progresses. This means that players can only afford a limited number of incorrect answer inputs and, thus, must perform their computations carefully and precisely.

In the *Just Keep Solving* apps, each player has the *freedom* to choose between two game modes and from a variety of levels that are dependent on the topic covered and its difficulty. This freedom also provides less knowledgeable learners the opportunity to achieve a certain level of competence before they are subjected to deliberate practice that focuses primarily on their weak areas (Pachman et al., 2013). Further, the availability of these modes and levels contributes to the game's *mystery* and might lead students to spending more time with the game. Since both apps have been designed to be single-player games, they have limited *sociability*. It is worth mentioning, however, that when used in the classroom setting and with teacher supervision, the *Just Keep Solving* apps may allow for friendly competitions and/or cooperative playing experiences.

The implementation into the *Just Keep Solving* apps of all the game-design factors mentioned above contributes to the game's value. In addition, students might find it more engaging or preferable to conduct their drills and practices, or assessments, using the games instead of pen-and-paper or flash-card modalities. Lastly, as previously mentioned, both *Just Keep Solving* apps are compatible with mobile technologies that can allow students to use the apps even outside the classroom. This enables them to have opportunities for independent or asynchronous learning.

5. INTEGRATION AND USE OF JUST KEEP SOLVING APPS

This section describes how the *Just Keep Solving* apps can be utilized as a pedagogical tool based on the RAT (Replacement, Amplification, Transformation) framework (Hughes, Thomas, & Scharber, 2006).

Using this framework, the *Just Keep Solving* apps can *replace* the traditional method of providing examples and exercises for students to solve without changing the learning goals of a lesson. The printed worksheets or exercises from the textbook are now replaced by the questions which appear in the more inviting game-like setting of the app.

Playing the apps can enrich or *amplify* a student's experience in solving problems. The design of the app enables students to choose to work at just the right level of difficulty and provides elements in which students can be excited and challenged to work on advanced levels. Furthermore, the teacher as a knowledgeable other (Lehtinen et al., 2017) may identify students' areas of strength and weaknesses in order to design appropriate deliberate practice activities for more knowledgeable students, or a combination of deliberate practice and drill-and-practice activities for less knowledgeable students (Pachman et al., 2013).

Finally, the *Just Keep Solving* apps may give impetus to *transform* instruction from being teacher-centered to student-centered. This is possible because the apps have been designed so that students can easily understand and play them with little to no guidance from a teacher or a guardian. Hence, the apps are suited for use by students who are learning asynchronously or remotely. Transformation can also happen when tasks are redefined. For instance, individual tasks can be changed to collaborative activities using the apps. This gives students an opportunity to discuss and teach each other the problem-solving techniques required to answer the questions.

The apps, particularly *JKS* can be used for remediation. As mentioned in the introduction, a *Basic Education Learning Recovery Plan* was set in place in the Philippines to address learning gaps brought about by the pandemic. The repeated solving of similar problems provided by the apps aims to have a transformative effect to help reduce the gaps in specific MELCs. Moreover, a gradual improvement in skills can provide encouragement to weak students and increase their confidence.

6. CONCLUSION AND OUTLOOK

The *JKS* was developed to provide a tool in which students can repeatedly solve certain mathematical problems on linear equations and inequalities while playing a game. Expanding on *JKS*, a second mobile game *JKS 2* was designed using similar mechanics but applied to topics on statistics and probability. The competencies covered by both apps are aligned to the Philippines' Department of Education Most Essential Learning Competencies. Both apps are based on the strategic application of deliberate practice through the incorporation of game design principles with the aim of maximizing the games' value and their potential to enrich students' learning experiences and help students develop the intended learning competencies.

The apps can be used as a pedagogical tool to provide opportunities for deliberate practice in a classroom setting and in an asynchronous or remote learning set-up. The app can also be further extended to include additional mathematical competencies that are identified to be areas of weakness among students. The next step is to study the effectiveness of the apps in addressing the MELCS and/or increasing students' engagement and interest towards learning.

ACKNOWLEDGEMENT

This paper is one of the outputs of the Ateneo Mathplus Resources Laboratory housed at the Department of Mathematics, School of Science and Engineering, Ateneo de Manila University. The authors thank the Department of Science and Technology-Philippine Council for Industry, Energy, and Emerging Technology Research and Development (DOST-PCIEERD) and the University Research Council (URC), Ateneo de Manila University for the support of the development of the mathematical resources for Grades 1 to 11 Mathematics. We thank our app developers Victor Antonio M. Ortega, Mr. Nigel Benedict Cargo, Mr. Jose Teodoro Lacson, and Mr. Amiel Damian F. Justiniani for the apps' assets. Much appreciation is extended to our partners the Department of Science and Technology - Advanced Science Technology Institute (DOST-ASTI), the Ateneo Innovation Center, our partner DepEd School Division Offices Rizal and Quezon City and the teachers and administrators of our partner schools.

REFERENCES

Castellar, E.N. et al., 2014. Improving Arithmetic Skills through Gameplay: Assessment of the Effectiveness of an Educational Game in Terms of Cognitive and Affective Learning Outcomes. *Information Sciences*, Vol. 264, pp. 19-31.

De Las Peñas, M. et al., 2022. Deployment of Mathematical Resources to a Philippine High School through a Community LTE Network. *Electronic Proceedings of the 27th Asian Technology Conference in Mathematics*. Mathematics and Technology LLC, Prague, Czech Republic.

De Las Peñas, M. et al., 2023. Mathematical Mobile Apps Via Rural Casting. To appear: *Proceedings of the Mobile Learning 2023 International Conference*. Lisbon, Portugal.

Department of Education (DepEd), 2016. *K to 12 Curriculum Guide: Mathematics*. https://www.deped.gov.ph/wp-content/uploads/2019/01/Math-CG_with-tagged-math-equipment.pdf

Department of Education (DepEd), 2020. *K to 12 Most Essential Learning Competencies with Corresponding CG Codes*. https://commons.deped.gov.ph/K-to-12-MELCS-with-CG-Codes.pdf

Department of Education (DepEd), 2013. *Senior High School Core Subject (Statistics and Probability)*. https://www.deped.gov.ph/wp-content/uploads/2022/02/SHS-Core_Statistics-and-Probability-CG.pdf

Department of Education (DepEd) Region I, 2022. *Regional Memorandum No. 664, s. 2022: Basic Education Learning Recovery Plan in light of the COVID 19 Pandemic*. https://depedro1.com/wp-content/uploads/2022/06/rm0664s2022.pdf

Ericsson, K.A., 2016. Summing up Hours of Any Type of Practice Versus Identifying Optimal Practice Activities: Commentary on Macnamara, Moreau, & Hambric. *Perspectives on Psychological Science*, Vol. 11, No. 3, pp. 351-354.

Ericsson, K.A., Charness, N., Feltovich, P.J., and Hoffman, R.R. (Eds.), 2006. *The Cambridge Handbook of Expertise and Expert Performance*. Cambridge University Press, Cambridge, England.

Foss, B. et al., 2014. Digital Game-Based Learning: A Supplement for Medication Calculation Drills in Nurse Education. *E-Learning and Digital Media*, Vol. 11, No. 4, pp. 342-349.

Fuchs, L.S. et al., 2010. The Effects of Strategic Counting Instruction, with and without Deliberate Practice, on Number Combination Skill among Students with Mathematics Difficulties. *Learning and Individual Differences*, Vol. 20, No. 2, pp. 89-100.

Hughes, J., Thomas, R., and Scharber, C., 2006. Assessing Technology Integration: The RAT – Replacement, Amplification, and Transformation - Framework. *Proceedings of Society for Information Technology & Teacher Education International Conference 2006*. Association for the Advancement of Computing in Education (AACE), Waynesville, NC USA, pp. 1616-1620.

Ke, F., 2008. A Case Study of Computer Gaming for Math: Engaged Learning from Gameplay? *Computers & Education*, Vol. 51, No. 4, pp. 1609-1620.

Kiili, K., 2005. Digital Game-based Learning: Towards an Experiential Gaming Model. *The Internet and Higher Education*, Vol. 8, No. 1, pp.13-24.

Lehtinen, E. et al., 2017. Cultivating Mathematical Skills: from Drill-and-practice to Deliberate Practice. *ZDM Mathematics Education*, Vol. 49, pp. 625-636.

Norris, C., Hossain, A., and Soloway, E., 2011. Using Smartphones as Essential Tools for Learning. *Education Technology*, Vol. 51, No. 3, pp.18-25.

Pachman, M. Sweller, J., and Kalyuga, S., 2013. Levels of Knowledge and Deliberate Practice. *Journal of Experimental Psychology: Applied*, Vol. 19, No. 2, pp. 108-119.

Shi, Y.-R. and Shih, J.-L., 2015. Game Factors and Game-based Learning Design Model. *International Journal of Computer Games Technology*, Vol. 2015, pp. 1-11.

Shuler, C., 2009. *Pockets of Potential: Using Mobile Technologies to Promote Children's Learning*. The Joan Ganz Cooney Center, New York, USA.

Yip, F.W.M., and Kwan, A.C.M., 2006. Online Vocabulary Games as a Tool for Teaching and Learning English Vocabulary. *Educational Media International*, Vol. 43, No. 3, pp. 233–249.

Ateneo de Manila University

Archīum Ateneo

Mathematics Faculty Publications | Mathematics Department

2006

Strategies in Maximizing the Use of Existing Technology in Philippine Schools

Debbie Marie Y. Bautista
Ateneo de Manila University

Ma. Louise Antonette N. De Las Peñas
Ateneo de Manila University, mdelaspenas@ateneo.edu

Follow this and additional works at: https://archium.ateneo.edu/mathematics-faculty-pubs

 Part of the Mathematics Commons

Custom Citation

Bautista, D. M. Y., De Las Peñas, M. L. A. N. (2006). Strategies in Maximizing the Use of Existing Technology in Philippine Schools. Proceedings of the Eleventh Asian Technology Conference in Mathematics (eds. S.C. Chu, S.H. Hou, and W.C. Yang), Mathematics and Technology, LLC, Blacksburg VA, USA. 308-316.

This Conference Proceeding is brought to you for free and open access by the Mathematics Department at Archīum Ateneo. It has been accepted for inclusion in Mathematics Faculty Publications by an authorized administrator of Archīum Ateneo. For more information, please contact oadrcw.ls@ateneo.edu.

Strategies in Maximizing the Use of Existing Technology in Philippine Schools

Debbie Marie Y. Bautista
debbie@mathsci.math.admu.edu.ph
Department of Mathematics
Ateneo de Manila University
Philippines

Ma. Louise Antonette N. De Las Peñas
mlp@mathsci.math.admu.edu.ph
Department of Mathematics
Ateneo de Manila University
Philippines

Abstract: One of the challenges that continue to confront teachers in Philippine schools is the accessibility of technology for the study and learning of mathematics. In this paper, we will look at several situations and actual experiences happening in Philippine schools. Strategies on how existing technological tools are to be maximized will be discussed, including the creation of lesson plans and classroom activities. The use of technology-based manipulatives in mathematics learning as alternatives to unavailable technology will also be looked at.

1. Introduction

There is an increasing importance of using technology in the teaching and learning of mathematics. It allows students to explore and discover concepts and it has the potential of extending the mathematics that can be taught. However, Philippine schools do not have access to some of these technologies. We classify these situations into four constraints. The first one is having Internet access. The second is having a computer without Internet. Third, there are also situations where computers are not available at all. Finally, we discuss the scenario where technology is available, but in very limited numbers. For each situation, we provide suggestions to use the available technology efficiently and to devise materials that could be alternatives to unavailable technology.

2. Using the Internet

Some schools have computers with internet access but some use it primarily for obtaining data or using email. However, the internet also provides several ways to enhance mathematics teaching and learning. It allows both teachers and students to access interactive resources, curriculum materials, and real-world data [3]. One important way by which the Internet can enhance mathematics learning is through interactive applications. A good example of this is the Mathsnet website (www.mathsnet.net). Created by Bryan Dye of Norfolk, England, this project is a source of free interactive math resources aimed at providing an enjoyable learning environment. Further, the applications require the user to fully participate, and not just be a passive viewer. The website includes an Interactive Construction Course where students use interactive figures to do geometric constructions (See Figure 1.1). Students may use their tools incorrectly in doing actual

compass and straightedge constructions. The interactive course, however, allows students to do precise constructions. There are different levels of difficulty to cater to individual needs. Another advantage is that the applet allows the students to move the original figure and view their own constructions in a dynamic environment. Students can see more clearly the role of these constructions in their own geometric proofs.

Another way by which the internet could be used is through the curriculum materials that it provides [3]. Materials could be found on the teaching and learning of mathematics for all levels. Lesson plans, as well as teaching strategies are available and could be revised to adapt to the Philippine setting. One example would be the lesson plans developed by through the Rational Number Project (http://education.umn.edu/rationalnumberproject/). It is widely recognized that children in Philippine schools have difficulty in learning fractions. The course materials produced

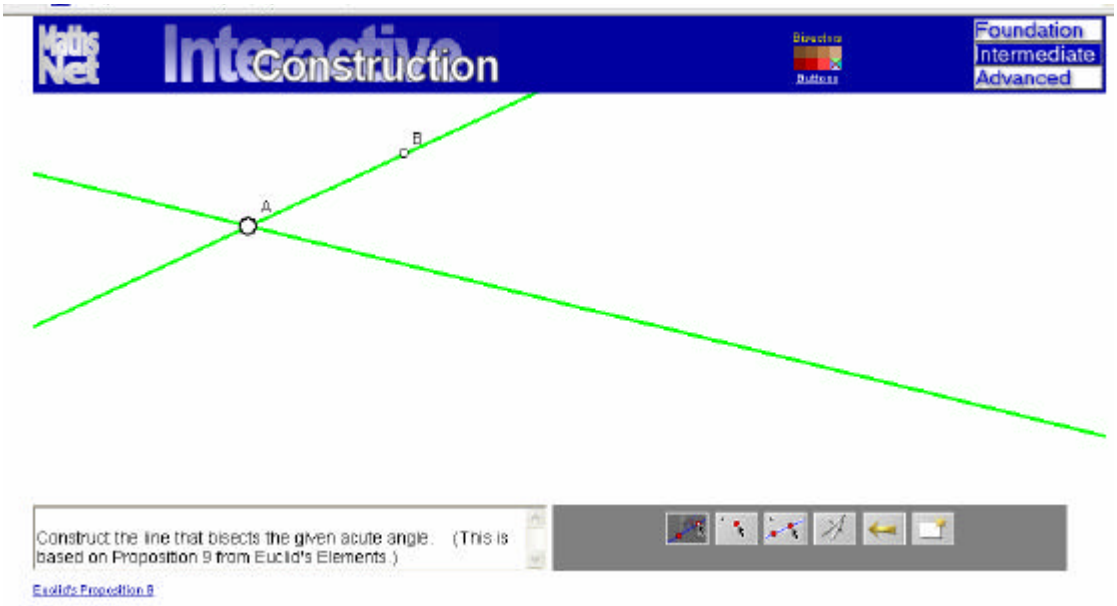


Figure 2.1 Doing Geometric Constructions on the Internet (http://www.mathsnet.net/campus/construction/ibb4.html)

by this project are the result of a long-term research on how children learn fractions, decimals, and proportionality, and were made accessible over the internet.

3. Using Free Software Packages

Another problem faced by teachers in Philippine schools is the unavailability of some software packages. Fortunately, the internet provides access to a number of shareware applications. Some of these programs are executable files and can be copied directly from one computer to another. Sharing them becomes even more convenient since no installation is required. Thus, even computers without internet access could have copies of these programs. One such resource is Wingeom (795 Kb), which can be downloaded for free from the Peanut Software Homepage (http://math.exeter.edu/rparris/). The program was developed by Rick Parris of Phillips Exeter Academy in New Hampshire. One example of how the program can be used is through investigating the analytic geometry of the parabola (See Figure 3.1). Another possibility is the

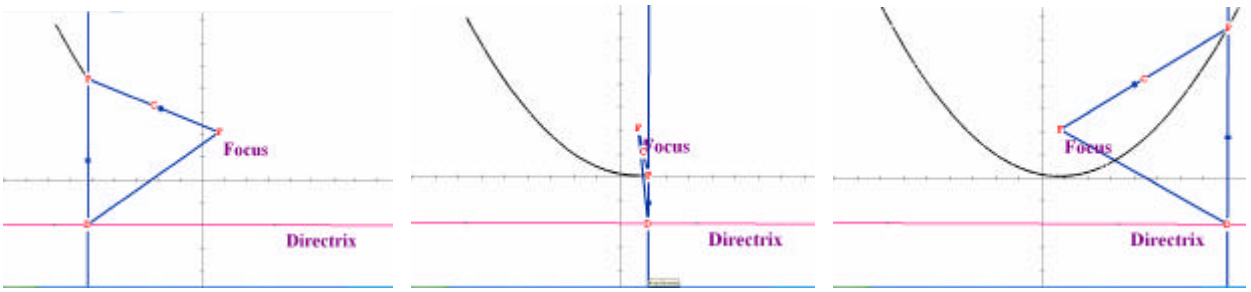


Figure 3.1 An Animation of the Locus of All Points Equidistant from a fixed point and a fixed line.

exploration of geometric art (See Figure 3.2). This is a Geometer's Sketchpad (GSP) project suggested in [8]. However, Wingeom could be used to perform this GSP project. Concepts related to the construction of such designs could then be analyzed. The same website by Parris is also a source of programs catered to Algebra, Discrete Mathematics, Linear Algebra, and Statistics.

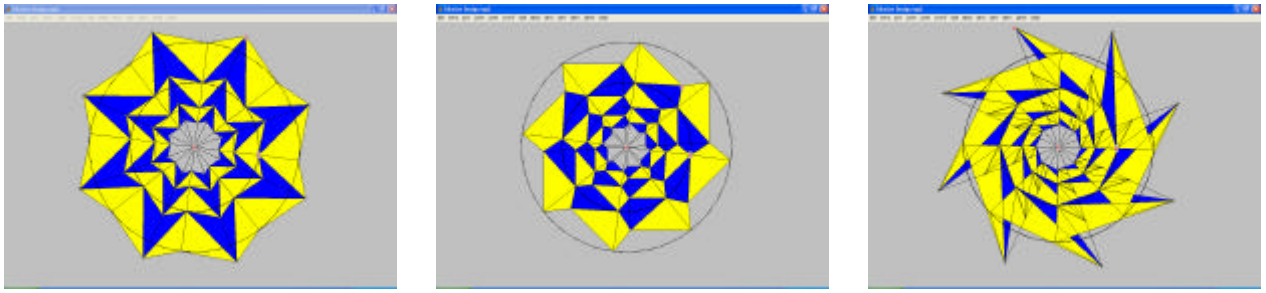


Figure 3.2 An Animation of Different Possible Designs Obtained by Dragging Vertices.

Another free software package is GRAPES, which was developed by Japanese mathematics teachers (http://www.criced.tsukuba.ac.jp/grapes/). This program has already been translated to English and Spanish. The program could be used to analyze how the derivative of a function is related to the function itself (See Figure 3.3).

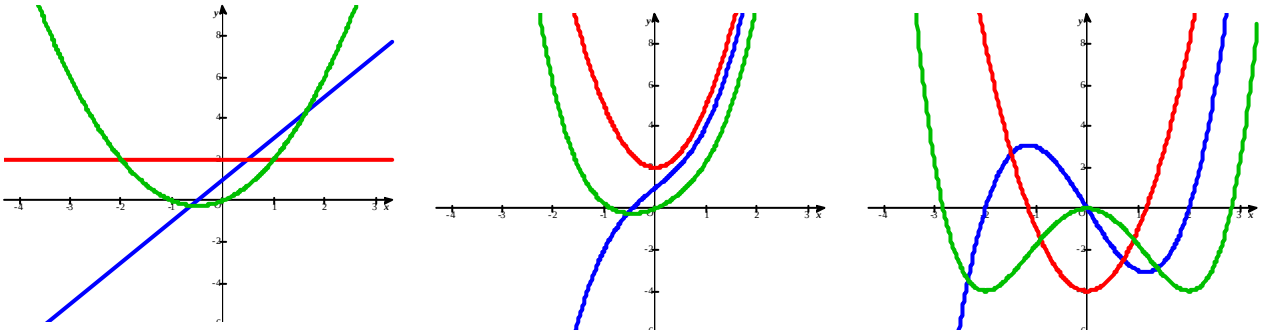


Figure 3.3 An Animation of a Function and its First and Second Derivatives

These free software packages are not exactly the same as commercial ones, though. For instance, some features in Geometer's Sketchpad, such as creating buttons, are not in Wingeom. The user interface of Wingeom is also simpler and less eye-catching than a commercial package. GRAPES, meanwhile is mostly a tool for graphing, and it cannot show ideas in geometry. However, by learning what a free software package can and cannot do helps the teacher come up with suitable activities for the classroom. It is also advised that teachers try to find free software packages which have features which are not present in the free software currently used. Thus, even if a free software package lacks certain features, it is still possible that some other program would carry that missing feature.

4. Using Technology-Based Manipulatives and Non-Graphic Calculators

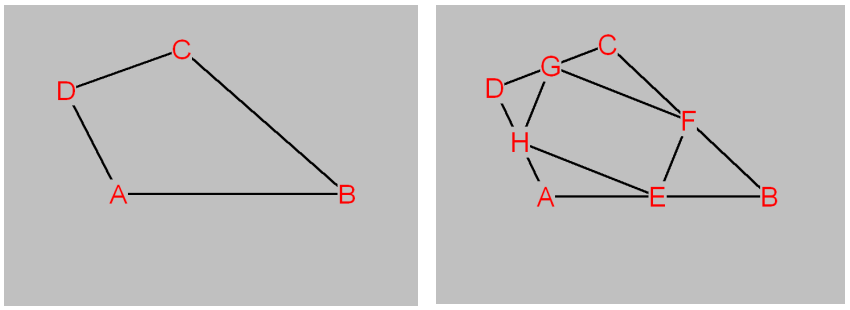
With the advent of newer and more advanced tools and software, one may not see much importance in the ordinary four-function and the scientific calculator. However, it these are still tools which further mathematical thinking and learning. Even though it cannot show a graphical representation, it still has numeric capabilities which could facilitate discussions and group work. These capabilities can be used to the fullest if the teacher has a good grasp of how the tool works.

One activity that utilizes the scientific calculator would be an analysis of real-world data using regression analysis. Students could be asked to make a survey on a relevant and interesting topic. Data gathered could be studied using regression analysis. There are also times when solutions to equations cannot be obtained feasibly by hand. Scientific calculators which can compute roots can then be used. Students get a feel for real-world problems which almost certainly would not result in equations with "clean" solutions. Another topic which can be discussed with the aid of a scientific calculators would be series. For instance, one can see numerically that an infinite series could yield a finite sum while an approximation for the number *e* using Taylor's Formula, gives the expected, yet amazing, result.

In fact, several studies, such as [7] illustrate the many benefits of a non-graphic calculator, even in primary school. Rather than spending a considerable amount of time drilling students on adding numbers, students may use calculators to explore patterns, and to develop skills such as estimation and analysis. Students then have more time to comprehend the problem, allowing them to be more involved in the problem-solving process. Whereas students sometimes answer questions by just guessing the arithmetic operation needed, calculators would give the teacher and the students more time to discuss the ideas and concepts behind the problem-solving task. Calculators would also allow students to explore patterns since students save a lot of time doing computational work and could thus focus more on recognizing existing patterns. Needless to say, appropriate changes must be made in the curriculum to implement the use of calculators even in lower elementary grades. The important thing is to realize that the calculators can be used not only to provide instant results, but they can be used to learn mathematical concepts. It is just a matter of learning how to make lesson plans which give students this opportunity. Traditionally, students may use pencil-and-paper computations to solve percents. Now, they may use calculators to investigate underlying concepts, such as, what happens if a number is repeatedly reduced to 75%. These investigations introduces more elaborate mathematics, such as exponential decay and limits, even though the topic involves elementary arithmetic.

If there is no accessible computer at all, some activities performed using dynamic geometry software could still be replicated using concrete models which can easily made at home. In particular, we construct manipulatives out of ordinary cardboard and pins.

Let us consider the following example, performed using dynamic geometry software. Construct a kite *ABCD* (See Figure 4.1(a)). Connect the midpoints of each side to construct a new quadrilateral, which we call the Midpoint Quadrilateral (See Figure 4.1(b)).



(a) (b)

Figure 4.1 A Kite and the Quadrilateral Formed by Joining the Midpoints of its Four Sides.

Using the drag mode, students are able to see properties of that for any kite, the midpoint quadrilateral is always a rectangle (See Figure 4.2).

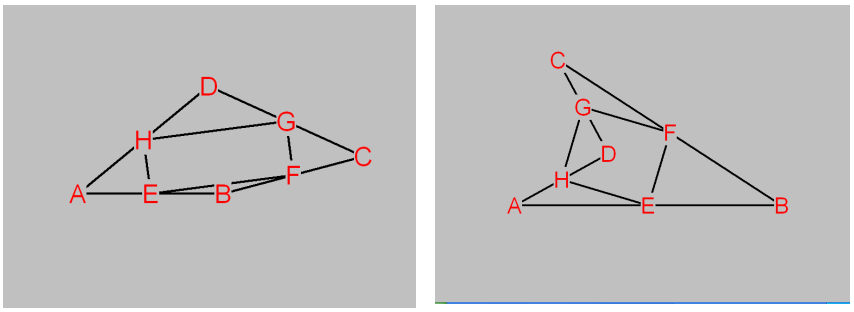


Figure 4.2 The Midpoint Quadrilateral is a Rectangle for any orientation, and even if the kite is convex.

The same property could be exhibited using easily available materials. Strips of cardboard are used for the sides, while cardboards with a slit in the middle could serve as sliders, used to connect the midpoints. These sliders are created to allow the lengths they represent to vary, as the kite's orientation varies itself. By alternatively pushing and pulling on the kite's vertices, one can see that the midpoint quadrilateral remains rectangular (See Figure 4.3)

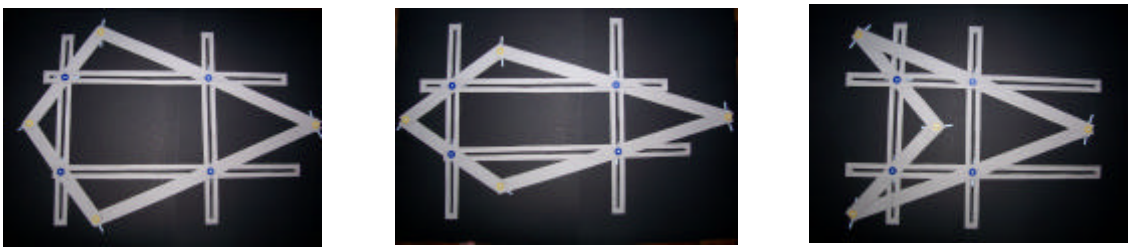
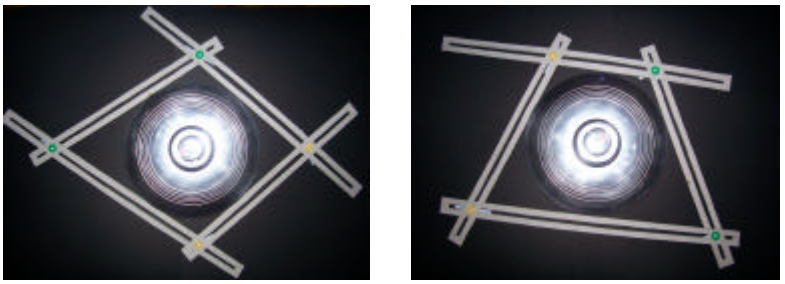


Figure 4.3 The Midpoint Quadrilateral as Seen Using a Concrete Manipulative.

The sliders themselves could be used to demonstrate some mathematical results. For instance, as suggested by [1], the sliders could be used to lead students to discover some properties of cyclic quadrilaterals. Use the sliders to construct an apparatus similar to Figure 4.4(a). Fit a circular object inside so that the circle is inscribed in the quadrilateral formed by the apparatus (See Figure 4.4(b). Groups of students may perform the activity using different circle sizes. The students should also be instructed to make different quadrilaterals which circumscribe their circles. This experiment would then allow students to form conjectures about cyclic quadrilaterals.



(a) (b)

Figure 4.4 Different Quadrilaterals Circumscribing a Circular Object

It is a challenge, therefore, to think of new instruments to help students visualize geometric concepts. Many more concepts could be shown using some variations on the above tools. It is up to the teacher to come up with inexpensive manipulatives which can show these geometric concepts.

5. Maximizing the Influence of a Limited Number of Computers

We now discuss the case where there are not enough technological tools for each individual student. This subject is treated last since it may be applicable to any of the situations described in Sections 2 to 4. For instance, several suggestions in these sections could be used easily if each student has his own computer or if not, then at least there should be one computer equipped with a video display. However, this is not always the case. Very often, not all students could use the tools simultaneously, while some schools do not have an LCD or even an overhead projector. In this section, we suggest ways by which to work out the problem.

For simplicity, let us consider the case where there are only five graphing calculators in a classroom of 50 students. It has been recommended by Alejandre [2], that the calculators could be used as one of the stations within a menu of stations. This is a technique wherein several areas are set up in the classroom, with each area having its own set of tasks or instructions to be completed by a group of students. Thus, in a class of 50 students, the chairs could be arranged facing each other in 11 groups of four, and two groups of three. Alejandre also suggested having one or two more areas than there are groups of students so that students could move around once they complete the activity in their station. In our scenario, since there are only five graphing calculators, then only 5 of the stations would be equipped with one. The activities for the rest of the station should be answerable by hand or by some other method.

As an illustration, consider the following activity where students use graphing calculators to learn graphing transformations such as translations and reflections. Some papers exploring the

possibility of teaching this topic through art designs are shown in [5] and [6]. Sample designs are shown in Figure 5.1. In these papers, students are tasked to create designs by graphing equations or inequalities. Ideally, each student should have his own graphing calculator so that he can do a design of his own. How can a teacher perform these activities in a classroom with only five graphing calculators?

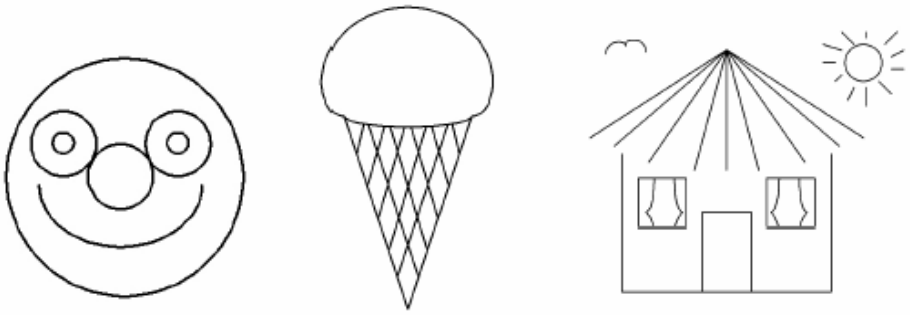


Figure 5.1 Some Designs that Could be Reproduced by Graphing Equations

One option is to divide students into groups of five, and organize several stations within the classroom. All the five calculators should be located on one station. The group assigned to this station would make their own design by graphing equations or inequalities. The other groups could be to assigned to all the other stations, where they would answer some paper-and-pencil tasks regarding transformations. They could also be asked to answer questions about previous topics, for review. One station could also be a recreational math station where students may answer any absorbing math problem. After a pre-specified amount of time, students would be asked to change stations, so that everyone may have a chance to use the graphing calculators.

Another option is to simply divide the class into five groups, with each group having one calculator. Each group would then be asked to create a design, such as a playground. Each member would then divide tasks among themselves. One person could create a slide, another could graph a sandbox, and so on. Collectively, their equations would be used to create a single playground scene. These activities are suitable for primary and secondary school students, depending on which type of graphs are to be used in the designs. Even college-level students could perform these activities when they study polar or parametric equations.

In the previous suggestion, the activity was primarily meant to serve as a way to reinforce a concept. It is also possible that these are used to facilitate mathematical discovery among students. The websites, computer activities, and even the concrete apparatus mentioned above could serve as one of the stations, while the rest of the class are involved in paper-and-pencil work. For instance, if the topic is about quadrilaterals, and there are only five computers (without internet access), then the teacher could load the free software packages beforehand. Each computer station would include tasks related to quadrilaterals that can be easily seen using dynamic geometry software. Each station would also provide students with questions that would lead them to discovering new properties about quadrilaterals. A number of other stations could have concrete models that could show other concepts on quadrilaterals. If a graphing calculator is available, then it could also be included as one of the stations. A good starting point in organizing these types of activities would

be the Multicultural Math Fair website (http://.mathforum.org/alejandre/mathfair/index.html), again suggested in [2].

Back to the original scenario where there are five graphing calculators, it is also possible to use these to help students check their work. For instance, groups of students may be asked to solve limits on their seats and verify their answers using the graphing calculator. In this case, students make use of the graphing calculator only after solving each question. Since they need only a few minutes to check their solution, the five calculators would be enough for the needs of the entire class. If the lesson were on derivatives, each group of students could be given a graph on paper. Then, as a group, they help each other compute the slopes of the tangent lines at several points, and estimate how the graph of the derivative function would look like. They then verify their answers using the graphing calculators. They may then be asked follow-up questions related to points where a function attains an extreme value, and points where a function becomes non-differentiable.

In fact, a class can benefit from having just one graphing calculator especially if it comes with a projection screen. In this case, the calculator could be used by the teacher to illustrate an example, to help students visualize mathematical concepts, or to facilitate problem-solving which involve non-integer solutions. This also provides an opportunity where a disadvantage could be turned into a benefit. A large student-to-class ratio, primarily thought of as a drawback, would actually take less time to perform simulation experiments. A large number of students would make data-gathering more efficient since several students could perform simultaneous experiments. The activity suggested by Alejandre [2] is to use the computer or graphing calculator to store data gathered by the entire class. For instance, each student could perform experiments on the Monty Hall dilemma, while a pre-specified student could be tasked to encode all data into the available technological tool. Moreover, the data and the interpolated function could easily be displayed by the spreadsheet as students interpret and analyze their results. This is one way of turning a disadvantage into something that benefits the entire group.

6. Concluding Remarks

While it is true not all technological tools may be available in Philippine schools, it is also true that teachers could still manage ways to come up with creative and ingenious solutions to such circumstances. This paper could serve as a springboard from which Filipino teachers could build a base on. The important thing is not to focus on what is lacking, but to concentrate on what is available and on what can be done. Students themselves learn to be more resourceful, and to make the most of what they have.

References

[1] Akiyama, J., Sakai, Toshinori, Torigoe, N., and Watanabe, Y. (2000) Nonstandard Ways of Teaching Standard Mathematics II. *Teaching and Learning Aids and Materials (Hands-on) in Mathematics Education (Topic Study Group 5), of the Ninth International Congress on Mathematical Education*, Tokyo / Makuhari, Japan.

[2] Alejandre, S. (2005) The Reality of Using Technology in the Classroom. *Technology-Supported Mathematics Learning Environments, Sixty-Seventh Yearbook*, National Council of Teachers of Mathematics, Inc.

[3] Galindo, E. (2005) Mathematics Teaching and Learning Supported by the Internet. *Technology-Supported Mathematics Learning Environments, Sixty-Seventh Yearbook*, National Council of Teachers of Mathematics, Inc.

[4] Isoda, M., Kakihana, K., Miyakawa, T., Aoyama, K., Yoden, Y., Yamanoi, K., Uehara, Eigo, and Chino, K. (2005) Mathematics Classroom Innovation with Tehnology Japanese Movement. *Proceedings of the Tenth Asian Technology Conference in Mathematics*, Cheong-Ju, South Korea.
[5] Kim, N. (2005). Designing and Mathematics Learning. *Proceedings of the Tenth Asian Technology Conference in Mathematics*, Cheong-Ju, South Korea.
[6] Miller, S. (2001). Understanding Transformations of Periodic Functions through Art. *Mathematics Teacher*, Vol. 94 No. 8 (pp. 632-635).
[7] Pomerantz, Heidi (1997). The Role of Calculators in Math Education. Retrieved from http://education.ti.com/sites/US/downloads/pdf/therole.pdf.
[8] Serra, M. (1997). Discovering Geometry, 2nd edition, Key Curriculum Press, California, USA (pp. 599-601).
[9] Websites:
a. GRAPES http://www.criced.tsukuba.ac.jp/grapes/volume.html
b. Mathsnet http://www.mathsnet.net/
c. Multicultural Math Fair http://mathforum.org/alejandre/mathfair/index.html
d. Peanut Software Homepage http://math.exeter.edu/rparris/
e. Rational Number Project http://education.umn.edu/rationalnumberproject/

Ateneo de Manila University

Archīum Ateneo

Mathematics Faculty Publications | Mathematics Department

12-1-2023

Technological Tools for the Teaching and Learning of Statistics

Mark Anthony C. Tolentino
Ateneo de Manila University, mtolentino@ateneo.edu

Juan Carlo F. Mallari
Ateneo de Manila University

Maria Alva Q. Aberin
Ateneo de Manila University

Ma. Louise Antonette De las Penas
mdelaspenas@ateneo.edu

Agnes D. Garciano
Ateneo de Manila University

See next page for additional authors

Follow this and additional works at: https://archium.ateneo.edu/mathematics-faculty-pubs

Custom Citation

Tolentino, M.A.C., Mallari, J.C.F., Aberin, M.A.Q., De Las Peñas, M.L.A.N., Garciano, A.D., Sarmiento, J.F., Versoza, D.M.B. (2023). Technological tools for the teaching and learning of statistics. 31st International Conference on Computers in Education, ICCE 2023 - Proceedings

This Conference Proceeding is brought to you for free and open access by the Mathematics Department at Archīum Ateneo. It has been accepted for inclusion in Mathematics Faculty Publications by an authorized administrator of Archīum Ateneo. For more information, please contact oadrcw.ls@ateneo.edu.

Authors

Mark Anthony C. Tolentino, Juan Carlo F. Mallari, Maria Alva Q. Aberin, Ma. Louise Antonette De las Penas, Agnes D. Garciano, Jumela F. Sarmiento, and Debbie Marie B. Verzosa

This conference proceeding is available at Archīum Ateneo: https://archium.ateneo.edu/mathematics-faculty-pubs/232

Shih, JL. et al. (Eds.) (2023). Proceedings of the 31st International Conference on Computers in Education. Asia-Pacific Society for Computers in Education

Technological Tools for the Teaching and Learning of Statistics

Mark Anthony C. TOLENTINO[a,*], Juan Carlo F. MALLARI[a], Maria Alva Q. ABERIN[a], Ma. Louise Antonette N. DE LAS PEÑAS[a], Agnes D. GARCIANO[a], Jumela F. SARMIENTO[a], Debbie Marie B. VERZOSA[b]

[a]*Department of Mathematics, Ateneo de Manila University, Philippines*
[b]*Department of Mathematics and Statistics, University of Southern Mindanao, Philippines*
*mtolentino@ateneo.edu

Abstract: Statistics and its applications form an integral part of STEM education. In the literature, it is shown that technology is valuable in supporting the teaching and learning of statistics. This paper discusses some technological tools that have been developed to support statistics education in the grade school, junior high school, and senior high school levels. It describes the design and pedagogical basis of these tools, and how these may be integrated in the classroom.

Keywords: Mathematical apps, statistics, STEM, technological tools in statistics

1. Introduction

Statistics continues to play an important role in STEM education as it can be applied to all STEM fields (Adams, 2017). In fact, Ben-Zvi and Garfield (2008) have stated how statistics is becoming a necessary area of study that teaches students how to react intelligently to quantitative information. Mastery in statistics allows students to apply different statistics concepts and techniques across STEM projects or within their disciplines when analyzing data generated from investigations (Watson et al., 2020). Thus, statistics is usually featured in the mathematics curricula of different countries and is often included in multiple grade levels (https://timssandpirls.bc.edu/timss2015/encyclopedia/). In the Philippines, for example, the K to 12 Basic Education Curriculum (Department of Education, 2016) includes *Probability and Statistics* as one of the five main content strands.

Given the importance of statistics education, efforts and resources to enhance or to support teaching and learning statistics remain valuable. The use of technology is usually emphasized in this regard; for instance, the Pre-K-12 Guidelines for Assessment and Instruction in Statistics Education II (GAISE II) highlights "the recognition that modern statistical practice is intertwined with technology, and the importance of incorporating technology as feasible" (Bargagliotti et al., 2020, p.2). There are several methods on how technology is used in teaching and learning statistics, and how these impact student learning (Chance et al., 2007). Studying statistics in an interactive environment using technology tools supports the understanding of statistical concepts (Prodromou, 2015).

While there are already various technological tools available for statistics education, most existing software (e.g., spreadsheet software, programming language) can be too sophisticated or have irrelevant functionalities for beginner learners. In the context of developing countries such as the Philippines, many of these software may also not be affordable or may be more demanding in terms of device requirements. Thus, we have developed a number of technological tools to support statistics education. In this paper, we discuss pedagogical bases and various features of these tools and how these tools can be concretely integrated into statistics classes.

2. Pedagogical Basis

Statistics is often seen to be a branch of mathematics, but it is a distinct field of study requiring both mathematical and non-mathematical skills (Wild et al., 2018). It covers five interrelated themes—variation, expectation, distribution, randomness and informal inference (Watson et al., 2020). Moore (1998, p. 1257) argued that statistical thinking "is a general, fundamental, and independent mode of reasoning about data, variation, and chance." To promote statistical thinking and develop the notion of statistical inference, it is important to provide simulation and randomization-based activities in the classroom.

Rossman and Chance (2014) compiled a list of textbooks that include randomization in the early parts of statistical courses, as well as technological tools that allow students to perform simulations. They also described their own introductory statistics course where students initially perform simulations of random processes using coin flipping, compare their simulations with those of their classmates, and later perform the same simulations using technology. In their curriculum, simulations not only illustrate statistical inference but become the principal mechanism through which they learn to perform statistical inference themselves.

Another important pedagogical consideration, as emphasized in GAISE II (Bargagliotti et al., 2020), is the role of context in statistics. Aside from dealing with the variability of data sets, students should also recognize the context(s) of these data sets so that they can craft interpretations and conclusions appropriately. Hence, to develop such competencies, it is vital that students have access to authentic data sets that are contextually clear and relatable to them. Using such data sets, students can practice procedural skills in statistical analysis and experience contextual learning (Samo et al., 2017).

In designing and developing technological tools, as far as possible, we were guided by Rossman and Chances's (2014) recommended list of features—ease of use, mimicking of by-hand simulations, and consistency of user interface. While the first and third features are doable, we had to do away with the second feature because we wanted the tools to run even on modest devices. To manage this limitation, the tools are accompanied by teachers' guides and student worksheets to enable teachers to use the tools more readily. These guides and worksheets are aligned with the most essential learning competencies (MELCs) specified in the official Philippine mathematics curriculum (Department of Education, 2020).

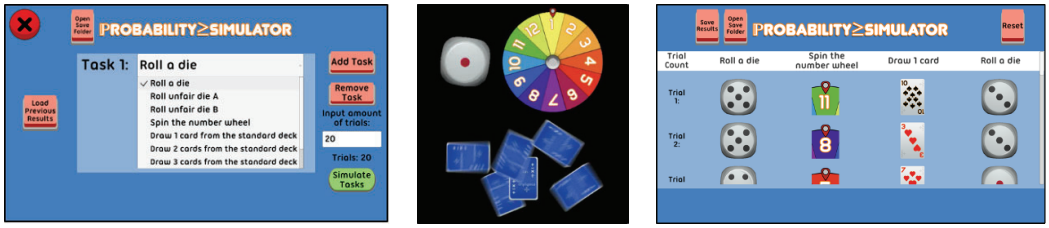
3. Technological Tools for Statistics

The technological tools we have developed include apps, applets, a web-based platform, and videos. The apps have been developed using Unity Editor Version 2021.3.25f1 and are available both for Windows and Android devices. The Windows versions can run on 32- or 64-bit systems and have modest requirements: Windows 7 or later, 1 GHz processor or higher, 4 GB RAM, and approximately 300 MB available storage. On the other hand, the Android versions can run on devices with Android 5.1 until 13, 2 GB RAM, and approximately 120 MB available storage. The applets and web-based platform do not require installations and can run on most internet browsers, with only the platform requiring internet connectivity.

3.1 *Tools for Simulation and Data Generation*

GAISE II emphasizes the importance of the context and variability of data sets that are used in statistics education (Bargagliotti et al., 2020). One aspect of this involves giving students opportunities or experiences of dealing with uncertainty and randomness. In the classroom, it is possible to achieve this by having activities in which students perform repeated trials of actual physical probabilistic experiments (e.g., rolling dice, drawing cards). However, in some cases (e.g., remote learning, due to time or resource constraints), these physical experiments may not be feasible. Thus, we have developed the app *Probability Simulator* that allows students to simulate different probabilistic experiments using only their electronic devices (e.g., computer, tablet, smartphone).

In *Probability Simulator*, the following probabilistic experiments are available for simulation (Figure 1(a)): rolling a fair die; rolling unfair dice; spinning a number wheel; and drawing 1 to 5 cards from a standard deck. In addition, multi-stage probabilistic experiments can also be simulated by setting up a sequence of different tasks. The student can then specify the number of times that the task(s) must be repeated.



(a) (b) (b)

Figure 1. Screenshots from *Probability Simulator*: (a) task selection, (b) animated depictions of tasks simulation, (c) outcomes window.

When the student taps/clicks the Simulate Tasks button, a short animation (Figure 1(b)) is played to depict the simulation of the selected tasks. After the animation, the outcomes of the tasks are displayed in a scrollable window (Figure 1(c)). If the number of trials is too many, the scrollable window may become inconvenient for viewing all the outcomes; thus, there is also an option to save the results in a spreadsheet file (csv) that can then be processed or analyzed further using other software. In the following section, we will illustrate how *Probability Simulator* can be used for learning activities across different grade levels.

Another tool that we have developed is the web-based platform *Senso Eskwela Pilipinas* (*SEP*) (De Las Peñas et al., 2020; Tolentino et al., 2022). In *SEP*, students themselves become part of the data collection process by answering the *SEP* survey and having their anonymous responses become part of the *SEP* database. In turn, the database becomes a source of readily available, authentic, and relatable data sets. Based on initial integration into two Grade 11 classes, *SEP* has shown positive benefits in students' learning (Tolentino et al., 2022).

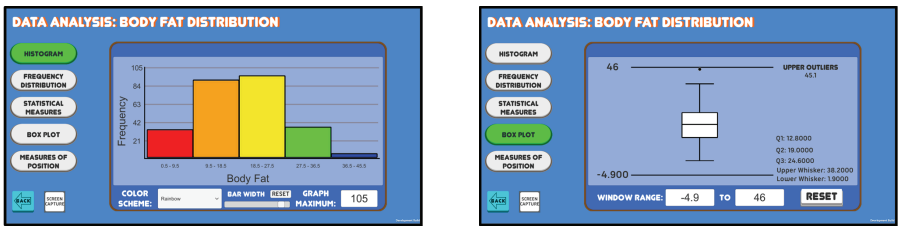
3.2 *Tools for Data Analysis*

In real-world contexts, analyzing different data sets, especially large ones, is usually done using different statistics software. Thus, it is also important to give students opportunities to perform different data analysis activities using technology.

Focusing on competencies for Grades 7, 8, and 10, we have developed a data analysis tool subject to two important considerations. First, we want the tool to focus only on selected functionalities that are relevant to the aforementioned grade levels. This is to make the tool relatively simple and easy to use so that students and teachers are not overwhelmed with many functionalities and complicated commands. Secondly, we want to maximize the accessibility of the tool; that is, it must be freely available, uses relatively low storage, and can run offline in a wide range of low-cost electronic devices.

Given the above considerations, we have developed the app *Mathplus Statistics*, a tool for computing descriptive statistics and generating data visualizations, which can be accessed via Windows or Android. Its functionalities include the generation of histograms (Figure 2(a)), frequency distribution tables, and box plots (Figure 2(b)), as well as the calculation of measures of central tendency and variability and measures of position.

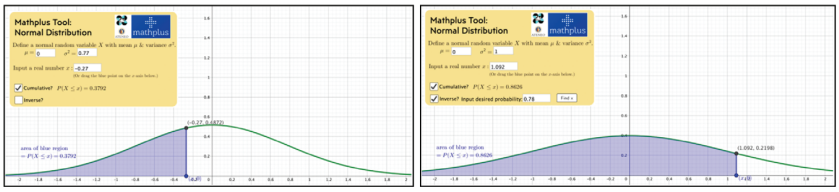
Students may upload their own data (in csv format) or use the app's built-in datasets. The app is also compatible with SEP in the sense that data sets downloaded from the *SEP* platform are already properly formatted and may be uploaded directly to the app. Data filtering is also available and can be applied to data to keep (or exclude) observations based on various variable-dependent criteria.



(a) (b)

Figure 2. Screenshots from *Mathplus Statistics*: (a) histogram, (b) box plot.

In addition to *Mathplus Statistics*, we have also developed two Grade 11 tools or applets using the free software *GeoGebra* (https://www.geogebra.org/): *Mathplus Tool for Normal Distribution* and *Mathplus Tool for t Distribution*. The applet *Mathplus Tool for Normal Distribution* allows students to compute probabilities involving the normal distribution for which the mean and variance are user-inputted (Figure 3(a)). A graph of the normal probability distribution function and the corresponding area under it are also displayed. Lastly, there is also an inverse function (Figure 3(b)) in which a student can input a desired probability then obtain the corresponding value at which the normal cumulative distribution function gives this probability. *Mathplus Tool for t Distribution* has similar functions but for the *t* distribution.



(a) (b)

Figure 3. Screenshots from the *Mathplus Tool for Normal Distribution* applet.

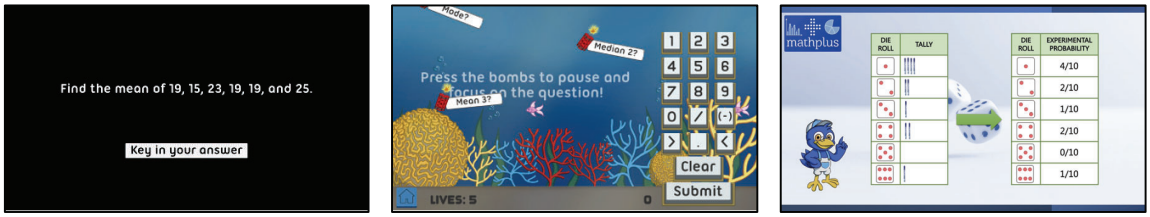
Aside from being an alternative to printed out distribution tables, the applets can also be used for strengthening students' understanding of these probability distributions. Particularly, the visualization of the graphs and probabilities-as-areas can help students master different properties of computing probabilities and of the probability distributions.

3.3 *Instructional Resources*

In addition to the data generation and analysis tools discussed above, we have also created resources that can contribute to statistics instruction. These have been made according to the list of most essential learning competencies of the Department of Education (DepEd, 2020).

One such resource is the app *Just Keep Solving 2 (JKS 2)*, a game that helps students develop skills in carrying out computations under chosen topics in probability and statistics. As the name implies, *JKS 2* is an expansion of an earlier app *Just Keep Solving!* (Garciano et al., 2023). The *Statistics* topic in *JKS 2* has three levels covering measures of central tendency, measures of variability, and the identification of correct statistical measures in data analysis and interpretation. In the game, questions (Figure 4(a)) are shown as bombs that will explode upon reaching the bottom of the screen (Figure 4(b)). Players tap on these bombs to pause their movement and see the questions they are challenged to answer. Players must answer a set number of questions before losing all lives or answer enough questions to survive within a specific time limit. Garciano et al. (2023) provides additional details of *JKS 2*.

Some videos have also been produced to complement some of the apps discussed above. The videos have been developed to include a demonstration of an app and a concise discussion of the relevant math lesson(s). The project's mascot *Banoy Bilang* plays the role of a teacher's assistant in most of the videos. A particular example is the *Experimental Probability with Banoy Bilang* video that is intended for Grade 8 students. In the video, the *Probability Simulator* app is used to review the computation of experimental probabilities (Figure 4(c)).



(a) (b) (c)

Figure 4. Screenshots from *Just Keep Solving 2*: (a) sample question, (b) main level screen; (c) screenshot from the video *Experimental Probability with Banoy Bilang.*

4. Integration of the Technological Tools

A possible obstacle in using any technological tool for statistics education is teachers' "lack of awareness and comfort with new technologies" (Chance et al., 2007, pp. 18-19). Additionally, teachers are often busy enough with their usual teaching responsibilities and other ancillary activities (Batilantes, 2021) to have sufficient time for learning or designing new activities that use technology. At the same time, students also need direction or guidance when it comes to learning activities that they can do independently.

Given the above considerations, we have also developed teachers' guides and student worksheets detailing learning activities that make use of the discussed technological tools. The teachers' guides contain complete information (e.g., competencies covered, materials needed, instructions) related to implementing concrete learning activities. On the other hand, the student worksheets contain instructions for doing the activities; the worksheets are ready-to-use and can also serve as students' answer sheets, if applicable.

In Grades 1-3, for example, one set of learning competencies under the *Probability and Statistics* strand (Department of Education, 2020) is focused on introducing and developing students' notion of uncertainty and likelihood. In the teacher's guide *Introduction to Uncertainty* and its corresponding student worksheet, we present a learning activity that makes use of the *Probability Simulator* app. This allows the students to perform probabilistic experiments more practically, compared to using physical objects (e.g., dice, cards) that may be too time-consuming. Other examples of teacher's guides and student worksheets can be accessed for free from mathplusresources.wordpress.com.

5. Conclusion and Future Direction

In this paper, we recognize the significance of statistics in students' STEM education and how the teaching and learning of statistics can be improved by using technology. We have developed a number of technological tools that can support different aspects of statistics education. The tools can be used for simulation and data generation (*Probability Simulator* app and *Senso Eskwela Pilipinas*) or for data analysis (*Mathplus Statistics* and *GeoGebra* applets for normal and *t* distributions). There are also resources (the game *Just Keep Solving 2* app and instructional videos) that can support instruction. These resources can easily be downloaded and used on low-cost mobile devices, making them accessible and suitable for the use of most students in the Philippines. We have also developed teachers' guides and student worksheets that present concrete learning activities that use the different technological tools or resources.

Among the activities for future work is Investigating the effectiveness of the tools presented in this paper in terms of increasing student engagement, interest in learning and strengthening statistical knowledge and skills. Another future activity is gathering data on usage and acceptance of the technological tools on the part of students and teachers. Moreover, other ways of integrating the use of these tools in the classroom can be further explored in collaboration with Filipino high school teachers. A greater knowledge of the

requirements for statistics education for both students and teachers will result from such collaboration.

Acknowledgements

This paper is one of the outputs of the Ateneo Mathplus Resources Laboratory housed at the Department of Mathematics, School of Science and Engineering, Ateneo de Manila University (ADMU). The authors thank the Department of Science and Technology-Philippine Council for Industry, Energy, and Emerging Technology Research and Development and the University Research Council, ADMU for supporting the development of the resources for Grades 1 to 11 Mathematics. We thank our app developers Logik, Inc, Mr. Victor Antonio M. Ortega, Mr. Nigel Benedict Cargo, Mr. Jose Teodoro Lacson, and our artist Mr. Amiel Damian F. Justiniani.

References

Adams, C. (2017). *The 7 most important STEM skills we should be teaching our kids*. WeAreTeachers. https://www.weareteachers.com/important-stem-skills-teaching-kids/

Bargagliotti, A., Franklin, C., Arnold, P., & Gould, R. (2020). *Pre-K-12 guidelines for assessment and instruction in statistics education II*. American Statistical Association.

Batilantes, S. P. (2021). Project VLOGI (Video Lectures on Giving Instructions): Effects on learners' performance in Probability and Statistics. *International Journal of Educational Studies in Mathematics*, *8*(4), 299-315.

Ben-Zvi, D., & Garfield, J. (2008). Introducing the emerging discipline of statistics education. *School Science and Mathematics*, *108*(8), 355–361.

Chance, B., Ben-Zvi, D., & Garfield, J., & Medina, E. (2007). The role of technology in improving student learning of statistics. *Technology Innovations in Statistics Education, 1*(1).

De Las Peñas, M. L. A. N, Verzosa, D. M. B., Sarmiento, J. F., Tolentino, M. A. C., & Loyola, M. L. (2020). Designing mobile apps to promote numeracy and statistical reasoning. In W.-C. Yang & D. Meade (Eds.), *Electronic proceedings of the 25th Asian technology conference in mathematics* (pp. 167-182). Mathematics and Technology, LLC.

Department of Education (2016). *K to 12 curriculum guide: mathematics*. www.deped.gov.ph/wp-content/uploads/2019/01/Math-CG_with-tagged-math-equipment.pdf

Department of Education. (2020). *K to 12 most essential learning competencies with corresponding CG codes*.

Garciano, A. D., Verzosa, D. M. B., De Las Peñas, M. L. A. N., Aberin, M. A. Q., Mallari, J. C. F., Sarmiento, J. F., & Tolentino, M. A. C. (2023). Practice through play using mobile technology. In P. Kommers, I. A. Sánchez, & P. Isaías (Eds.), *Proceedings of the international conferences on e-society 2023 and mobile learning 2023*. International Association for Development of the Information Society.

Moore, D. S. (1998). Statistics among the liberal arts. *Journal of the American Statistical Association*, *93*(444), 1253–1259.

Prodromou, T. (2015). Teaching statistics with technology. *Australian Mathematics Teacher*, *71*(3), 32-40.

Rossman, A. J., & Chance, B. L. (2014). Using simulation-based inference for learning introductory statistics. *Wiley Interdisciplinary Reviews. Computational Statistics, 6*(4), 211-221.

Samo, D. D., Darhim, Kartasasmita, B. (2017). Developing contextual mathematical thinking learning model to enhance higher-order thinking ability for middle school students. *International Education Studies*, *10*(12), 17-29.

Tolentino, M.A.C., De Las Peñas, M.L.A.N., Aberin, M.A.Q., Garciano, A.D., Loyola, M.L., Sarmiento, J.F., Mallari, J.C.F., & Verzosa, D.M.B (2022). Engaging learners through data: Senso Eskwela Pilipinas. In W.-C. Yang, M. Majewski, D. Meade, & W.K. Ho (Eds.), *Proceedings of the 27th Asian technology conference in mathematics* (pp. 351-362). Mathematics and Technology, LLC.

Watson, J., Fitzallen, N., & Chick, H. (2020). What is the role of statistics in integrating STEM education? In J. Anderson & Y. Li (Eds.), *Integrated Approaches to STEM Education*. Springer, Cham.

Wild, C.J., Utts, J. M., & Horton, N. J. (2018). What is statistics? In D. Ben-Zvi, K. Makar, & J. Garfield (Eds.), *International handbook of research in statistics education* (pp. 5-36). Cham, Switzerland, Springer.

Technology-Based Manipulatives

Debbie Marie Y. Bautista
debbie@mathsci.math.admu.edu.ph
Mathematics Department
Ateneo de Manila University

Ma. Louise Antonette N. De las Peñas
mlp@mathsci.math.admu.edu.ph
Mathematics Department
Ateneo de Manila University

Abstract: One of the challenges confronting a Philippine mathematics teacher is the unavailability/lack of technology for use in the classroom. In this paper, we present technology-based manipulatives that provide opportunities for experimentation and discovery in the mathematics classroom as alternatives to the absence of technology. Models on Euclidean geometry, analytic geometry, trigonometry, and calculus will be discussed that can be used by students to observe and make conjectures on formulas and theorems.

1. Introduction

It is widely recognized that technology plays a major role in the teaching and learning of mathematics. Unfortunately, many public schools in the Philippines do not have access to these technology. Further, many teachers do not have experience in using computers and graphics calculators. This paper then provide ways by which teachers can innovate on readily available materials to simulate at least some of the benefits of using technology. Various technologies such as graphics calculators (GC), computer algebra systems (CAS), and dynamic geometry (DG) software each have an impact on how mathematics can be taught [5]. However, this paper shall focus on DG. According to the van Hiele Theory, the main reason traditional geometry curriculum fails is that it is presented at a higher level than those at which students are operating. The Van–Hiele Theory distinguishes between five levels of thought; the use of technology-based manipulatives is for example significant, in the levels of *ordering* (students understand interrelationships of figures) and *deduction*, (students begin to understand the significant of deduction, role of axioms, theorems and proof) [10]. The use of manipulatives based on DG hopes to benefit students in understanding mathematics to a larger extent. It has been reported that DGs provides positive impact not only on students' learning of geometric concepts [8] but on their mathematical reasoning as well [9]. In fact, throughout the activities mentioned, some possible open-ended questions are suggested to allow students to develop reasoning abilities with the aid of the visual aids. It is the aim of this paper to provide the same benefits to students, even with the absence of technology for use in the classroom. Further, DG lends itself naturally to the creation of technology-based manipulatives or instruments. These activities and manipulatives may not be new, but with the advent of DG software, it is also wise to go revisit these concrete and dynamic teaching aids.

This paper supports the *theory of active construction of knowledge* as attributed to Jean Piaget [11], where learners are actively engaged in building concepts. Students discover and gain

understanding of mathematical concepts by exploration and investigation, using technology-based manipulatives. In this paper, we also try to show that through technology, activities may be created that promote the use of multiple representations in acquiring mathematical knowledge, as suggested in [4]. This is done on the cognitive level—where for instance, graphs, geometric figures and diagrams represent abstract concepts. In the absence of technology, we model the ideas using indigenous materials that are available in the community.

To enable this work to be even more applicable to Filipino teachers, each electronic construction was done using Geogebra [15] and GRAPES [16], which are open-source dynamic geometry software. To access the files created using Geogebra, another free software Java [14], version 1.4.2 or later, should be installed in the computer. The HTML files can be viewed using Internet Explorer. To access the files created using GRAPES, the reader should first download the executable GRAPES file from [16].

2. Examples in Geometry

A. Discovering Kite Properties

An investigation involving the midpoint quadrilateral of kites is found in [3]. Further, explorations on the midpoint quadrilateral of other types of quadrilaterals were performed using dynamic geometry software. In these activities, students were led to discover properties of the midpoint quadrilateral. Some expected student conclusions are that the midpoint quadrilateral of a quadrilateral is always a parallelogram, and that the midpoint quadrilateral of a kite is always a rectangle. Students were further challenged to find sufficient conditions on a quadrilateral so that its midpoint quadrilateral is always a rectangle. These activities were done using commercial DG software. Here, we do the same investigations using Geogebra. Open ***kite.html*** [18] and ***kite2.html*** [19].

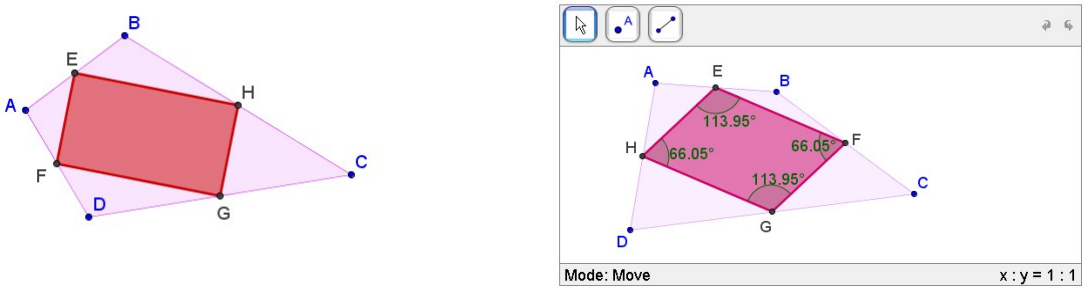


Figure 1: Midpoint Quadrilaterals (Plots produced with Geogebra, [18] and [19])

Alternatively, a simple manipulative using ordinary cardboard strips and pins can be used to demonstrate the same concepts. The idea of using strips to illustrate mathematics concepts can be seen in [1]. View the following movie files: ***kite.mov*** [20] and ***kite2.mov*** [21]. Some still shots of the manipulatives are shown below.

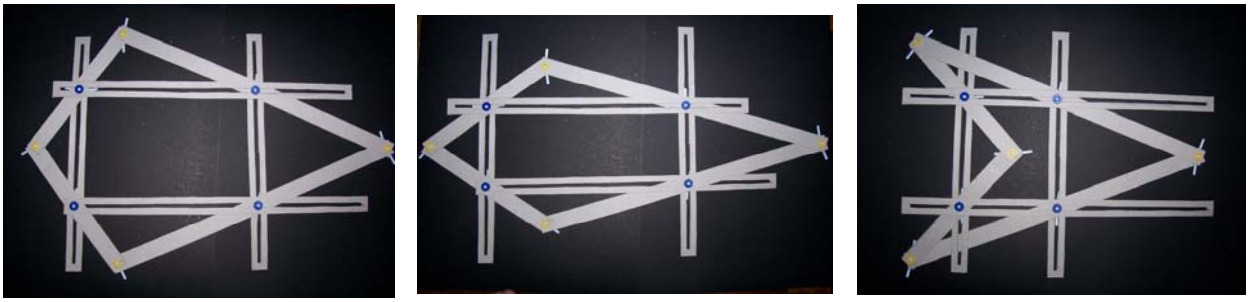


Figure 2: Illustrating the Midpoint Quadrilateral of a Kite Using Cardboard

B. Angle Measurements in a Circle

It is a well-known theorem that angles which intercept the same are are congruent, and is their measures are half the measure of the intercepted arc. This is illustrated using a Geogebra construction. Further, these files can also be used to demonstrate the Theorem of Thales: that an angle inscribed in a semicircle is a right angle. Open ***inscribed_angle_worksheet.html*** [22] and ***inscribed_angle_practice.html*** [23]**.**

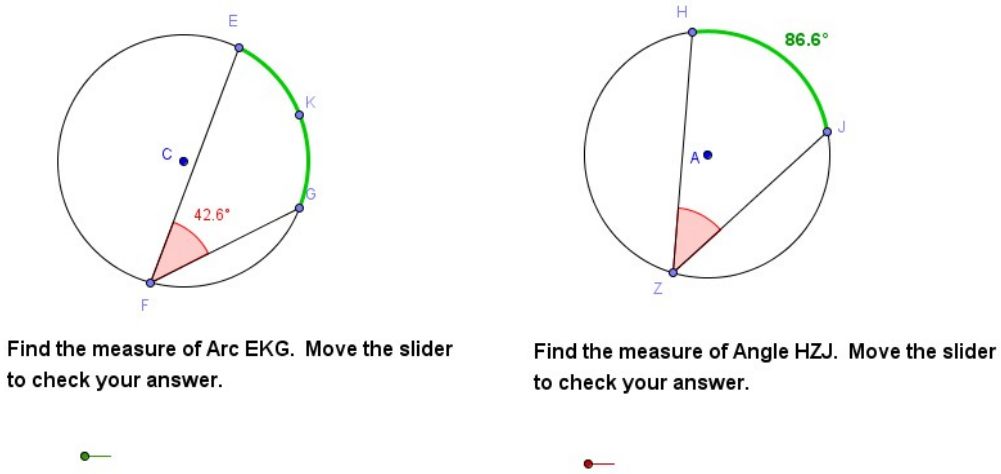


Figure 3: Angle Inscribed in an Arc, (Plots produced with Geogebra [23])

In the same way, an ordinary cardboard can be used to show that angles which intercept the same arc are congruent. View the following movie file: ***semicircle.mov*** [24]. The still shots are shown in Figure 4. Further, by attaching two more strips with endpoints at the center, one can also investigate the measure of the inscribed angle in relation to the intercepted arc.



Figure 4: Angles intercepting the same arc are congruent.

C. Loci

DG Software could easily show a locus of points. For instance, the points equidistant from two points *A* and *B* are on the perpendicular bisector of the segment *AB*. Take the example from [6] of finding a point equidistant from four towns. Students are first asked to determine a point, if any, which is equidistant from four given towns, represented in Figure 5 by *A*, *B*, *C*, and *D*. Judging from experience, students will most likely perform this task by using the drag tool to drag point *P* so that it is equidistant from the four towns. Their approach is essentially by trial and error. However, by proper questioning, students can be led to discover that *P* must lie on the perpendicular bisector of each of segments *AB*, *BC*, *CD*, and *DA*. This exploration can be done on commercial DG software, but it could also be performed using Geogebra. See ***Water1.html*** [25]. Indeed, students were also able to see that the four towns lie on a circle, and this has led to interesting discussions. From an initial brute force method, this free software package allowed students to develop reasoning, as envisioned by Jiang [9].

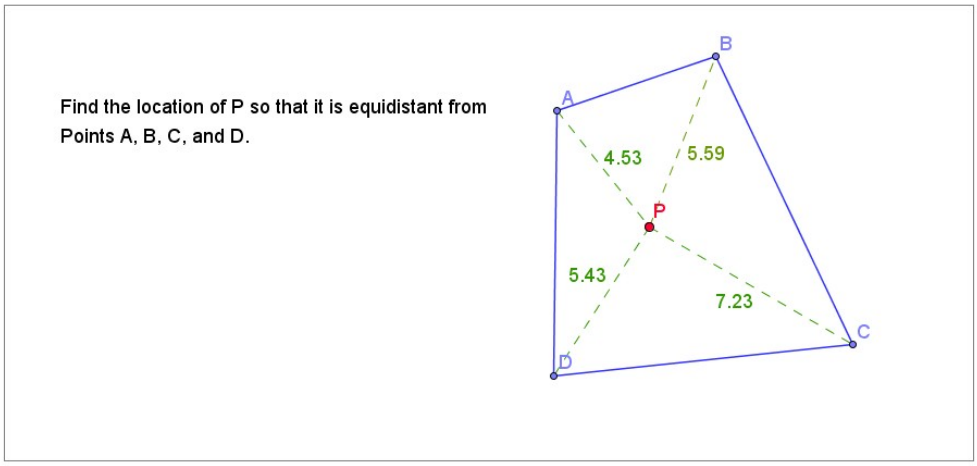


Figure 5: Finding a point equidistant from four given points (Plot produced using Geogebra, [25])

Teaching the locus of points can also be done creatively without using a computer. In [7], a treasure hunt activity was done to help students learn this topic. One given example is as follows. Points *A*, *B*, and *C* are fixed landmarks in the school yard that could represent points (like a flagpole, tree, or trashcan). Small nails, representing treasures, are hidden below the ground.

Students are then given clues to find the nails. Sample clues are "One treasure is 15 feet from *A* and 22 feet from *C*," or "One treasure is equidistant from *B* and *C*." Sometimes, as in both examples above, more than one point could satisfy the given clues and students have to check these points for the hidden treasure. This activity could be used as a motivational activity to introduce locus of points. The following activity is a modification of this Treasure Hunt, and its main problem is similar to the problem on the four towns given above. The teacher initially selects four landmarks, making sure that they all lie on a circle. Students could use tape measures to determine the point equidistant from these four landmarks. From here, students would again most likely resort to trial and error, but this activity motivates further discussion on perpendicular bisectors, thereby achieving a benefit from DG. This activity is well-received by students who enjoy doing activities outside the classroom.

3. Examples in Analytic Geometry

A. Parabola

The parabola is defined to be the set of all points equidistant from a fixed point, called the focus, and a fixed line, called the directrix. A Geogebra file can facilitate visualization and analysis of the locus of a parabola. See the following files: ***parabola1.html*** [26], ***parabola2.html*** [27], and ***parabola3.html*** [28].

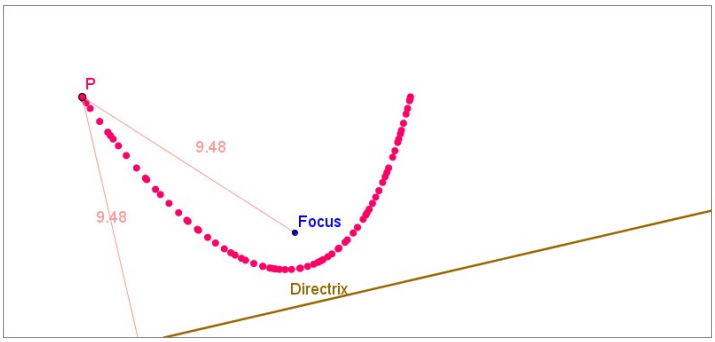
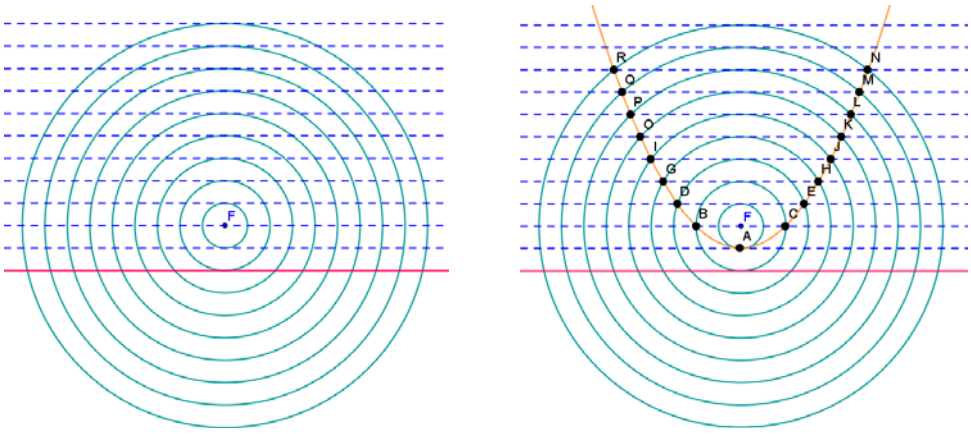


Figure 6: Screenshot of a Parabola (Plot produced with Geogebra [27])

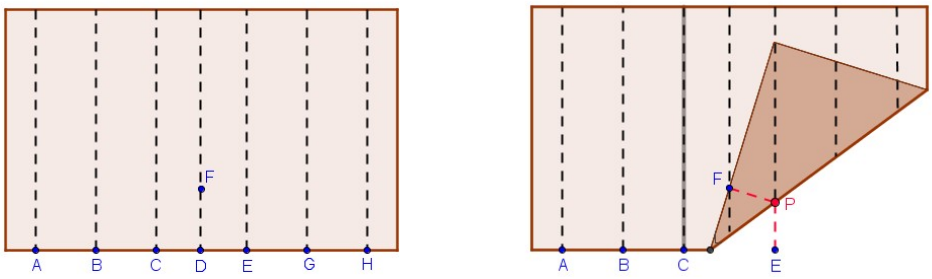
One can also illustrate this definition even when a computer is unavailable. One can use concentric circles and parallel lines drawn on paper, as shown in Figure 7a. The red line is the directrix, and the focus *F* is at the common center of each circle. It is then easy to construct points equidistant from *F* and the directrix, as seen in Figure 7b. This idea was taken from [12].



(a) (b)

Figure 7: Using concentric circles to investigate the parabola (Image Produced with Geogebra)

Another way to demonstrate the definition of the parabola even without a computer is by paper-folding. Indeed, many concepts in mathematics can be illustrated by folding paper. As in Figure 8a, one can start with a piece of pad paper with parallel segments, and a designated point *F* to represent the focus. A sheet of pad paper would work well for this activity. The base of the paper can be considered to be the directrix. Fold the paper so that the bottom of each parallel segment (*A*, *B*, etc.) meets *F*. One such fold is demonstrated in Figure 8b. The intersection *P* between the dotted line and the crease made would be equidistant from the base of the paper and the focus. Hence, it is a point on the parabola.



(a) (b)

Figure 8: Using paper-folding to generate a parabola (Image Produced with Geogebra)

Much like the DG software, this paper-folding activity can allow students to visualize the definition of the parabola, which they might initially find abstract. Further, when students do this activity, they develop the sense that each point on the parabola satisfies certain conditions.

The reader interested in using concentric circles or paper-folding to investigate the ellipse and hyperbola could refer to [12].

B. Translations

One useful benefit of DG Software is that it allows the user to see effects of changing parameters in an expression. Because of this, one can use DG software to illustrate translations of graphs. For an example, see the GRAPES file ***translations.gps*** [29]. Another example would be an investigation on how changing the graph of a function changes the graph of its derivative. For reference, open the GRAPES file ***derivatives.gps*** [30]. Screen shots are shown in Figure 9.

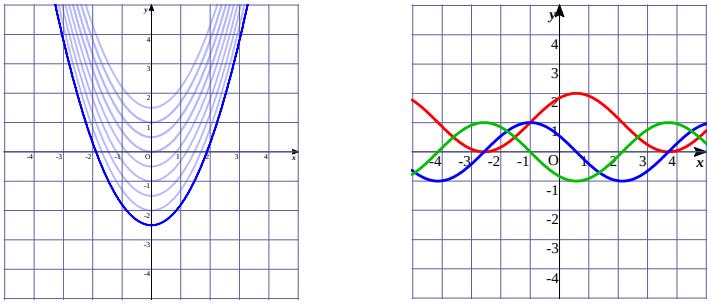


Figure 9: Translations and Derivatives (Plots produced with GRAPES [29], [30])

Unfortunately, only translations can be replicated if a computer is not available. One can use an overhead projector, and two transparency sheets to illustrate motion. One slide contains a fixed grid, while another slide contains a moving graph. See Figure 10.

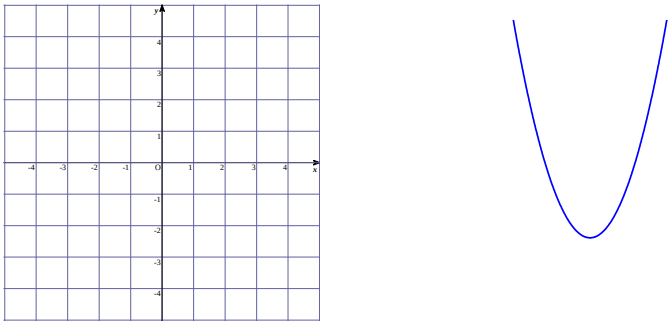


Figure 10: A fixed grid on the left, and a graph which may be moved horizontally or vertically over the fixed grid to illustrate translations. (Plots produced with GRAPES)

One other idea is by using Aerobigraphs [13]. Here, students themselves illustrate transformations of graphs using bodily gestures. This is especially applicable to Filipino students, who are known to enjoy action-based activities, such as dancing. This method also has the advantage of being able to show graphical transformations other than translations.

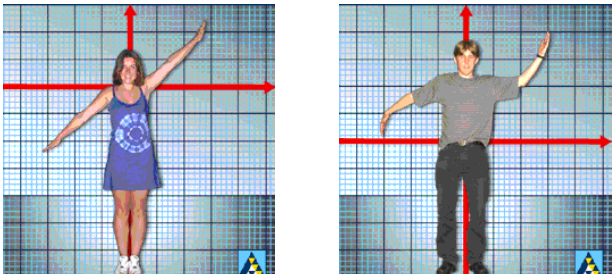


Figure 11: Using bodily gestures to demonstrate graphical transformations. (Image taken from [13])

4. Examples in Trigonometry

A. The Unit Circle and the Sine and Cosine Ratios

In this section, some concepts in trigonometry are presented using DG Software. Toward the end, an alternative that does not need any computer is demonstrated.

The unit circle is often used to illustrate the six trigonometric values. This convenient choice is based on the fact that the trigonometric ratios remain the same, even when circles with larger or smaller radii are used. Geogebra can illustrate this concept. See ***UnitCircle.html*** [31].

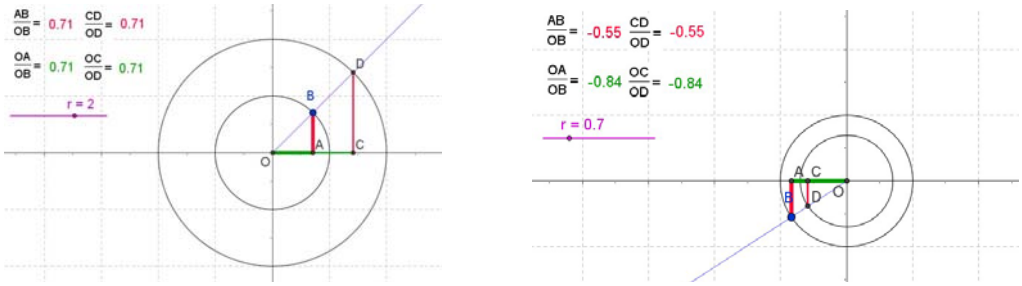


Figure 12: The Sine and Cosine Ratios Taken from Circles with Different Radii (Plot produced using Geogebra [31])

B. A Geometric Interpretation of the Trigonometric Functions

As a second example, one may look at [14], which shows an applet that simultaneously shows the six trigonometric functions in relation to the unit circle. A similar applet, showing four of the trigonometric functions, was created using Geogebra. Open ***FourTrigFunctions.html*** [32].

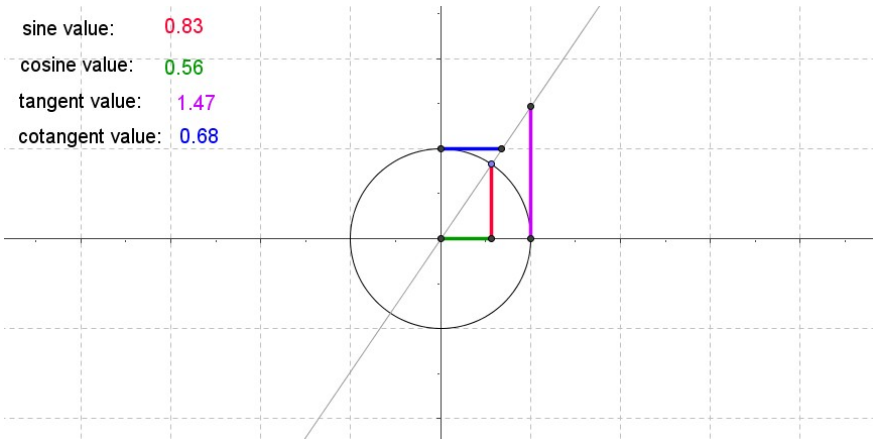


Figure 13: Four Trigonometric Functions (Plot produced using Geogebra [32])

A similar idea is shown in [2]. This article was converted into a dynamic webpage using Geogebra. Open ***SixTrigFunctions.html*** [33].

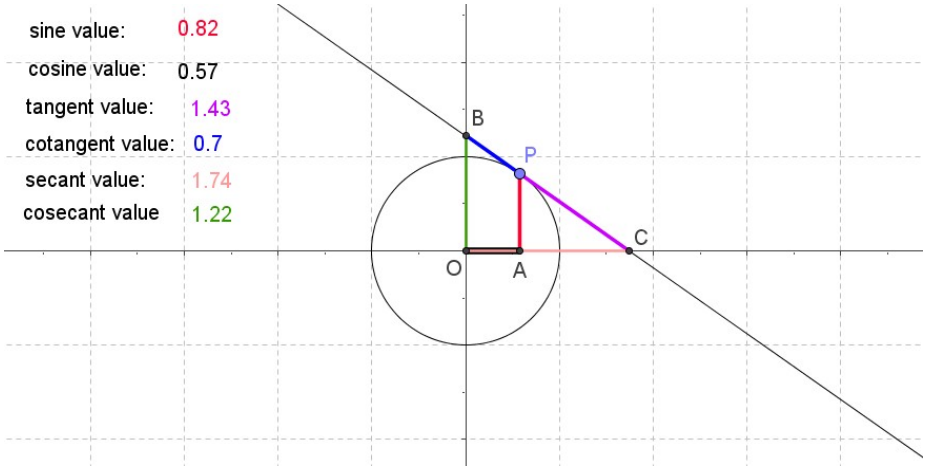


Figure 14: Six Trigonometric Functions (Plot produced using Geogebra [33])

Alternatively, one can access Casio's ClassPad demonstration [34] for a dynamic illustration of the trigonometric functions.

At first, it may seem that simulating the applets from these examples without a computer is impossible. However, these dynamic constructions can be replicated by using a transparency containing a grid containing only the unit circle (see Figure 15), and a barbecue stick or thin chopstick.

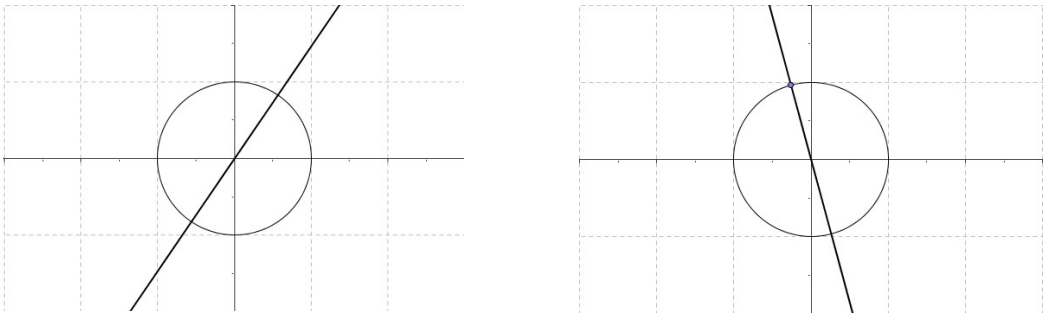


Figure 15: Using a transparency and a thin stick to reconstruct DG applet (Image produced using Geogebra)

By rotating the barbecue stick about the origin, one can already simulate what the applet does. To show that the trigonometric ratios remain unchanged if the points were taken from circles with different radii from the unit circle, the teacher has to prepare two additional grids: one containing a circle with radius larger than one, and another containing a circle with radius less than one. The teacher then rotates the stick around each circle and manually computes trigonometric values. Students would see that the ratios remain the same, even for different circles. This is similar to the activity given in Part A. The teacher could also use ordinary water-based pens to show the six trigonometric functions as shown in Part B.

Of course, the numerical values shown in the applet will not change dynamically. However, like DG software, this method could still allow the teacher to pose questions that foster critical thinking in students: Over what quadrants is the sine or cosine value positive? When is the tangent

value undefined? What are the maximum and minimum values, if any, for each of the trigonometric functions? These questions can be answered, discussed, and visualized using this manipulative.

5. Examples in Calculus

A. The Derivative

The derivative is one important concept in Calculus. GRAPES can illustrate that the slope of the tangent line may vary as a point moves along the curve. Open ***tangentlines.gps*** [35].

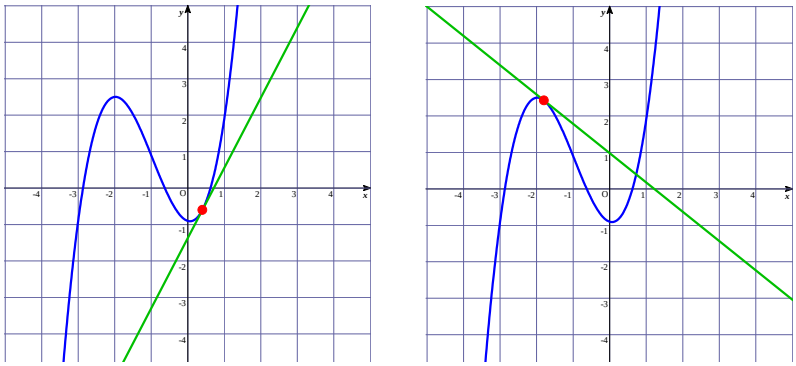


Figure 16: Tangent Lines at Different Points of a Cubic Function (Plot produced using GRAPES [35]).

This software could also illustrate how a function can fail to have a derivative at a certain point as shown in ***nondiff.gps*** [36].

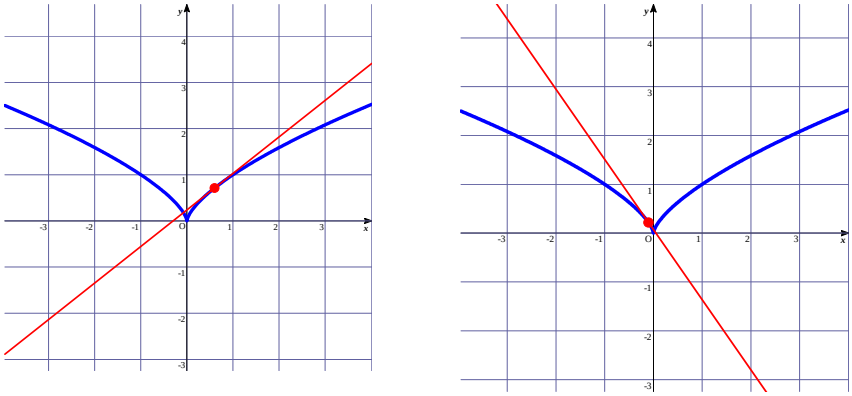


Figure 17: Non-differentiability at a Point (Plot produced using GRAPES [36]).

However, these effects can be shown even without DG software. One can use a transparency sheet and move a thin stick over it to show the tangent lines dynamically. Although this method has a disadvantage of not being able to show numerical slopes dynamically, students are still able to see clearly that its value *changes*. Again, students can still benefit from the usual questions afforded by DG software: At which point is the tangent line horizontal? Are there

tangent lines that pass through the origin? What is the limit of the derivative from the left and from the right at the origin?

B. The Fundamental Theorem of Calculus

The First Fundamental Theorem of Calculus is not easily grasped by students: From experience, it is very hard for students taking up calculus for the first time to understand the equation

$$\frac{d}{dx}\int_a^x f(t)dt = f(x).$$

Fortunately, DG software can show this theorem visually. Open the file ***function1.html*** [37]**.** By using the Previous and Next links in this *html* file, five other files may also be accessed, totaling six. Some screenshots are shown below.

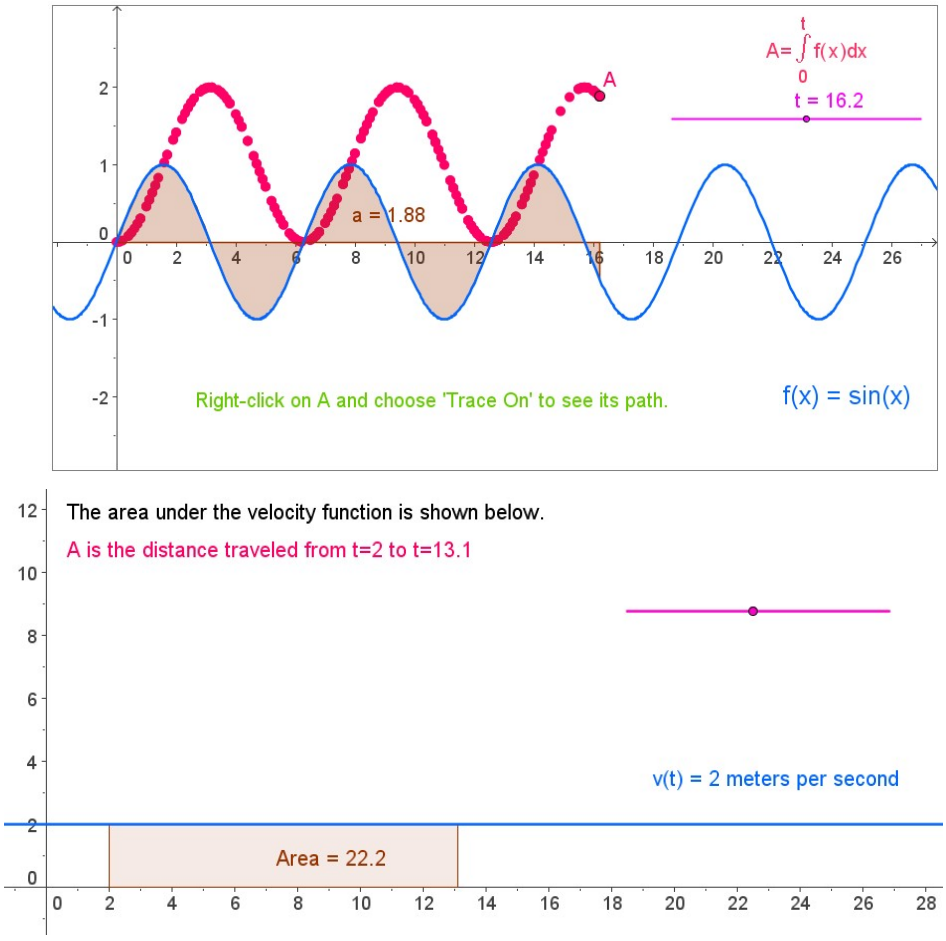


Figure 18: The Fundamental Theorem of Calculus (Plot produced using Geogebra [37]).

The first three applets illustrate the important idea behind the Fundamental Theorem of Calculus. Suppose that *f* is a continuous function of *x*, and one attempts to find the definite integral

of *f* over the interval $[a, x]$. Next, the values of this definite integral $\int_a^x f(t)dt$ are taken as functions of *x* as well. This is a complicated idea and the use of DG helps students visualize this concept. By using the slider and the trace tool, students can see that the path formed by the values of $\int_a^x f(t)dt$ consists precisely of values of the definite integral, and that this path turns out to be an antiderivative of the original function! These types of activities then allow students to see the important relationship between the definite integral and antiderivatives.

The last three applets meanwhile are examples of constructions that allow students to see that the area under the nonnegative velocity graphs given represents the distance traveled. A teaching suggestion would be to ask students to find the product of velocity and time during different time intervals, given the velocity curve. They should then be guided to discover that the products they computed can geometrically be represented by the rectangles in the construction. By doing so, students would see that in computing the displacement, one can use the strategy of finding the area under the velocity curve.

To illustrate this idea without using a computer, one may use transparencies again. The teacher prepares a transparency that contains the graph of a function *f*, and the graph of $\int_a^x f(t)dt$ underneath it (refer to Figure 19a). Initially, these two graphs are covered using a long sheet of paper so that students cannot see them. Next, in Figure 19b, parts of the graph are slowly revealed to the students as the teacher slides the long sheet of paper to the right.. It must then be explained that the graph below represents the area under *f* starting at $t = 0$. The paper is continuously moved slowly to the right. Students could see that as long as *f* remains above the *x*-axis, then the graph of $\int_a^x f(t)dt$ continues to increase. It can also be observed that the area starts to decrease once the graph of *f* falls below the *x*-axis. Finally, when the whole graph is revealed to students, it could be seen that *f* is the derivative of the area function shown underneath. This technique could be used to help students *discover* the theorem, rather than just *listen* to it being told to them.

These activities not only help students visualize concepts but these also help create in the students a positive disposition towards mathematics. Through these activities, students see mathematics as something that makes sense, and not just a set of formulas to read and memorize. For instance, students are able to sketch the graph of $\int_a^x f(t)dt$ if they are given the graph of *f*. They are also able to observe that the derivative of the graph of $\int_a^x f(t)dt$ (which they sketched) is the same as that of *f*. This is evidence that they understand the Fundamental Theorem.

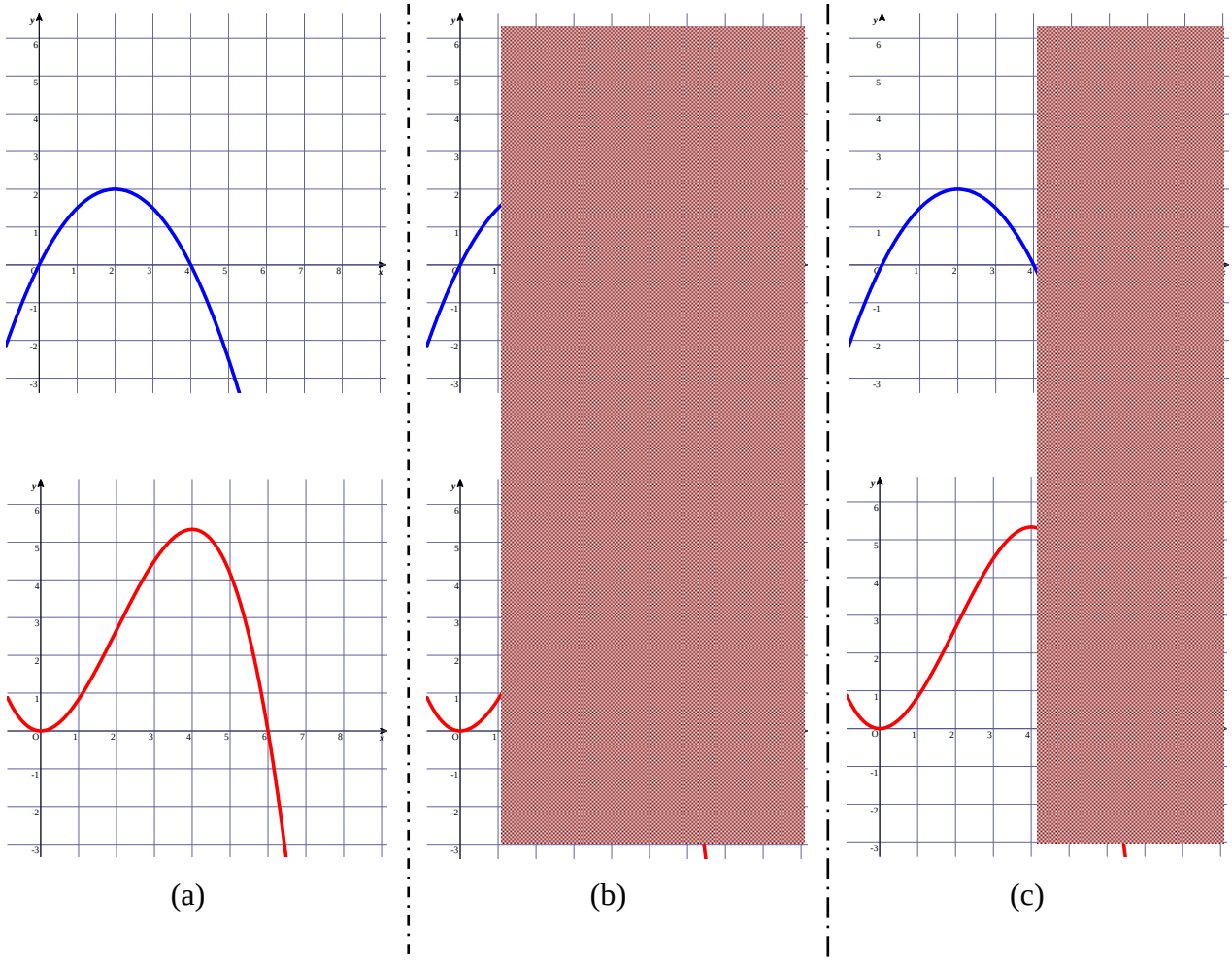


Figure 19: The Fundamental Theorem of Calculus using a transparency (Image produced using GRAPES).

6. Conclusions and Future Work

It is widely recognized that the most schools in the Philippines do not have access to computers in the classrooms. However, this should not stop teachers from thinking of innovative ways to bring about the benefits of DG software to their students. Students may further be challenged to invent and construct their own manipulative and visual aids. This paper is not to be taken as a solution to all possible problems. Rather, it provides several examples so that teachers may use these as a springboard for more ideas of their own. Teachers might even utilize a mixture of both DG software and technology-basaed manipulatives, if these are available. It is hoped that future work would be done to come up with more teaching innovations that would benefit students the same way that DG can. The important thing is that teachers think of creative ways to use available resources, and to use these techniques to pose questions that would foster discovery and critical thinking. Indeed, one could use countless manipulatives and visual aids. However, if used solely to help students visualize, there is a danger that this would encourage even more passive learning. Thus, while teachers are encouraged to use these technology-based manipulatives, they are also urged to use these techniques as motivation for more meaningful class and group

discussions. It is further recommended that teachers use action research to study effectiveness of using these types of activities in their own classrooms.

References

[1] Akiyama, J., Sakai, Toshinori, Torigoe, N., and Watanabe, Y. (2000) Nonstandard Ways of Teaching Standard Mathematics II. *Teaching and Learning Aids and Materials (Hands-on) in Mathematics Education (Topic Study Group 5), of the Ninth International Congress on Mathematical Education* , Tokyo / Makuhari, Japan.

[2] Bansangue, M. (1993) A Geometric Approach to the Six Trigonometric Ratios. *Mathematics Teacher, 86(6),* p. 496.

[3] Bennet, D. (1993) *Exploring Geometry with The Geometer's Sketchpad.* Berkeley, CA: Key Curriculum Press.

[4] De Jong, T., Ainsworth, S. Dobson, M., Van der Hulst, A., Levonen, J., Reimann, P., Sime, J., Van Someren, M., Spada, H., and Swaak, J. (1998). Acquiring Knowledge in Science and Mathematics: The Use of Multiple Representations in Technology-Based Learning Environments. In Van Someren, M., Reimann, P., Boshuizen, H., and De Jong, T. (Eds.) *Learning with Multiple Representations (Advances in Learning and Instruction.* Oxford: Elsevier Science, Ltd.

[5] De Las Penas, M., and Yang, W.C. (2005) The Use and Influence of Technology in Mathematics Education. *Proceedings of the Tenth Asian Technology Conference in Mathematics*, Cheong-Ju, South Korea.

[6] De Villiers, M. (1999). *Rethinking Proof with the Geometer's Sketchpad*. CA: Key Curriculum Press.

[7] Hayek, L. (1993) Using a Treasure Hunt to Teach Locus of Points. *Mathematics Teacher, 86(2),* pp. 133-134.

[8] Hannafin, R., Burruss, J., Little, C. (2001). Learning with dynamic geometry programs: perspectives of teachers and learners. *The Journal of Educational Research, 94,* pp. 132-144.

[9] Jiang, Z. (2005). Using the Dynamic Geometry Software to Enhance Students' Mathematical Reasoning and Proof Abilities. *Proceedings of the Tenth Asian Technology Conference in Mathematics*, Cheong-Ju, South Korea.

[10] Pegg, J. (1985). How Children Learn Geometry: The Van Hiele Theory. *Australian Mathematics Teacher 41(2),* pp. 5-8.

[11] Piaget, Jean. (1950). *The Psychology of Intelligence.* New York: Routledge.

[12] Posamentier, A. and Hauptman, H. (2006) *101 Great Ideas for Introducing Key Concepts in Mathematics: A Resource for Secondary School Teachers.* CA: Corwin Press.

[13] Aerobigraphs, created by Dye, B. URL: http://www.mathsnet.net/aerobics/aerobics.html.

[14] Six Trigonometric Functions. URL: http://www.ies.co.jp/math/java/trig/sixtrigfn/sixtrigfn.html

Software Packages

[15] Geogebra, developed by Markus Hohenwarter, 2001. http://www.geogebra.org/cms/

[16] GRAPES, developed by Tomoda Katsuhisa, http://www.criced.tsukuba.ac.jp/grapes/volume.html

[17] Java, a product of Sun Microsystems. http://java.com/en/

Supplemental Electronic Material

[18] Bautista, D. *HTML file created using Geogebra, illustrating the midpoint quadrilateral of a kite*, 2007.
kite.html
[19] Bautista, D. *HTML file created using Geogebra, providing a guided investigation on the conditions whereby the midpoint quadrilateral of a quadrilateral is a kite*, 2007.
kite2.html
[20] Bautista, D. *Movie file on using cardboard to illustrate the midpoint quadrilateral of a kite*, 2006.
kite.mov
[21] Bautista, D. *Movie file on using cardboard to illustrate the midpoint quadrilateral of a concave kite*, 2006.
kite2.mov
[22] Barmoha, G. *HTML file created using Geogebra, illustrating the measurement of an inscribed angle within a circle.* Also accessible (February, 25, 2007) through http://www.geogebra.org/en/upload/files/english/Guy/Circles_and_angles/Inscribed_angle.html
inscribed_angle_worksheet.html
[23] Barmoha, G. *HTML file created using Geogebra, a worksheet on the measurement of an inscribed angle within a circle.* Also accessible (February 25, 2007) through http://www.geogebra.org/en/upload/files/english/Guy/Circles_and_angles/Inscribed_angle_practice.html.
Inscribed_angle_practice.html
[24] Bautista, D. *Movie file on using cardboard to illustrate the measurement of an angle inscribed in an arc*, 2006.
semicircle.mov
[25] Bautista, D. *HTML file created using Geogebra, on a point equidistant from four given points*, 2007.
Water1.html
[26] Barmoha, G. *HTML file created using Geogebra which investigates the locus of a parabola.* Previously accessed through http://www.geogebra.org/en/wiki/index.php/Main_Page.
parabola1.html
[27] Bautista, D. *HTML file created using Geogebra, illustrating the locus of a parabola*, 2007.
parabola2.html
[28] Bautista, D. *HTML file created using Geogebra, illustrating the locus of a parabola*, 2007. Based on the GRAPES file available (February 25, 2007) on http://www.criced.tsukuba.ac.jp/grapes.
parabola3.html
[29] Bautista, D. *GRAPES file illustrating the effect of changing parameters on a graph*, 2006.
translations.gps
[30] Bautista, D. *GRAPES file illustrating the effect on a function's derivative when changing parameters*, 2006.
derivatives.gps

[31] Bautista, D. *HTML file created using Geogebra, illustrating the sine and cosine ratios in different circles*, 2007.
UnitCircle.html
[32] Bautista, D. *HTML file created using Geogebra, illustrating four of the trigonometric functions*, 2007.
FourTrigFunctions.html
[33] Bautista, D. *HTML file created using Geogebra, illustrating the six trigonometric functions*, 2007.
SixTrigFunctions.html
[34] Yang, W.C. and Lewin, J. *Casio ClassPad dynamic demonstration of the definition of cosine and sine.*
http://mathandtech.org/CASIO/unitcircle/unit-circle.html
[35] Bautista, D. *GRAPES file illustrating the tangent line at different points of a cubic function*, 2007.
tangentlines.gps
[36] Bautista, D. *GRAPES file illustrating non-differentiability at a point*, 2007.
nondiff.gps
[37] Bautista, D. *Geogebra file illustrating the First Fundamental Theorem of Calculus*, 2007.
function1.html, function2.html, function3.html, function4.html, function5.html, function6.html

ResearchGate

See discussions, stats, and author profiles for this publication at: https://www.researchgate.net/publication/355181294

USING MOBILE TECHNOLOGY TO PROMOTE HIGHER- ORDER THINKING SKILLS IN ELEMENTARY MATHEMATICS

Conference Paper · March 2021

CITATIONS
6

READS
267

6 authors, including:



Ma. Louise Antonette Navarro De Las Peñas
Ateneo de Manila University
79 PUBLICATIONS **217** CITATIONS
SEE PROFILE



Jumela Sarmiento
Ateneo de Manila University
9 PUBLICATIONS **51** CITATIONS
SEE PROFILE



Mark Anthony Cayanan Tolentino
Ateneo de Manila University
23 PUBLICATIONS **36** CITATIONS
SEE PROFILE



Alva Aberin
Ateneo de Manila University
20 PUBLICATIONS **18** CITATIONS
SEE PROFILE

All content following this page was uploaded by Ma. Louise Antonette Navarro De Las Peñas on 12 October 2021.

The user has requested enhancement of the downloaded file.

USING MOBILE TECHNOLOGY TO PROMOTE HIGHER-ORDER THINKING SKILLS IN ELEMENTARY MATHEMATICS

Debbie Marie B. Verzosa
Department of Mathematics and Statistics
University of Southern Mindanao, Philippines

Ma. Louise Antonette N. De Las Peñas
Department of Mathematics
Ateneo de Manila University, Philippines

Jumela F. Sarmiento
Department of Mathematics
Ateneo de Manila University, Philippines

Maria Alva Q. Aberin
Department of Mathematics
Ateneo de Manila University, Philippines

Mark Anthony C. Tolentino
Department of Mathematics
Ateneo de Manila University, Philippines

Mark L. Loyola
Department of Mathematics
Ateneo de Manila University, Philippines

ABSTRACT

The problem of rote-based learning in mathematics is well documented. Mobile technology can provide a potential solution, especially when application (app) design is based on sound pedagogical principles and gamification elements. However, an inventory of available mobile apps for mathematics reveals that many of the available apps are guided by a behaviorist perspective that favors repetition over meaningful learning. This paper reports on the design of mobile mathematics apps that harness gamification techniques to promote higher-order thinking skills (HOTS) even in basic elementary school concepts such as number comparison, and addition and subtraction. The integration of these apps in the classroom is also discussed.

KEYWORDS

mobile technology in teaching mathematics, mobile learning, numeracy apps, higher-order thinking skills, HOTS

1. INTRODUCTION

Despite the long history of reform education in mathematics, rote learning and low-level thinking are still prevalent in mathematics classrooms in many countries (Cox, 2015; Nag et al., 2014). Teachers struggle to teach mathematical concepts meaningfully and instead resort to explicating procedures disconnected from important elementary school concepts such as place value or number magnitude (Verzosa, 2020). Traditional

instruction focuses on teaching rules rather than on promoting critical thinking or problem solving, which are the twin goals of Philippine mathematics education (Department of Education, 2016).

Mobile technology offers a potential solution for mathematical learning, particularly if it is designed well (Calder, 2015). It can provide an additional medium for helping students visualize abstract mathematical concepts and can promote confidence and enjoyment in doing mathematics (Fabian, Topping, & Barron, 2018). A meta-analysis of research on the use of mobile technology demonstrates its potential for developing higher-order thinking skills (HOTS), independent learning, and reflective thinking (Ahmad et al., 2020). Unfortunately, a large proportion of the available apps perform the same function as practice worksheets; and are guided by a behaviorist perspective favoring repetition over meaningful learning (Highfield & Goodwin, 2013; Papadakis, Kalogiannakis, & Zaranis, 2018). This is despite the plethora of research that promotes sense-making and higher-order thinking in mathematics (Burns, 2007; Van de Walle, Karp, & Bay-Williams, 2015).

This paper reports on three numeracy apps that target a wide range of competencies in elementary school mathematics. The design of these apps was guided by the literature on mathematics education and are intended to promote higher-order thinking. The apps are designed in a game like environment. The use of games in mobile devices within lessons in primary mathematics has resulted in improved student engagement and made mathematics more enjoyable (Attard, 2018). Game-based learning is said to provide good exposure and experience in helping the learning process (El-Nasr & Smith, 2006).

These apps are a product of an ongoing government-funded project entitled "Technology Innovations for Mathematical Reasoning, Statistical Thinking and Assessment". In this project, mathematical apps also help teachers address the shift in modalities in education brought about by the pandemic. The apps serve as rich resources for both synchronous and asynchronous lessons in blended learning. Because the apps can be installed on mobile devices, these can be used by more school children and teachers who have access to cellphones and tablets, rather than computers and laptops.

2. HIGHER-ORDER THINKING SKILLS

A classic framework for analyzing student tasks is provided by Bloom's Revised Taxonomy (Anderson & Krathwohl, 2001). Remembering and understanding comprise the lowest level of thinking. These consist of tasks that require students to copy, listen, memorize, or compute. By contrast, verbs associated with the higher levels of mathematics require students to investigate, represent, predict, construct or create (Van de Walle et al., 2015). Students who demonstrate higher-order thinking do not just know how to manipulate numeric symbols. They can form their own strategies and apply their knowledge to solve non-routine tasks, or tasks that cannot be solved by a specific algorithm.

2.1 Foundational Ideas in Elementary Mathematics

Higher-order thinking requires knowledge of foundational mathematics concepts. In the elementary grades, some of the most important ideas involve number sense (Yang, Li, & Lin, 2008), which includes various components such as number magnitude (Siegler & Lortie-Forgues, 2014), the effect of operations, and conceptual place value (Ellemor-Collins & Wright, 2011). These concepts provided the pedagogical basis for the apps designed through our project.

3. NUMERACY APPS

3.1 Ordering Game

The *Ordering Game* is based on a game described by a recognized expert in mathematics education, Marilyn Burns (cited by Silva, n.d.). Burns' game involves whole numbers, but our team's mobile app extends the game to include fractions and integers (Figure 1). In this game, one or two numbers are generated at random (Figure 1a). The user must choose one number and drag it to a row. As the process repeats, the user places more and

more numbers on the row with the condition that the numbers appear in ascending order. The goal of the game is to fill-up the entire row within the allowed number of chances, indicated at the bottom part of the screen. The game becomes more difficult as it progresses because it becomes more likely that a randomly selected number cannot be correctly placed in any of the remaining slots.



(a) (b) (c)

Figure 1. Screenshots from the *Ordering Game* for (a) Whole Numbers, (b) Integers, and (c) Fractions

The game offers opportunities to develop higher-order thinking skills. First, it pushes the user to reason with numerical magnitude, which is one of the key learning competencies in elementary mathematics (Siegler & Lortie-Forgues, 2014). Numerical magnitude understanding is necessary for estimating and comparing the sizes of numbers. Second, the game provides a natural environment to learn probability concepts. For example, if the randomly generated numbers are within the range 11 to 66 and the randomly generated number is 60, then the user must consider the likelihood that the next generated numbers will be higher or less than 60. Since there is a relatively low probability that a succeeding number will be greater than 60, then it is wise to place 60 somewhere near the right end of the row.

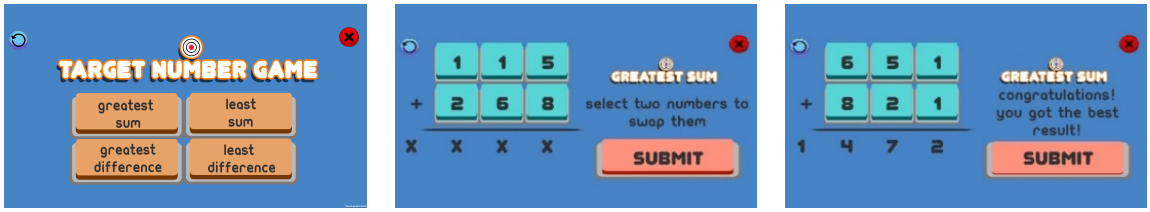
The app also presents appropriate representational media that enable users to reason mathematically. The game presents a variant of a number line estimation task, which is one of the tasks often used to measure symbolic numerical magnitude understanding (Fazio et al., 2014). In the case of fractions (Figure 1c), a regional model of each fraction is also shown, to support the visualization and comparison of fraction magnitudes.

Additional options are also available for each number type. For whole numbers (Figure 1a), users may choose among varying number range choices from 0 to 99,999. For integers (Figure 1b), users may choose to play within a number range between from -99,999 to 99,999. In the case of fractions (Figure 1c), users may choose to work with fractions with denominators from 1 to 6 or denominators from 1 to 9. In addition, users may also choose among Easy, Medium, or Challenging levels, which determine the number of chances available in each game.

3.2 Target Number Game

In the *Target Number Game,* users can choose one of the following targets: Greatest Sum, Least Sum, Greatest Difference or Least Difference (Figure 2a). Two sets of three-digit numbers are given and the goal is to rearrange the six digits, by swapping two at a time, so that the answer, either the sum or difference, shows the target. The game provides feedback on whether the user is able to get the target answer. It also gives hints so that the user can eventually reach the optimal solution.

To reach the target, users must have the knowledge of numerical magnitudes and the effect of operations. Higher-order thinking emerges from the number of considerations required to solve the problem. How do you form two three-digit numbers, from the given six digits, to form the largest or smallest possible sum? For example, to get the greatest sum in Figure 2b, the user should place the two largest digits in the hundreds place of each of the three-digit numbers. Then the next two largest digits should be in the tens places. The user can also discover that although the maximum sum is unique the two sets of three-digit numbers are not unique. In Figure 2c, another possible choice for the addends are 621 and 851. To get the least sum, the user should use the opposite strategy, that is, to place the two largest digits in the ones place, and so on.

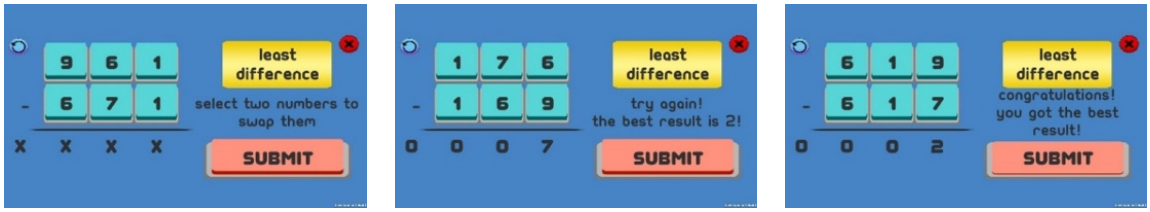


(a) (b) (c)

Figure 2. Screenshots from the *Target Number Game* (a) Main Menu, (b) Example of Greatest Sum, and (c) An Optimal Solution to an Example of Greatest Sum

Finding the greatest or least difference requires even more thinking. To find the greatest difference of two three-digit numbers formed using six digits, the user should realize that the first three-digit number must be as large as possible and the second three-digit number as small as possible. The most challenging target is finding the least difference. Unlike the other problems, there is no simple strategy such as placing the largest digits in the hundreds place or the ones place. The reason for the difficulty is that the difference must be a positive number. Thus, it is not only a matter of making the first three-digit number as small as possible and making the second three-digit number as large as possible. For example, in Figure 3a, the user must rearrange the digits to get the least difference. The user will try out different possibilities such as in Figure 3b, which will not give the target answer. Thus, the user will try out other arrangements until the target is achieved (Figure 3c). By playing the game a number of times, the user will be able to find a strategy to find the least difference.

Pilot testing revealed that even adults need to think to solve problems presented by the *Target Number Game*. But since the app involves only basic addition and subtraction, it can present the same thinking opportunities for elementary school children.



(a) (b) (c)

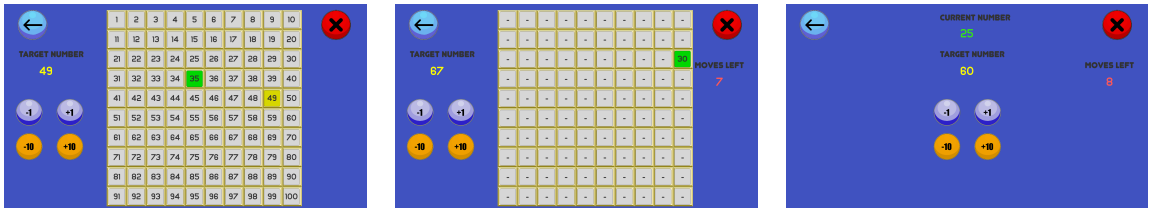
Figure 3. Screenshots of finding the least difference from the *Target Number Game*

3.3 Grid Game

In *Grid Game,* a 10 by 10 grid is shown on the screen, together with a beginning number and a target number. The user has to click the buttons (e.g., +1, -1, +10, -10) in order to reach the target number. For example, if the beginning number is 35 and the target is 49 (Figure 4a), then a possible solution is to click +10 once and +1 four times. The learning objective is to support children's structuring of multi-digit numbers by training them to increment and decrement by tens starting any number, such as knowing that 56 and 10 more is 66 (Ellemor-Collins & Wright, 2011).

There are several levels available. In the *Explore* level, any solution is accepted, as long as the target number is reached. At the more advanced levels, the user has to complete the task in the shortest or most efficient way possible, as indicated by the number of allowed moves ("Moves Left" in Figures 4b and 4c). These levels promote higher-order thinking skills by forcing the user to move from counting by ones or just pressing +1 or -1 multiple times to reach the target. An efficient strategy requires knowledge of incrementing and decrementing by 10 and then pressing +1 or -1 a few times.

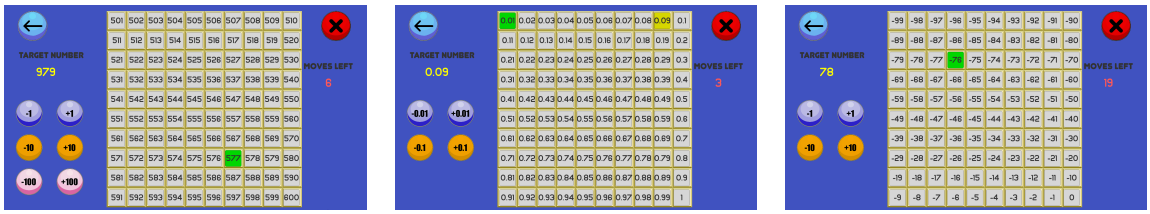
There are also variations in the type of visual support offered by the app. In the *Advanced* level, only a blank grid is shown (Figure 4b); in the *Challenging* level, no grid is shown (Figure 4c).



(a) (b) (c)

Figure 4. Screenshots from the *Grid Game* for Whole Numbers (a) Explore, (b) Advanced, and (c) Challenging levels

Because of the base-10 structure of the number system, the app can also be extended to larger whole numbers (in the range of 1-1000; Figure 5a), decimals (Figure 5b), and integers (Figure 5c). As in the previous app, the *Grid Game* involves only addition and subtraction and yet offers opportunities for higher-order thinking. For example, to get from 577 to the target 979 (Figure 5a) in 6 moves requires a correct combination of buttons +1, -1, +10, -10, +100, and -100. If the user fails to solve the problem, the app presents the same problem again so that the user always has chances to think of the most effective strategy.



(a) (b) (c)

Figure 5. Screenshots from the *Grid Game*: (a) Whole Numbers (range 1-1000), (b) Decimals, (c) Integers

4. GAMIFICATION FEATURES

Employing gamification, or the application of game design elements, techniques, and principles, in education across different subjects and different learning activities, has been prevalent in the past years. Primarily, gamification is used to motivate and engage students better, with the hope of increasing the effectiveness of learning materials, as well as to hone skills such as creative thinking and independent learning (Caponetto et al., 2014). Moreover, video and mobile games enjoy widespread popularity among the youth; for instance, in the Philippines, it is reported that there are approximately 40 million gamers with around 75% of them playing games on mobile devices (Elliot, 2020). Thus, it is opportune to use gamified environments in education to take advantage of students' familiarity with and disposition towards games.

The design of the *Ordering Game*, *Target Number Game*, and *Grid Game*, as described in the previous section, was guided by gamification principles and took advantage of game-like elements and settings to maximize their effectiveness and encourage student participation. More specifically, these three apps adhere to the Educational Games Design Model (Ibrahim & Jaafar, 2009), which consists of Game Design, Pedagogy, and Learning Content Modelling. Table 1 provides the details of the apps' Game Design.

Table 1. Game Design of *Ordering Game*, *Target Number Game*, and *Grid Game*

	Element	Implementation
Usability	Satisfaction	The vibrant color theme, large font styles, and visual designs in the three apps are catered to its target users, who are primarily grade school students. The apps feature different modes and difficulty levels for more diverse experiences.
	Efficiency	The three apps feature simple menus and interfaces. Users can immediately go into a game level after a few menu selections. Transitions between levels are quick and the controls are simple and responsive.
	Effectiveness	The three apps simulate learning activities that can help students develop their numeracy skills. Each app allows the users, with the guidance of an adult, to customize the app's topic and level to match the students' level.
Multimodal	Multimedia	The apps make use of texts, icons, figures, simple animations, and sound prompts.
	Interaction	The apps are interactive with users being able to tap, click, or drag objects on the screen directly using touch gestures, or indirectly using an input device.
Fun	Challenge	The difficulty level of the games can be adjusted so that users can start with easy levels and progress to more challenging levels on their own or as instructed by their teachers. The topics covered as well as game elements can be adjusted.
	Clear goals	Each app has a single clear goal. The user is prompted once the game is done or the goal is achieved.
	Uncertain outcome	The problem in each game is randomly generated but strategically planned. Moreover, solving each problem requires higher-order thinking. For example, in *Ordering Game*, there is some uncertainty in the sequence of numbers that will be generated, so users are potentially confronted with a new set of challenges each time.
	Self-esteem	The customization of the difficulty levels allows the user to build their self-esteem as they successfully win more difficult levels.

On Pedagogy, the previous section details how the strategy for each game requires knowledge and skills on numeracy. To summarize, the strategy for *Ordering Game* requires estimation skills and knowledge of number magnitude; for *Target Number Game*, the strategy requires knowledge of number magnitude and the effect of operations; and for *Grid Game*, the strategy requires knowledge of incrementing/decrementing by ones and tens. With the tiered difficulty levels implemented in the apps, users can develop knowledge and skills as they encounter the need to formulate new strategies for the increasingly difficult levels. Moreover, the three apps can be used independently, requiring little guidance from adults or teachers.

On Learning Content Modelling, the three apps were designed so that the math topics involved therein are aligned with some of the most essential learning competencies for Grades 1 to 6 (Department of Education, 2020) prescribed by the Philippine Department of Education. These apps can also be used as remediation or enrichment activities for students in higher grade levels. As previously described, the contents and game structure of each app were designed to promote higher-order thinking skills so that even older children or adults can be challenged by the apps.

5. INTEGRATION OF THE APPS INTO THE CLASSROOM

The nature and gamification features of the apps described in the previous sections ensure opportunities for students to engage in non-routine problems which require analysis, estimation, making predictions and conclusions, and reasoning. Such engagement promotes active learning and the prospect of developing higher-order thinking skills. There are several approaches in which the app can be integrated in the classroom to support student learning and develop higher-order thinking skills among students.

All the three apps (*Ordering Game*, *Target Game* and *Grid Game*) can be used to provide input for meaningful mathematical activities. When these are utilized, it should be directed towards (1) reasoning; (2) inference making; (3) creation of a strategy and (4) collaborative interaction with classmates. To illustrate, in

the instance of *Ordering Game* shown in Figure 1a, the teacher can ask questions such as: *Which of the numbers 56 and 65 should be used? In which position should it be placed? Why?* These questions engage students and can develop reasoning skills, form inferences and eventually devise a winning strategy.

Similarly, in the *Target Game,* the teacher can pose questions as a scaffolding technique towards stronger understanding and reasoning. In the problem shown in Figure 2b, teachers can ask: *Which two numbers can be placed in the hundreds place to get a four-digit sum? What numbers should be placed in the tens digit to get the greatest sum? How many possible answers are there?* After several examples, the teacher can lead the students to make a generalization on how to create the greatest sum from the given digits.

In the Grid game task shown in Figure 5a, teachers can ask: *Which of the buttons should be used to go from 577 to 979 within 6 moves? Is it possible to do it if the +1 button is used? Why? How about if the +10 button is used? How many solutions are possible to get the target number?*

The team has also conducted a series of webinars and training for teachers to introduce them to the apps and highlight its potential for developing higher-order thinking skills. Initial feedback is promising. Some comments indicate that the apps are "very enjoyable and motivating" and "will help students ignite their interest" through "its game like features", and that learners "will be able to enjoy, think and [be] challenged a lot". A more robust implementation will form the next phase of this ongoing project (although initial implementation has started for the apps in October 2020 in selected Department of Education School Divisions). It is anticipated that the apps, which are designed on the basis of research in mathematics education (Ellemor-Collins & Wright, 2011; Siegler & Lortie-Forgues, 2014; Van de Walle et al., 2015) and mobile technologies (Fabian, 2018; Ibrahim & Jaafar, 2009) can facilitate mathematical thinking.

6. CONCLUSION

The numeracy apps presented here address an urgent need, especially because many of the available apps focus on rote learning rather than on developing deep conceptual understanding and higher-order thinking (Papadakis, Kalogiannakis, & Zaranis, 2018). The three apps described in this paper were designed on the basis of mathematical and pedagogical frameworks, and on gamification strategies. Further, the apps are aligned with official learning competencies, while also promoting higher-order thinking skills. The apps were designed to run on mobile technologies to encourage and enhance the student learning experience. Another intention was to maximize the benefits of mobile technologies such as mobility and portability, to overcome time and space constraints in the learning environment. Beyond pedagogical and gamification designs, the role of the teacher and the educational context are crucial elements in the success of integrating the mathematical apps in learning.

The development of the mathematical apps described in this paper is funded by a government agency, and partnerships with various school divisions had been forged. The next step in the project is to study the effectiveness of the apps for student learning, and possibly how these affect the students' attitude toward mathematics, which will be conducted with partner Department of Education School divisions.

ACKNOWLEDGEMENT

The authors would like to thank the Department of Science and Technology-Philippine Council for Industry, Energy, and Emerging Technology Research and Development (DOST-PCIEERD) and the University Research Council (URC), Ateneo de Manila University for the support of the development of the interactive resources discussed in this paper. Acknowledgement also goes to Dr. Ma. Mercedes T. Rodrigo and the Ateneo Laboratory for the Learning Sciences (ALLS) for providing technical assistance. Deep gratitude is extended to our app developers: Mr. Victor Antonio M. Ortega and Mr. Nigel Cargo; and app designer Mr. Amiel Damian F. Justiniani.

REFERENCES

Ahmad, M. et al., 2020. Mobile Technology in Enhancing Students' Higher Order Thinking Skill. *Journal of Physics: Conference Series*, Vol. 1529, 042057.

Anderson, L.W. & Krathwohl, D. R. (Eds.), 2001. *A Taxonomy for Learning, Teaching, and Assessing: A Revision of Bloom's Taxonomy of Educational Objectives: Complete Edition*. Addison Wesley Longman, New York, USA.

Attard, C., 2018. Mobile Technologies in the Primary Mathematics Classroom: Engaging or Not? In N. Calder, K. Larkin, & N. Sinclair (Eds.), *Using Mobile Technologies in the Teaching and Learning of Mathematics,* Springer, pp. 51-65.

Burns, M., 2007. *About Teaching Mathematics: A K-8 resource* (3rd ed.)*.* Math Solutions Publications, Sausalito, CA.

Calder, N., 2015. Apps: Appropriate, Applicable, and Appealing? In T. Lowrie, R. Jorgensen (Zevenbergen) (Eds.), *Digital Games and Mathematics Learning*. Springer, Dordrecht, the Netherlands, pp. 233-250.

Caponetto, I. et al., 2014. Gamification and Education: a Literature Review. *Proceedings of the 8th European Conference on Games-Based Learning - ECGBL 2014*, pp 50-57.

Cox, R. D., 2015. "You've Got to Learn the Rules:" A Classroom-level Look at Low Pass Rates in Developmental Math. *Community College Review*, Vol. 43, No. 3, pp. 264-286.

Department of Education (DepEd), 2016. K to 12 Curriculum Guide: Mathematics. [Online]. Available: https://www.deped.gov.ph/wp-content/uploads/2019/01/Math-CG_with-tagged-math-equipment.pdf

Department of Education (DepEd), 2020. K to 12 Most Essential Learning Competencies with Corresponding CG Codes. [Online]. Available: https://commons.deped.gov.ph/K-to-12-MELCS-with-CG-Codes.pdf

El-Nasr, M. S. & Smith, B. K., 2006. Learning through Game Modding. *Computer Entertainment*, Vol. 4, No. 1, Art. 3B, pp. 1-20.

Ellemor-Collins, D. & Wright, R., 2011. Developing Conceptual Place Value: Instructional Design for Intensive Intervention. *Australian Journal of Learning Difficulties*, Vol. 16, No. 1, pp. 41-63.

Elliott, R., 2020. The Philippines' Games Market: Data and Insights. [Online]. Available: https://newzoo.com/insights/articles/data-and-insights-on-the-philippines-games-market/#:~:text=Share%3A,Southeast%20Asia's%20overall%20games%20market.

Fabian, K., Topping, K.J, & Barron, I.G, 2018. Using Mobile Technologies for Mathematics: Effects on Student Attitudes and Achievement. *Education Tech Research Dev*, Vol. 66, pp.1119-1139.

Fazio, L.K. et al., 2014. Relations of Different Types of Numerical Magnitude Representations to Each Other and to Mathematics Achievement. *Journal of Experimental Child Psychology,* Vol. 123, pp. 53-72.

Highfield, K. & Goodwin, K., 2013. Apps for Mathematics Learning: A Review of 'Educational' Apps from the iTunes App Store. In V. Steinle, L. Ball & C. Bardini (Eds.), *Mathematics Education: Yesterday, Today and Tomorrow (Proceedings of the 36th Annual Conference of the Mathematics Education Research Group of Australasia).* Melbourne, Australia, pp. 378-385.

Ibrahim, R. & Jaafar, A, 2009. Educational Games (EG) Design Framework: Combination of Game Design, Pedagogy and Content Modeling. *Proceedings of the 2009 International Conference on Electrical Engineering and Informatics, ICEEI 2009*, pp. 293-298.

Nag, S. et al., 2014. Literacy, Foundation Learning and Assessment in Developing Countries: Final Report. *Education Rigorous Literature Review*. Department for International Development, United Kingdom.

Papadakis, S., Kalogiannakis, M., & Zaranis, N., 2018. Educational Apps from the Android Google Play for Greek Preschoolers: A Systematic Review. *Computers and Education*, Vol. 116, pp. 139-160.

Siegler, R. S. & Lortie-Forgues, H., 2014. An Integrative Theory of Numerical Development. *Child Development Perspectives*, Vol. 8 No. 3, pp. 144-150.

Silva, M. (n.d.) Place Value Activity Package. [Online]. Available: http://www.mathematicshed.com/uploads/1/2/5/7/12572836/place-value-activity-pack.pdf

Van de Walle, J.A., Karp, K S., & Bay-Williams, J. M., 2015. *Elementary and Middle School Mathematics: Teaching Developmentally* (9th ed.)*.* Pearson, Essex, UK.

Verzosa, D. M. B., 2020. Pre-service Elementary Teachers' Reasoning in Introducing Multidigit Subtraction. *International Journal of Mathematical Education in Science and Technology,* Vol. 51, pp. 876-892.

Yang, D. C., Li, M. N., & Lin, C. I., 2008. A Study of the Performance of 5th Graders in Number Sense and its Relationship to Achievement in Mathematics. *International Journal of Science and Mathematics Education,* Vol. 6*,* pp. 789-807.

View publication stats

method will you use to find your answer?" "Show me how to get your answer and tell me what you are thinking as you do it." "Write down your answer." "Tell me how you think you could check whether your answer is correct?"

Using Word Problem Solving Prompts to support NESB students



Debbie Verzosa
Ateneo de Manilla University
The Phillipines
<dverzosa@ateneo.edu.au>

Joanne Mulligan
Macquarie University
<joanne.mulligan@mq.edu.au>

In this article Debbie Verzosa and Joanne Mulligan suggest a new approach to addressing the difficulties that NESB students encounter when solving word problems.

Many young children may find it difficult to solve mathematical word problems even if they are capable of carrying out the necessary operations required by the problem. It is common even for children whose first language is English to be impeded by the linguistic demands of a problem (Perso, 2009). Children who can untangle the linguistic complexity of a problem may still fail if they cannot apply their mathematical knowledge to the problem situation. In this article, we propose a new approach to scaffolding mathematical word problems in a way that makes them appropriate to a child's stage of language acquisition.

To investigate the sources of children's difficulties, Newman (1977; refined by White, 2005) proposed individual structured interviews that consist of five stages or prompts:

- Read the question to me.
- What is the question asking you to do?
- What method will you use to find your answer?
- Show me how to get your answer, and tell me what you are thinking as you do it.
- Write down your answer.

While the Newman interview has been used extensively across many countries for a long time (e.g., Clements & Ellerton, 1992; White, 2009), its use is limited if a child is in the early stages of learning English. This is the case for many children in Australia, such as Indigenous children in remote communities or recent immigrant children who have yet to experience schooling in English. Children learning a second language may be expected to go through five stages of language acquisition (Krashen & Terell, 1983; see Table 1), so it is crucial to give questions or prompts that are appropriate for their stage (Hill & Björk, 2008).

It is easy to see that the Newman prompts do not pose any problems for a child in the intermediate fluency or advanced fluency stages described here. However, these same prompts may not give useful information when a child is still in the first three stages of language acquisition.

A new approach

We propose a set of prompts that provide more opportunities for children in the first three stages of language acquisition to engage in mathematical word problems and rich challenging tasks. Without such prompts, these tasks are often inaccessible due to these children's language difficulties. Our approach was drawn from our work with second-grade Filipino children who learn

Table 1. Stages of language acquisition, with corresponding prompts (Krashen & Terell, 1983; Hill & Björk, 2008).

Stage	Characteristics	Appropriate prompts
Pre-production	Understands few words. Seldom speaks, mostly nods "yes" or "no", draws, or points.	Show me… Point to the… Who is the…?
Early production	Can produce one-word responses.	Yes/no questions Either/or questions Who? How many?
Speech emergence	Can produce simple sentences for daily conversation (e.g., for buying items), but makes grammatical errors for more complex sentences.	Why? How? Explain…
Intermediate fluency	Can produce longer sentences and more complex statements (e.g., for communicating thoughts or opinions).	Why do you think…? What if…?
Advanced fluency	Attains proficiency comparable to native speaking peers.	Decide if… Retell…

mathematics in English even when they had not acquired conversational fluency in the language (Verzosa & Mulligan, 2011). The same situation applies to many Australian students whose home language is not English. For Indigenous students, this language may be an Indigenous language, Aboriginal English, or a creole. For recent immigrants, it may be a major language in their country of origin. The conditions for learning available to these students are very limited, considering that they may not fully engage in a task only because they have limited knowledge of English. Teachers seeking to engage these students fully need to find ways to present learning tasks through a more accessible medium.

Of course, an obvious solution is simply to translate word problems to the child's first language. This removes language obstacles and can enable children to access the problem task. However, this scaffold alone was often found to provide minimal help, especially when children could not conceptualise the structure of the word problem. To illustrate, consider the missing addend problem:

> Jolina had 8 pencils. Then Alma gave her some more pencils. Now Jolina has 14 pencils. How many pencils did Alma give her?

The first column in Figure 1 shows how many children in our study failed to solve this problem even when it was written or narrated in their first language (Filipino). These children could not conceptualise a set as being part of another set, and some children insisted on first knowing what the missing addend was, before they could proceed with a solution. In this problem, they failed to think of 8 as part of the 14. Some of them also applied a default strategy: they solved all word problems by adding the two given numbers. One can hardly blame them, considering that they do not understand relatively simple sentences such as, "Jolina had 8 pencils." These limitations prevented them from solving a missing addend problem even when linguistic demands had been significantly eased.

It is evident that linguistic prompts are not always adequate to guarantee success. Our teaching strategy therefore provided a variety of representational tools in order to communicate mathematical tasks and concepts. We provide an illustration through representations of a missing addend task (see Table 2). We chose this task because many children in our study initially could not solve it.

The questions that we presented for each of these representations could be further modified so that they are appropriate to the child's stage of language acquisition (see Table 3).

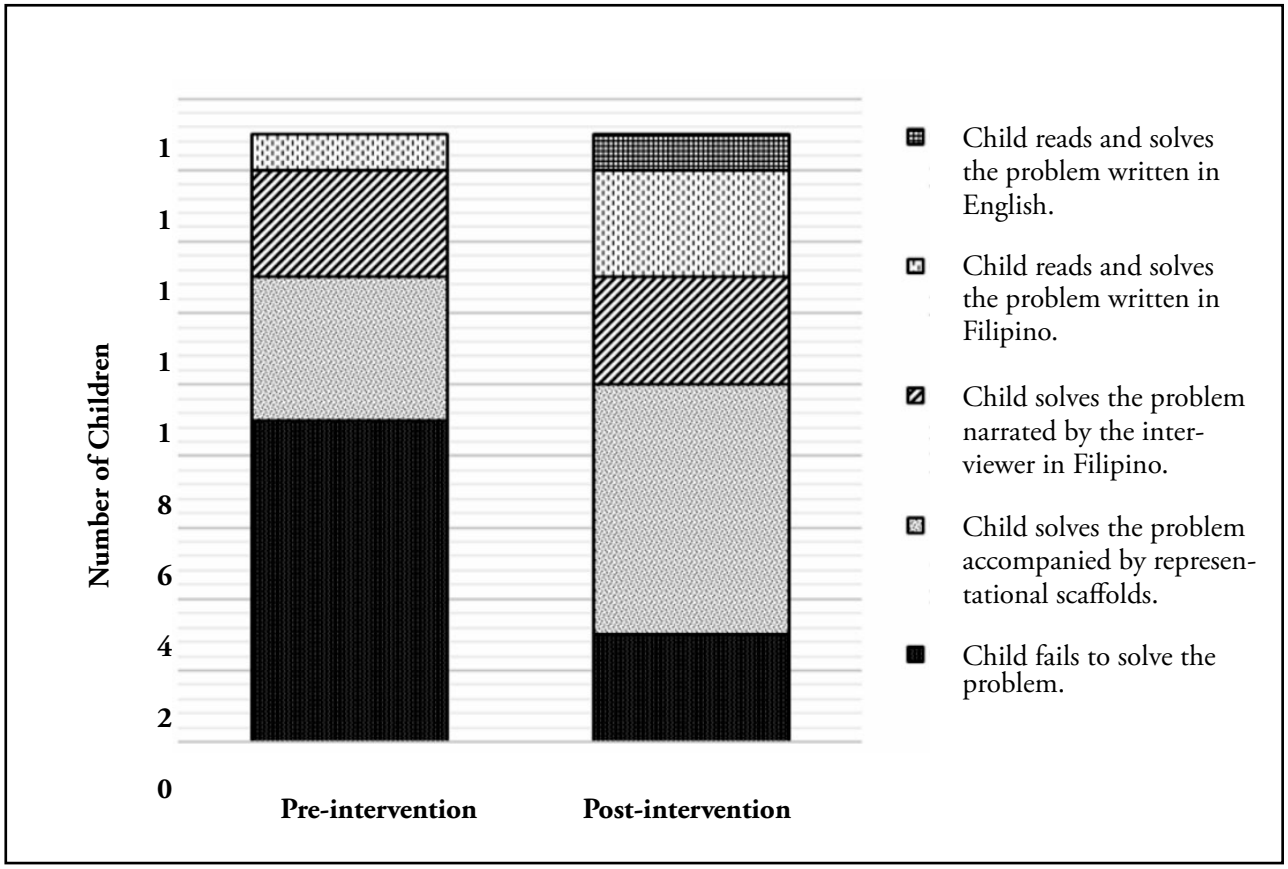


Figure 1. Enabling scaffolds for a missing addend problem.

Table 2. Various representations for the missing addend task 8 + ☐ = 14.

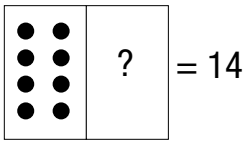
Mode of representation	Typical tasks or activities
Concrete	Screening task (Wright, Martland & Stafford, 2000): Briefly display 8 blocks. "I will join some blocks to the 8, but I will not tell you how many." Join 6 blocks to the original 8, without showing the child the number of additional blocks. "Now, there are 14 blocks altogether. How many blocks are in the bag?" [presented in Filipino]
Pictorial	 = 14
Verbal–pictorial	"Wish ko lang [I wish I had]" task (Kolson, Mole & Silva, 2006): Show 8 dots. "I have 8 dots. I wish I had 14. How many dots do I need?" [presented in Filipino]
Symbolic	8 + ☐ = 14

Table 3. Modified prompts for the first three stages of language acquisition.

Stage	Appropriate prompts
Pre-production	Draw how many dots are needed. Get the number of blocks that show your answer. Use the blocks to check your answer.
Early production	Do you think that 7 is the correct answer? How many dots are needed?
Speech emergence	Tell me how you could check whether your answer is correct.

Affordances of representational tools

Admittedly, our use of representational scaffolds and modified questions can reduce the difficulty of word problems. It also limits the opportunities for children to read and interpret questions, which are necessary components of numeracy (Perso, 2009). Nevertheless, we encourage the use of these scaffolds, at least until children reach higher stages of language acquisition. Otherwise, it will become impossible to expose them to a range of problem structures, and there may be a tendency to expect them to answer the same kind of low-level question over and over again.

We used our scaffolds during a four-week teaching intervention focused on different types of additive word problems, one of which was the missing addend problem. These scaffolds were often accompanied by a range of prompts that were appropriate for children in the first three stages of language acquisition. As mentioned earlier, some children could not solve this problem even when the problem situation had already been translated to Filipino or narrated to them (see the first column in Figure 1). This failure suggested that the source of their difficulties was not just linguistic but also mathematical. The representational tools allowed children to develop the mathematical knowledge required for conceptualising the situation described in the missing addend problem.

We now describe one teaching episode where such prompts were successful in conveying mathematical structures that children initially could not conceptualise. In this episode, the teacher presented three children the following problem in Filipino:

> Ed had 4 pieces of bread. Then he bought more bread. Now he has 9 pieces of bread. How many pieces of bread did Ed buy?

Although the children had no trouble reading the words, they did not comprehend the text fully. Even when the teacher narrated the problem to them in Filipino, they produced 9 as the answer because as one child reasoned, "Bumili siya ng nine [She bought nine]." They clung to their answer even when the teacher prompted them to draw 4 pieces of bread and a bag and to think how many pieces are needed in the bag to make 9. It was only when the teacher drew a piece of bread inside the bag and asked them if there were already 9 pieces that they began to conceptualise a set having an unknown quantity.

Prompting children to draw or use blocks often led to success, even for other types of word problems. In the problem, "There are 43 children, and 15 are girls. How many are boys?", children were prompted to first draw a representation of the 43 children, and to encircle 15 of these (see Figure 1). This strategy also discouraged children from adopting their default strategy of performing an addition. Notice that children were also encouraged to draw in groups of ten.

As representational scaffolds were frequently provided to the children during the intervention, these eventually enabled more children to solve a missing addend problem (see Figure 2). These scaffolds were critical because linguistic scaffolds alone did not always facilitate correct solutions. Further, these scaffolds, when presented using appropriate language, allowed children to think about new mathematical situations.



Figure 2. One child's drawn representation of a problem.

Conclusions

Children in all stages of language acquisition have a right to access quality mathematics teaching that differentiates for their abilities and needs. Teachers should ensure that a child's lack of familiarity in English does not prevent him or her from accessing new mathematical knowledge. In this article, we proposed a process for modifying mathematical tasks in a way that makes them appropriate to a child's stage of language acquisition. We also showed that one should not dismiss all difficulties of NESB students as stemming from language difficulties. As such, translating word problems to a child's home language does not always result in correct solutions. The challenge is how to help children conceptualise a larger range of mathematical situations while also providing prompts that are appropriate to their level of language acquisition. In this sense, representational tools can supplement oral language and convey mathematical structures that are initially beyond the learner's grasp. Thus, these representational tools can provide children with opportunities to develop their mathematical understandings independently of their linguistic difficulties.

References

Clements, M. A. & Ellerton, N. F. (1992). Over-emphasising process skills in school mathematics: Newman error analysis data from five countries. In W. Geeslin & K. Graham (Eds), *Proceedings of the 16th conference of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 145–152). Durham, NH: PME.

Hill, J. & Bjork, C. L. (2008). *Classroom instruction that works with English language learners: Facilitators guide.* Alexandria, VA: Association for Supervision and Curriculum Development.

Kolson, K., Mole, S. & Silva, M. (2006). *Dot card and ten frame activities.* Retrieved 2 October 2008 from http://www.wsd1.org/PC_math/Dot%20Card%20and%20Ten%20Frame%20Package2005.pdf

Newman, A. (1977). An analysis of sixth-grade pupils' errors on written mathematical tasks. In M. A. Clements & J. Foyster (Eds.), *Research in mathematics education in Australia, 1977* (Vol. 2, pp. 269–287). Melbourne: Swinburne College Press.

Krashen, S. D. & Terrell, T. D. *The natural approach: Language acquisition in the classroom.* Englewood Cliff, NJ: Alemany Press/Pergamon Press.

Perso, T. (2009). Cracking the NAPLAN code: Numeracy and literacy demands. *Australian Primary Mathematics Classroom, 14*(3), 14–18.

Verzosa, D. & Mulligan, J. (2013). Learning to solve addition and subtraction problems in English as an imported language. *Educational Studies in Mathematics*, 82, 223–244.

White, A. L. (2005). Active mathematics in classrooms: Finding out why children make mistakes—and then doing something to help them. *Square One, 15,* 15-19.

White, A. L. (2009). Counting on 2008: Diagnostic and remedial mathematics program for middle years students. In R. Hunter, B. Bicknell & T. Burgess (Eds), *Crossing divides (Proceedings of the 32nd annual conference of the Mathematics Education Research Group of Australasia,* (Vol. 2, pp. 579–586). Palmerston North, NZ: MERGA.

Wright, R. J., Martland, J. & Stafford, A. K. (2000). *Early numeracy: Assessment for teaching and intervention.* London: Paul Chapman Publishing.

Journal of Science and Mathematics
Education in Southeast Asia
2009, Vol. 32 No. 2, 131-160

Young Filipino Students Making Sense of Arithmetic Word Problems in English

Debbie Bautista
Macquarie University, Sydney

Joanne Mulligan
Macquarie University, Sydney

Michael Mitchelmore
Macquarie University, Sydney

Young Filipino children are expected to solve mathematical word problems in English, a task which they typically encounter only in schools. In this exploratory study, task-based interviews were conducted with seven Filipino children from a public school. The children were asked to read and solve addition and subtraction word problems in English or Filipino. Analysis focused on how language influences problem solution, and on the reading and mathematical strategies used. Results showed that children (a) were better able to comprehend and solve problems given in Filipino, (b) rarely used reading strategies, (c) were not familiar with using drawings or objects to represent word problems, and (d) employed unitary counting procedures for problems involving multi-digit numbers.

Key words: Word problems; English language learners; Reading comprehension

Introduction

The 1974 Bilingual Education Policy in the Philippines mandated the use of English as the medium of instruction in mathematics. Mathematics textbooks and materials are written in English for all grade levels. Thus, Filipino children are expected to solve arithmetic word problems in English even from their first year of schooling. This expectation poses a very challenging task, particularly because children from non-affluent families typically

encounter English only in school and may not grow up proficient in English (Gonzales, 1989; National Statistical Coordination Board [NSCB], 2007; Young, 2002).

Although supported by scant literature, Filipino children appear to find mathematical word problems difficult (Bernardo, 1999; Brawner et al., 1999). These difficulties are frequently attributed to their inability to make sense of the problem due to lack of English language proficiency. Some studies suggest that the use of Filipino facilitates word problem solution (e.g. Bernardo, 1999). The Trends in International Mathematics and Science Studies (TIMSS) National Research Coordination Office (cited by Carteciano, 2005) asserts that language barriers and comprehension problems were among the "what-else-is-new problems" facing Filipino students who are learning mathematics in a second language.

There is extensive literature on how young children solve addition and subtraction word problems (for reviews, see Fuson, 1992b; Verschaffel, Greer, & De Corte, 2007). A well-established theoretical base has informed the design and implementation of successful professional development programs aimed to apply research to classroom instruction. However, there are no such initiatives specifically focused on helping Filipino children solve arithmetic word problems. There is a pressing need for intervention since an understanding of addition and subtraction is a foundation for later mathematics achievement (Irwin & Irwin, 2005; Pearn & Stephens, 2007).

We are currently planning a project aimed at designing an intervention program focused on addition and subtraction word problems for public school children in Metropolitan Manila. Given the limited research in this area, it is important to understand children's strategies in solving word problems and factors that influence success. To this end, the first author administered a written test and conducted student interviews among young Filipino students. This paper reports on the student interviews, while the findings from the written test are reported elsewhere (Bautista, Mitchelmore, & Mulligan, 2009).

Background and Theoretical Perspective

The extensive research into children's thinking in addition and subtraction situations has led to an understanding of how addition and subtraction concepts develop. Solving word problems competently involves more than looking for key terms (such as *altogether* or *more*) and mindlessly performing

an arithmetic operation on the given numbers in the problem. Rather, problem solvers need to attend to the "structure" of the word problem, or the relationships between the given quantities. Based on the word problem structure, children must then (i) construct an internal representation of the problem, and (ii) select a solution strategy (De Corte & Verschaffel, 1991; Mayer, 2003). We base our analysis on these two phases of word problem solving.

First, in constructing an accurate representation, the child has to read and make sense of the text. Reading is not the same as decoding, or the ability to recognise words (Stothard & Hulme, 1996). Instead, reading is the process of constructing meaning from the text; it is active and strategic (Eunice Kennedy Shriver National Institute for Child Health and Human Development [NICHHD], 2000). Reading strategies such as thinking aloud, visualising, and connecting text to prior knowledge have been found to be effective for students when solving arithmetic word problems (Fogelberg et al., 2008).

Second, a solution strategy is selected based on the internal representation constructed from the child's interpretation of the problem text. Although children's internal representations are not observable, they can be inferred on the basis of their external representations such as the strategies they use to solve problems (Goldin & Shteingold, 2001). In the present study, solution strategies were analysed according to their level of abstraction (Carpenter & Moser, 1984); drawn representations were analysed according to the type of component signs present (Thomas, Mulligan, & Goldin, 2002); and strategies for tasks involving multidigit numbers were analysed according to children's multidigit conceptions (Fuson et al., 1997).

Since Filipino children solve word problems in a second language, our analysis will also draw attention to how the language of the problem influences problem solution. There is extensive literature on the influence of the language of the problem on the solution performance of students learning mathematics in a second language (e.g. Abedi, Courtney, Leon, Kao, & Azzam, 2006; Clarkson, 2007; Dale & Cuevas, 1987; Secada, 1991). Research indicates that there is improved student performance when word problems are given in the students' first language (Adetula, 1990; Bernardo, 1999). However, children's failure to solve word problems cannot be wholly attributed to the inability to comprehend what was read. In our own study of 75 Filipino schoolchildren (Bautista et al., 2009), it was found that

presenting problems in Filipino improves, but does not guarantee, solution accuracy.

With the above theoretical perspectives, we aim to investigate how young Filipino public school children solve word problems. Most empirical data on Filipino children's performance on word problem solving were obtained from large-scale written assessments (e.g. Brawner et al., 1999). We supplement findings from written tests with an exploratory study utilising individual task-based interviews typical of cognitive-based studies (Goldin, 2000). Two key research questions are investigated:

1. What problem-solving strategies do Filipino children demonstrate when solving addition and subtraction word problems in English or Filipino?
2. How do children respond when encouraged to use different modes of representation?

In common with other single or multi-case studies (Yin, 2003), the aim of this study is not to make generalised conclusions but to explore students' activity and expose problem-solving strategies. The interviews reported here, together with the results of the written test (Bautista et al., 2009), will provide a more in-depth analysis of the differences between solving English and Filipino word problems.

Method

Participants

The participants were seven students studying in a public school in one of the most densely populated and poverty-stricken areas of metropolitan Manila. Malnutrition is a persistent problem for the students. Resources are inadequate, and it is common for classes to consist of 70 students. All participants were part of a community-based organisation which provides informal educational assistance to school-age members. The participants were 3 boys and 4 girls, ranging from Grades 1 to 4. Participant details are provided in Table 1. Pseudonyms are used for all participants. The interviewer is a long-time volunteer in the community, and the participants were already familiar with her.

At the time of the interviews, the participants had received instruction in addition and subtraction of whole numbers. The main difference between grade levels with respect to these operations is in the magnitude of the

numbers used, as determined by the Philippine Basic Education Curriculum (Department of Education Bureau of Elementary Education, 2003). Word problems were also included in the curriculum for the purpose of applying computational skills.

Table 1
Participant Details at the Time of the Interview

Name	Sex	Age (years/months)	Grade level
Mila	Female	6/1	1
Ramil	Male	7/1	1
Alma	Female	7/4	2
Teresa	Female	9/0	2
Vivian	Female	9/3	3
Jake	Male	10/0	4
Noel	Male	10/5	4

Although the curriculum documents include word problems as an application of an arithmetic operation, research shows that word problems may be solved using strategies that closely model the problem situation (Carpenter & Moser, 1984; De Corte & Verschaffel, 1987). Thus, the purpose of our interviews was not to determine whether children "applied the correct operation" but to assess their understanding of additive structures on the basis of their strategies.

Instruments

A set of 13 tasks was devised for the interview. Selected tasks are shown in Table 2. Eight of these tasks were constructed to represent the semantic categories of Riley, Greeno, and Heller (1983): Change problems 1-6; Combine problems 1 and 2; and Compare problems 1-6. The eight tasks from Riley et al's categories involved small numbers—all sums were less than 12. Two non-routine tasks which are not solved by applying one arithmetic operation were included to elicit drawn or concrete representations (Kamii, Rummelsburg, & Kari, 2005). Two more tasks involved multidigit numbers. Finally, one Retelling task was included to reveal how the problem text was interpreted and how the problem representation was constructed (Verschaffel, 1994).

An English and Filipino version for each task was created, and back-translated to check for consistency.

Table 2
Selected Interview Tasks

Task	Problem
Retelling task (Compare 3)	Kiko has 7 books. Kikay has 3 more books than Kiko. [*May 7 libro si Kiko. Mas marami ng 3 ang libro ni Kikay kaysa kay Kiko.*]Please retell the problem to me. How many books does Kikay have?
Change 3 (from Riley, Greeno, & Heller)	Rico had 3 rice cakes. Tina gave Rico some more rice cakes. Now, Rico has 8 rice cakes. How many rice cakes did Tina give Rico? [*May 3 puto si Rico. Binigyan pa siya ni Tina ng mga puto. Ngayon, may 8 puto na si Rico. Ilang puto ang binigay sa kanya ni Tina?*]
Compare 1 (from Riley, Greeno, & Heller)	Nato and Tino went with their father to catch some crabs. Nato caught 8 crabs and Tino caught 5 crabs. How many crabs did Nato catch more than Tino? [*Sumama sina Nato at Tino sa kanilang ama para manghuli ng alimango. Nakahuli ng 8 alimango si Nato at nakahuli ng 5 alimango si Tino. Ilang higit na alimango ang nahuli ni Nato sa nahuli ni Tino?*]
Equalise 1 (from Riley, Greeno, & Heller)	Ruben picked 3 guavas. David picked 8 guavas. How many more guavas does Ruben have to pick for him to have equal number of guavas with David? [*Nakapitas si Ruben ng 3 bayabas. Nakapitas si David ng 8 bayabas. Ilan pang bayabas ang kailangang pitasin ni Ruben para magkaroon sila ni David ng parehong dami ng bayabas?*]
Drawing representation task (from Kamii, Rummelsburg & Kari)	People formed a line to buy lunch. I was standing in line and counted 3 people in front of me and 6 people behind me. How many people were in line altogether at that time? [*Pumila ang mga tao para bumili ng pananghalian. Habang ako'y nasa pila, nabilang ko na may 3 tao sa aking harapan at 6 sa aking likuran. Ilan lahat kaming nakapila?*]
Concrete representation task (from Kamii, Rummelsburg & Kari)	I have 12 pieces of candy. If I gave 2 pieces to my mother, 2 pieces to my father, and 2 pieces to my sister, how many pieces would I have left? [*Mayroon akong 12 na kendi. Kung magbibigay ako ng 2 kendi kay nanay, 2 kendi kay tatay, at 2 kendi kay ate, ilang kendi ang matitira sa akin?*]

Procedures

Since all participants were members of a community-based organisation which provides some form of support for its members, it was appropriate to conduct the interviews in the organisation's centre. All interviews were conducted by the first author. Problems were written on cards and presented to the child one at a time. Since reading strategies were included in our analysis, the participants in this study were expected to read the problems on their own. Red and green cubes, unifix cubes, counters, paper, and pencils were available. The interviewer explained to the child that they could use any of these materials if they thought it would help them reach a solution. The child was also asked to explain his or her solutions.

If a child gave an incorrect response, the interviewer provided a Filipino translation of the problem, or suggested the use of blocks, or pen and paper. Whenever possible, children were given as many attempts as they needed to solve the problem, and children moved on to a new task only after having successfully solved the previous one. The assistance provided by the interviewer was intended to determine potential approaches which could facilitate problem solution and inform the design of a subsequent intervention.

In some instances where a child failed to solve a problem, a task (cf. Wright, Stanger, Stafford, & Martland, 2006) involving counters corresponding to the problem's mathematical structure was given. For example, the corresponding task for the Change 3 problem (see Table 2) involved displaying 3 counters, and joining 5 more counters without allowing the child to see how many were added. The child was then informed (in Filipino) that there were now eight counters, and was asked how many counters were added.

During the course of the interviews, some children gave no response after they have read the problems written in English, and some lacked vocabulary knowledge in the English version of the problem. In these cases, they were not asked to attempt a solution and were given the Filipino version instead. Only Filipino versions of the problem were given for all succeeding tasks. All interviews were completed in less than 35 minutes. The entire interview was either audio- or video-recorded, and completely transcribed afterwards.

Results and Analysis

Children's solutions to each problem task were analysed with respect to the language of the problem, reading strategies, and mathematical strategies. We provide summary descriptions followed by a selection of some protocols to illustrate the results.

Language Effects

The results demonstrate that the language of the problem led to differences in children's solutions. First, there was a higher occurrence of non-attempts and errors when problems were given in English. Some problems not solved in English were solved in Filipino. Second, results show difficulties in solving Compare problems. A related finding was that children found Compare 1 more difficult than the Equalise 1 problem, even though both required finding the difference between two disjoint sets. Third, for five of the seven children, only Filipino versions of the problems were presented after one or two tasks because the use of English clearly prevented them from proceeding further along a solution. They either gave no response or did not understand some words in the text.

In Protocol 1, Teresa (Grade 2) gave no response when presented with the English Change 3 problem, "Jose caught eight fishes in the river. Jose gave Lourdes five fishes. How many fishes does Jose have now?" However, she gave a correct solution after she read the Filipino version.

Protocol 1

Teresa:	(starts reading the given problem) Jose (stops on 'caught').
Interviewer:	Caught.
Teresa:	Caught—caught eight fishes in the river. Jose gave Lourdes five fishes. How many fishes does—does Jose have now?
Interviewer:	*Naintindihan mo ba* [Do you understand]? *Yung – alam mo ibig sabihin ng* caught [Do you know what *caught* means]? (child shakes head) *Nakahuli* [caught]. River, *alam mo yung* [do you know] river? (child nods) *O sige* [Alright], Jose caught—
Together:	Jose caught eight fishes in the river.

Teresa:	Jose gave Lourdes five fishes. How many fishes does Jose have now?
Interviewer:	*Kaya mo ba* [Can you do it]? (child nods) *Naintindihan mo na* [Do you understand]? (child nods) *O ilan* [How many]? (9 seconds pause) *Subukan natin Tagalog ha* [Let us try the Filipino version]. *Ito* [Here].
Teresa:	[reads the given problem] *Nakahuli ng walong isda si Jose sa ilog* [Jose caught eight fishes in the river]. *Binigyan ni Jose si Lourdes ng lima – limang isda* [Jose gave Lourdes five – five fishes]. *Ilang isda ang natira kay Jose* [How many fishes does Jose have now]? (one second pause) *Tatlo* [Three].
Interviewer:	O, *tatlo* [Three]. *Ang bilis a* [That was fast]. *Pa'no mo nakuha yung tatlo* [How did you get three]?
Teresa:	*Nag*-minus [Used minus].

Teresa's poor English language knowledge prevented her from solving the problem. However, when the problem was presented in Filipino, she was able to solve the problem without having to resort to using fingers or concrete objects. She immediately recognised that the problem represented a subtraction situation, and accordingly used her knowledge of subtraction facts to solve the problem.

Protocol 1 also suggests that children's limited English vocabulary may explain why there are several non-attempts for English problems. Another possible explanation is that children fail to relate particular words to the problem situation. As Protocol 2 shows, Noel (Grade 4) knew the meaning of the word *change* but in an inappropriate context.

Protocol 2

Noel:	Arman had one hundred pesos in his pocket. He bought a pizza for eighty-four pesos. How much is his change? One hundred eighty-four. (indistinct speech) *Mahirap* [Difficult].
Interviewer:	*Mahirap ba* [Was it difficult]? *Pero naintindihan mo yung tanong* [But do you understand the question]*? Alam mo ibig sabihin ng* change [Do you know what *change* means]?
Noel:	*Opo* [Yes].

Interviewer:	*Ano* [What]?
Noel:	*Papalit* [Swap].
Interviewer:	*Hindi, ano yan* – sukli [No, that's the balance of money returned].
Noel:	*Ay, sukli* [Ah, balance of money returned].
Interviewer:	Ok.
Noel:	Ah, *alam ko na* [Now I know]! [...] Sixteen *sukli niya* [was his change].
Interviewer:	*Sa'n mo naman nakuha ang* sixteen [Where did you get sixteen]?
Noel:	*Kasi yung* eight, *dinagdagan ko ng sampu* [Because I added ten to eight]. *Tapos yung* four, *dinagdagan ko ng anim* [Then I added six to four]. *Para maging* [to make] one hundred.

In the above protocol, Noel initially gave a quick answer but appeared unsure whether it was correct. However, when he was told what *change* meant in the problem context, he understood what to do. This protocol highlights another obstacle which Filipino students face. They are exposed to English primarily in school (Young, 2002). This situation deprives them of a rich opportunity for English vocabulary acquisition. Reading experts (e.g. see Armbruster, Lehr, & Osborn, 2001 for reviews) generally agree that children learn vocabulary when they hear and see words in multiple contexts, especially words with multiple meanings (Nelson & Stage, 2007). As a result, the limited exposure of Filipino children to the English language impedes their word problem solution.

The interviews also showed that there was general difficulty in solving Compare problems in both languages. For example, in the Retelling task, the children either failed to comprehend the problem, and those who did required many attempts. The only exception was Jake who managed to comprehend the Filipino version of the Recall task and solve the Filipino version of the Compare 1 problem. For the rest of the participants, Compare problems were difficult even when translated to Filipino. This finding is consistent with the results we obtained from our written test (Bautista et al., 2009).

The most common form of miscomprehension was interpreting a relative quantity to be an absolute quantity. This happens when children miscomprehend or ignore the comparative term and assign the relative quantity to one person (see Protocol 3). Here, Jake (Grade 4) recalled an English Compare 5 statement in the Retelling task.

Protocol 3

Jake:	(reads the text) The king has 7 gold coins. The queen has 3 more gold coins that, than the king.
Interviewer:	Ok. *Kaya mo bang sabihin sa'kin kung ano ang nakasulat dito* [Can you tell me what is written here]*? Kunwari siya, hindi siya marunong magbasa* [For example, he cannot read] (pointing to a doll). *Kaya mo bang ikwento sa kanya kung anong nakasulat diyan* [Can you tell him (the doll) what is written there]?
Jake:	*Opo* [Yes].
Interviewer:	*O anong nakasulat dito* [What is written here]?
Jake:	*Ang hari ay mayroong pitong, pitong gintong* [The king has seven, seven gold] —
Interviewer:	Ok.
Jake:	Coins.
Interviewer:	Ok.
Jake:	*At ang reyna ay may tatlong* [And the queen has three] gold — *gintong* [gold] coins *mas* [more] than the king.
Interviewer:	Ok. *E kung tinanong ko sa 'yo, ilang* [If I asked you how many] gold coins *meron yung hari* [does the king have]?
Jake:	Seven.
Interviewer:	*E yung reyna* [And the queen]?
Jake:	Three.
Interviewer:	Three. How many gold coins...
Jake:	Does the queen have?
Interviewer:	*Ang sagot mo* [Your answer is]?
Jake:	Three.

In the above protocol, observe that Jake chose to verbally retell the problem in Filipino. He did not have any problem translating the first statement. However, when he tried to retell the second statement, he was only able to translate the phrase, "The queen has three gold," and followed this with the Filipino-English phrase "coins *mas* than the king." Further, the word *mas* [more] is not normally said after a noun, and this makes the translation senseless. He was employing a literal translation, translating words one at a time, and failed to relate the comparative term *more than* to its referents. The result was a nonsensical translation which did not help him understand the text and solve the problem. His final solution was in fact the correct solution to his own interpretation of the problem. He assigned one quantity (7) to the king, and another quantity (3) to the queen. The difficulties may be attributed to language because he was able to solve both the Compare Retelling task and the Compare 1 problem when these were presented in Filipino, but not when these were presented in English.

The only other child who successfully solved a Compare problem (in Filipino) was Teresa. The rest of the children, however, had difficulties. Levi misinterpreted the difference as an assigned quantity. Alma added the two numbers in the problem. Vivian gave no response. These difficulties were not found with the similar Equalise 1 problem. These three children successfully solved the problem, and two of them solved it in their first attempt. The difficulties associated with Compare problems have been found in monolingual students as well. Like the participants in our study, monolingual students perform better when solving Equalise problems than Compare problems (De Corte, Verschaffel, & De Win, 1985). Several researchers (Cummins, 1991; Fan, Mueller, & Marini, 1994) agree that these two problems require the solver to find the difference between two disjoint sets, and that these differ only in the problem text. Cummins (1991) argues that the Compare problems are particularly difficult because children may not have the linguistic knowledge to comprehend comparative language.

Finally, the children's unfamiliarity with the English language is another important finding. It is well-acknowledged that children encounter English only in schools (Young, 2002), and national achievement tests confirm that majority of the students have not mastered English (NSCB, 2007). Our study shows similar results. When problems were presented in English, all participants, except for Jake and Noel (Grade 4), did not attempt to read the text. They may have been overwhelmed by the text, or lacked vocabulary

knowledge of common English words. For example, some specific words which were not understood were "said," "arriving," and "caught." Not only did they have problems with mathematical terms, but also with common English words. For these children, the advantages of being bilingual (e.g. Clarkson, 2007) were not realised because they were not proficient in one language. It was thus necessary to present all succeeding problems in Filipino, so that the children had a better chance of understanding the problem's mathematical structure.

Reading strategies

In this section, we highlight three findings. First, some children had difficulties in decoding English text. This was especially true for the younger participants. Second, the ability to decode text did not guarantee text comprehension, even when problems were given in Filipino. In some instances, children who could not solve a word problem were able to produce a correct solution when the task was presented using counters. Third, reading strategies were rarely used. We now present examples of these general findings.

For some children, it was more difficult to decode English text than it was to decode Filipino text. Protocol 4 shows the difficulties which Ramil (Grade 1) encountered when decoding the English Change 1 problem: "There were 6 people who came to my place for dinner. Seven more people rang up and said they were coming. How many people am I having for dinner?"

Protocol 4

Ramil:	[reads the given problem] There were six—*ano 'to* [what is this]?
Interviewer:	People.
Ramil:	People. (silence)
Interviewer:	Who.
Ramil:	Who. (silence)
Interviewer:	Came.
Ramil:	Came to my ha—*ano 'to* [what is this]?
Interviewer:	Place.

Ramil: Place for din—ner.
Interviewer: Dinner.
Ramil: Dinner. Se—six?
Interviewer: Seven.
Ramil: Seven ma.
Interviewer: More.
Ramil: Pee—
Interviewer: People.
Ramil: People ra-rang up and sa—
Interviewer: Said.
Ramil: Said they were com—
Interviewer: Coming.
Ramil: Coming. Hu—
Interviewer: How.
Ramil: How many pee—
Interviewer: People.
Ramil: People on—am ee
Interviewer: I.
Ramil: I ha-having for dinner?

By contrast, Protocol 5 shows how Ramil read the Filipino version of the same problem.

Protocol 5

Ramil: (reading accurately, but with frequent pauses) *May six na taong dumalo sa bahay ko upang maghapunan. Pitong tao pa ang tumawang – tumawag sa telepono para sabihin – hing darating sila. Ilan lahat ang mga bisita na mag – magsasalo-salo sa hapunan*?

Protocols 4 and 5 highlight the difference in Ramil's ability to decode English and Filipino text. Based on the types of reading miscues identified in the recent Handbook of Reading Assessment (Bell & McCallum, 2008), the 15 miscues in reading the English problem were as follows: four helper-supplied words, three pauses, and eight substitutions. There were 13 miscues in reading the Filipino version: all consisting of pauses, except for one self-correction. Although there was almost the same number of miscues, the types of miscue were completely different. Observe, for example, that Ramil was able to decode the Filipino text without any assistance from the interviewer.

These episodes also suggest one possible reason why Filipino students find it difficult to decode English text. Ramil primarily decodes text one syllable at a time. This may be an adequate strategy when reading Filipino because spelling corresponds very closely to how words are spoken (Gonzales & Rafael, 1981). But English text can only be decoded properly when "the target word is in the learner's oral vocabulary" (NICHHD, 2000, p. 4-3). Since the children in this study are not familiar with many English words, they apply Filipino rules for decoding English text. An example of this is found in Protocol 4. If the word *said* is pronounced the way it was written, it would be read as *sah-eed*–which was how Ramil started to decode this word. Students with decoding difficulties may likewise find word problems difficult because they read "so slowly and laboriously that before they come to the words at the end of the sentence, they have forgotten those at the beginning" (Helwig, Rozek-Tedesco, Tindal, Heath, & Almond, 1999, p. 114).

Presenting the problem in English not only impeded decoding ability but also text comprehension. Protocol 6 shows how Vivian (Grade 3) failed to construct meaning from the Change 3 problem, "Rico had three rice cakes. Tina gave Rico some more rice cakes. Now Rico has eight rice cakes. How many rice cakes did Tina give Rico?"

Protocol 6

Vivian:	*Sulat* [Write]?
Interviewer:	*Hindi pwede mong* [No, you can] (indistinct word) *pwede mo nang – pwede mo nang subukang ano – sagutan* [you can already try to solve]. *Naintindihan mo ba* [do you understand]? (child nods) *Sige nga, ano – anong – Pwede ka*

	mag-drawing, pwede mong gamitin 'to para tulungan kang sumagot [You can draw, you can use any of these to help you solve] (interviewer points to the blocks on the table).
Vivian:	*Anong gagawin ko* [What will I do]?
Interviewer:	*O, kagaya niyan* [Like this]. *Rico had three rice cakes*.
Vivian:	*Susulat ko dito* [Will I write here]?
Interviewer:	*Ikaw, kaya mo na bang sagutin mag-isa* [Up to you, can you already solve on your own]?
Vivian:	(shakes head)

In Protocol 6, Vivian did not know how to proceed. The child offered no response other than to ask whether she needed to write on the paper provided to her. In contrast, when the Filipino version of the problem was given, she was able to recognise that she was being asked how many rice cakes Tina gave Rico. Although Vivian ultimately failed to provide a correct solution, she manifested partial comprehension of the problem. Because she did not provide a correct solution, she was subsequently given a screened task involving counters (cf. Wright et al., 2006) which modelled the problem structure. Vivian was able to solve this task without any observable gesture. When asked how she solved the task, she reasoned that there must have been four additional counters because the initial five counters became nine.

Although Vivian had no difficulties in solving the screened task with the same mathematical structure as the initial problem, she still failed to solve the word problem when it was presented again in Filipino. She repeatedly mentioned that Tina gave Rico eight rice cakes, which is an incorrect interpretation of the text. Failure in comprehending the text is the most likely reason for her inability to solve the problem because she had the ability to solve a similar screened task when it was not embedded in text. Attempts to comprehend the problem text, even when it was read to her, were unsuccessful. The inability to retrieve information from written text was evident when she was asked for the number of rice cakes Rico has at present—she merely had to read the problem because this information was provided. However, she did not look for this information in the text. Instead, she added the two given numbers.

The example above is an illustration of a lack of interaction with the text, even when presented in Filipino. Protocol 70 provides a further illustration

of this lack of interaction. Here, Ramil (Grade 1) was solving the Filipino version of the problem "There are 6 visitors in my house. Seven more people said they were arriving. How many visitors will I have?"

Protocol 7

Ramil:	(reading the problem in Filipino) *May anim na bisita sa aking bahay. Pito (...) tao pa ang nagsabing darating sila. Ilan lahat ang aking mga bisita*?
Interviewer:	*Mm. Kaya* [So]?
Ramil:	*Anim* [Six].
Interviewer:	*Anim ang bisita* [Six visitors].
Ramil:	*Bisita* [Visitors].
Interviewer:	*Ngayon* [Now].
Ramil:	*Ngayon* [Now].
Interviewer:	*E ilan pa daw yung darating* [How many more are coming]?
Ramil:	*Pito, pito* [Seven, seven].
Interviewer:	*O, basahin mo ulit yung tanong* [Read the question again].
Ramil:	*May anim* [There are six]—
Interviewer:	(...) *yung huli, yung huli lang* [Just the last part].

There was no attempt to reread the problem, and an answer (six) was produced immediately. When he was asked to read the question so that he would know what to do with the information, he proceeded to read the text from the beginning. Reading, for him, meant decoding the text once again. There was no attempt to select relevant information or construct meaning from the text.

With the exception of Jake (Grade 4), children read the text only once for every problem (English or Filipino). They did not reread or refer back to the text, except when prompted by the interviewer. When they could not understand what they had read, they either gave no response, or they provided a guess. During the few times they asked questions, these were not meant to clarify meaning but to learn how to decode particular words, as shown in Protocol 4. The only participant who displayed reading strategies was Jake (Grade 4; see Protocol 8).

Protocol 8

Interviewer: *Eto* [Here]. (interviewer shows the word problem)

Jake: (child reads the problem) Ruben and David went to pick some ripe guavas. Ruben picked 3 guavas. David picked 8 guavas. How many more guavas does Ruben have to pick for him to have equal number of guavas with David? (indistinct word) David—*pumunta* [went], *kumuha* [got] *ng guavas si* Ruben, *kumuha ng tatlong* [three] *guavas, si* David, *walo* [eight]. *Ilang* [How many]—how many more—*ilang, ilan pa yung* guavas does Ruben mmm, *maku*—does Ruben have before (indistinct word) uhmm, Ruben got...*tatlo* [three] *nakuha ni Ruben*. David, eight. *Ilan pa daw* [how many] *kail...ni Ruben para ma-magkagaya sila ni* [same with]—equal—*ibig sabihin* [meaning], *magkaparehas* [same] *kay Ruben.* Five.

(To avoid confusion with Jake's own translation, we have translated only selected words in this protocol.)

Protocol 8 shows how Jake constructed meaning from the text. He paused regularly to construct meaning while he was reading the text. He thought aloud and paraphrased the text to facilitate understanding. He reread the text, and pointed to some parts of it to help organise the information. Even without being asked to, he translated the text line by line into Filipino in his effort to make sense of the problem. He did not devise a solution plan as soon as he decoded the text. Rather, he used various reading strategies which facilitated problem comprehension before deciding on a solution strategy. There was evidence of thinking during and after reading. Eventually, he was able to construct an appropriate problem representation and produce the correct solution.

Mathematical Strategies

The strategies used in the children's first solution attempt for each task were recorded in a matrix, and coded using categories developed by Carpenter and Moser (1984) and Fuson et al. (1997). Several common features of the children's preferred strategies were found. First, there were only two occasions where the counting sequence was used. Second, children did not know how the use of blocks or drawings could help them solve the problems.

Third, the strategies children used for multi-digit numbers were indicative of unitary conceptions. It was not uncommon for children to count objects by ones, even for two-digit numbers. Finally, children used their own strategies, and not those encouraged by classroom instruction. We now present protocols and children's work samples to describe these results in more detail.

Only Jake used the counting sequence to solve problems. But although he could do this efficiently for smaller numbers, he could not perform this strategy for larger numbers where he resorted to using concrete objects. The limited use of the counting sequence was also reported by De Corte and Verschaffel (1987), who felt the reason was that the curriculum "entirely disregarded or even discouraged [counting strategies]" (p. 370). The same situation applies in the Philippines—the curriculum jumps directly from the use of physical models to the application of arithmetic operations to solve word problems (Department of Education Bureau of Elementary Education, 2003). Fuson (1992a) argues that it is an erroneous oversimplification to assume that children can move directly from using concrete objects to using number facts.

The second finding was that the children preferred to work mentally, choosing not to use the blocks or drawings, even when these have been shown to help children organise their thinking and solve problems (Fan et al., 1994; Kamii, 2001; Outhred & Sardelich, 2005). Even when the use of blocks was suggested as a way to correct initial errors, some children did not know how the blocks could help them solve the problem, as exhibited in Protocol 9. This interaction occurred after the interviewer suggested the use of blocks following an incorrect response by Teresa (Grade 2).

Protocol 9

Teresa:	*Si Rico ay may tatlong puto* [Rico had three rice cakes].
Interviewer:	*Tatlong puto* [Three rice cakes]. *Sige nga, hanap ka diyan ng pwedeng pakita mo sa'kin ano yung tatlong puto* [Look for something there that can show me three rice cakes] (child gets three green blocks). Ok. *O, lagay mo diyan* [Place it there]. Ok. *Si Rico ay may tatlong puto. Binigyan pa siya ni Tina ng puto. Ngayon, ito yung puto n – nino* [Rico had three rice cakes. Tina gave him some more rice cakes. Now,

	whose are these rice cakes] (interviewer points to the three green blocks)?
Teresa:	Rico.
Interviewer:	Rico. *Ngayon, dahil binigyan siya ni Tina ng puto, mayroon na siyang* [Now, because Tina gave him more rice cakes, he now has]—
Teresa:	*Walo* [Eight].
Interviewer:	*Walong puto* [Eight rice cakes]. Ok. *Ngayon mayroon na siyang walong puto. Sige nga* [Now he has eight rice cakes. Go on] (child gets eight red blocks).

The above protocol shows that when children used blocks, they did not necessarily understand how they could use these to model the quantities and relations in the problem. Here, the interviewer read the problem line by line, allowing Teresa to model the action in the problem in segments. When the interviewer stated that there are now eight rice cakes, Teresa picked eight more blocks instead of adding more blocks to the original set of three blocks. It appears that Teresa used the blocks primarily to represent the two numbers in the problem, and not the relationship between these numbers. This may be interpreted as another form of "number grabbing" where the focus is directed only to the numbers and not to the problem structure.

As well as not using the blocks to represent the problem structure or its solution, using drawings also seemed to be a foreign strategy for the children. It was remarkable that children chose not to draw even in the Drawing Task where they were specifically instructed beforehand to draw. Despite being prompted, many were unsure of what to do.

As an example, consider the case of Ramil (Grade 1) who was prompted to draw when he initially failed to answer the Change 1 problem (refer to Protocol 7). His drawing (Figure 1) did not include any mathematical features, such as tally marks or symbols, and was quite idiosyncratic (Thomas et al., 2002). It appears that his attempt to draw was not focused on finding a solution to the problem. Much effort was placed on drawing superficial features and some additional details which were not given in the text, such as a house with a chimney and a person holding a mobile phone. This idiosyncratic drawing represented the situation, but it did not represent the problem's mathematical structure. "Decorative pictures" such as this, do

not enhance understanding or facilitate problem solution, and may even lead to negative results (Berends & van Lieshout, 2009; Elia, Gagatsis, & Demetriou, 2007).

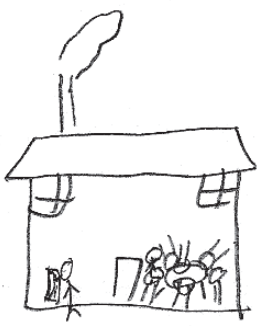


Figure 1. Ramil's drawn representation.

One possible explanation for why the children in this study failed to use blocks or drawings is that the curriculum (Department of Education Bureau of Elementary Education, 2003) emphasises symbolic representations of word problems. Verbal, concrete, or pictorial representations are largely ignored. Children's limited experience with various modes of representation could hinder the development of mathematical strategies for additive word problems because mathematical understanding relies heavily on the flexible use of multiple representations (Lesh, Post, & Behr, 1987; Maclellan, 2001).

The solution procedure recommended by the curriculum is also highly prescriptive. When children solve word problems, they are required to state what is asked and what is/are given, look for the word clues, select the correct arithmetic operation, and transform the word problem into a number sentence. These adult-imposed strategies impede and ignore children's own strategies and compel them to use strategies which may be meaningless to them.

The third finding from the analysis of mathematical strategies is that these strategies were indicative of a unitary conception of multi-digit numbers. For example, Ramil (Grade 1) and Jake (Grade 4) used blocks to solve the multi-digit problems. To find 100 – 84, Ramil constructed a set of 100 blocks, set aside 84 blocks, and counted the remaining blocks to obtain

the answer. Jake employed a similar strategy when solving 42 + 36. He first counted on by six to obtain 48. To proceed from 48, he counted 30 blocks and again counted on from 48 with the aid of the blocks. Both of them counted the blocks one by one, without reference to any grouping. This strategy is indicative of a unitary conceptual structure (Fuson et al., 1997) which leads to error-prone and time-consuming procedures. Indeed, in this study, Ramil miscounted the remaining number of blocks.

Multi-digit problems also prompted Teresa and Vivian to draw sticks or circles which they could count by ones. Figure 2 shows Vivian's solution to the Filipino version of the problem: "The teachers had a meeting. Out of 65 teachers, only 48 attended. How many did not attend?" She drew 65 sticks in rows of ten, and wrote the cumulative number of sticks at the end of each row. Next, she looked for the 48th stick by counting, "ten, twenty, thirty, forty" while she pointed at the numbers in synchrony. When she reached forty, she counted eight more sticks by ones to reach the 48th stick, and marked all the remaining sticks. Finally, she counted the marked sticks by ones to obtain the correct solution to the problem.

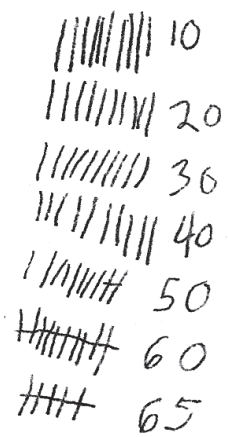


Figure 2. Vivian's solution strategy.

Vivian's solution showed evidence of grouping. Her skip count by tens was indicative of a sequence tens and ones conception. However, she was only able to use the groupings of ten to find the 48th stick, but not to solve 65 – 48. She did not realise that the 51st to 60th sticks formed a group of ten. Instead, she counted the sticks by ones. Thus, Vivian had constructed a partial sequence tens and ones conception.

Vivian's strategy also demonstrated a reliance on visible objects to count. She made sense of the count sequence, *10, 20, 30,..* by relating it to groups of ten sticks which she could perceive. Also, she did not use the economical counting on procedure for these subtraction tasks even when the two given numbers were relatively close to each other. This inefficient strategy suggests a lack of understanding of addition and subtraction as inverse operations.

Finally, episodes from the interviews demonstrate that the children relied on their own strategies rather than on those imposed by classroom instruction. No participant wrote down what was asked, looked for word clues, or wrote a number sentence, as recommended by the curriculum. One can thus question whether it is appropriate to include word problems only as an application of an arithmetic operation, and whether it is appropriate to prescribe a rigid list of steps to follow. Children who solved tasks involving multidigit numbers relied on unitary counting or mental strategies, and not on the standard algorithm taught in the classroom. It is likely that children fail to recognise how the mathematics being taught in class can "contribute to learning and problem solving" (Maclellan, 2001, p. 74).

Implications

Our analysis of children's strategies suggests that minimal interventions such as those used in this study (e.g. providing questioning strategies, suggesting the use of blocks or drawings) do not, by themselves, facilitate problem solution. Efforts need to be made to ensure comprehension of the language of the problem and to enrich the limited range of reading and mathematical strategies.

Our findings raise several considerations for designing an intervention program aimed to help students use efficient strategies to solve addition and subtraction word problems in English. First, because children had difficulties with common English words, vocabulary instruction and engaging children in English conversation must be an integral part of the

program. Although the children in this study were only asked whether they understood individual words in the problems, it is also possible that they may not understand phrases or semantics. To guide the design of an intervention, it would thus be worthwhile to ask children, in future interviews, whether they understand particular phrases or sentences in word problems.

Second, because decoding difficulties make word problem solving unnecessarily more arduous, the intervention must consider how children who still cannot decode text may benefit from mathematics instruction. They would probably learn best by working with oral word problems as a means to develop addition and subtraction concepts (cf. Zhou & Peverly, 2005).

Third, our findings indicate that children do not have a wide range of reading strategies for comprehending English or Filipino text. These reading strategies have to be developed. Modelling effective reading strategies has been shown to improve comprehension (Armbruster et al., 2001) and word problem solving (Foster, 2007). By having reading strategies modelled for them, children could learn how they themselves should interact with the text.

Fourth, the reliance on unitary counting and the absence of advanced strategies such as the use of derived facts highlight the need for developing numeracy. The intervention should provide emphasis on strategies based on the counting sequence (Fuson, 1992a) and on structuring numbers (Wright, Ellemor-Collins, & Lewis, 2007) because these lead to more sophisticated addition and subtraction strategies.

Finally, children's limited use of representations in problem solving reveals the need to provide them with diverse ways of representing word problems. Transforming word problems into symbolic form is not the only strategy to solve problems. Role playing or using drawings or concrete objects to model the action in the problem should be encouraged. Children should also be expected to communicate their strategies and justify their solutions. Further, there is a need to teach children to focus their representations on the problems' mathematical structure rather than on superficial features.

It is conjectured that by providing children with the representational and arithmetical tools outlined above, word problem solving performance would improve. Ultimately, the aim is to determine potential interventions

to help children make sense of word problems and to improve problem solving performance in the early grades, before linguistic and mathematical demands become more complex. An intervention program based on insights from this study is currently being trialled for small-group tutoring of young Filipino children.

Although this study has focused on improving the word problem solving performance of Filipino students, the results also have implications for any context where large numbers of students learn mathematics in a language they do not ordinarily encounter outside school, for example, in Malaysia where students or teachers are not fluent in the second language (Barwell, Barton, & Setati, 2007; Clarkson & Idris, 2006). The present research becomes even more valuable because most studies on mathematics for second-language learners have been focused on immigrant communities in developed countries.

References

Abedi, J., Courtney, M., Leon, S., Kao, J., & Azzam, T. (2006). *English language learners and math achievement: A study of opportunity to learn and language accommodation. Technical report 702*. Los Angeles, CA: National Center for Research on Evaluation, Standards, and Student Testing (CRESST).

Adetula, L. O. (1990). Language factor: Does it affect children's performance on word problems? *Educational Studies in Mathematics, 21*(4), 351-365.

Armbruster, B. B., Lehr, F., & Osborn, J. (2001). Put reading first: The research building blocks for teaching children to read: Kindergarten through grade 3. Retrieved 4 January, 2009 from http://www.nifl.gov/partnershipforreading/publications/PFRbookletBW.pdf

Barwell, R., Barton, B., & Setati, M. (2007). Multilingual issues in mathematics education: Introduction. *Educational Studies in Mathematics, 64*(2), 113-119.

Bautista, D., Mitchelmore, M., & Mulligan, J. (2009). Factors influencing Filipino children's solutions to addition and subtraction word problems. *Educational Psychology, 29*(6), 729-745.

Bell, S. M., & McCallum, R. S. (2008). *Handbook of reading assessment*. Boston, MA: Pearson Education.

Berends, I. E., & van Lieshout, E. C. D. M. (2009). The effect of illustrations in arithmetic problem-solving: Effects of increased cognitive load. *Learning and Instruction, 19*(4), 345-353.

Bernardo, A. B. I. (1999). Overcoming obstacles to understanding and solving word problems in mathematics. *Educational Psychology, 19*(2), 149-163.

Brawner, F. G., Golla, E. F., Ibe, M. D., De Guzman, F. S., Ogena, E. B., Talisayon, V. M., et al. (1999). *TIMSS-Repeat Philippine report, volume 2: Mathematics*. Manila, Philippines: Department of Education, Culture, & Sports, Department of Science and Technology - Science Education Institute, & University of the Philippines National Institute of Science and Mathematics Education.

Carpenter, T. P., & Moser, J. M. (1984). The acquisition of addition and subtraction concepts in grades one through three. *Journal for Research in Mathematics Education, 15*(3), 179-202.

Carteciano, J. A. (2005). NRCP/DOST addresses dire state of S&T education in the regions. Retrieved 23 December, 2008 from http://nrcp.dost.gov.ph/index.php?option=com_content&task=view&id=31&Itemid=106

Clarkson, P. C. (2007). Australian Vietnamese students learning mathematics: High ability bilinguals and their use of their languages. *Educational Studies in Mathematics, 64*(2), 191-215.

Clarkson, P. C., & Idris, N. (2006). Reverting to English to teach mathematics: How are Malaysian teachers and students changing in response to a new language context for learning? *Journal of Science and Mathematics Education in Southeast Asia, 29*(2), 69-96.

Cummins, D. D. (1991). Children's interpretations of arithmetic word problems. *Cognition and Instruction, 8*(3), 261-289.

Dale, T. C., & Cuevas, G. J. (1987). Integrating language and mathematics learning. In J. Crandall (Ed.), *ESL through content-area instruction: Mathematics, science, and social studies.* (pp. 9-54). Englewood Cliffs, NJ: Prentice Hall.

De Corte, E., & Verschaffel, L. (1987). The effect of semantic structure on first graders' strategies for solving addition and subtraction word problems. *Journal for Research in Mathematics Education, 18*(5), 363-381.

De Corte, E., & Verschaffel, L. (1991). Some factors influencing the solution of addition and subtraction word problems. In K. Durkin & B. Shire (Eds.), *Language in mathematical education.* (pp. 117-130). Buckingham, UK: Open University Press.

De Corte, E., Verschaffel, L., & De Win, L. (1985). Influence of rewording verbal problems on children's problem representations and solutions. *Journal of Educational Psychology, 77*, 460-470.

Department of Education Bureau of Elementary Education (2003). *Basic education curriculum (Philippine learning competencies): Mathematics*. Pasig City, Philippines: Author.

Elia, I., Gagatsis, A., & Demetriou, A. (2007). The effects of different modes of representation on the solution of one-step additive problems. *Learning and Instruction, 17*(6), 658-672.

Eunice Kennedy Shriver National Institute for Child Health and Human Development (2000). Report of the National Reading Panel: Teaching children to read: Reports of the subgroups. Retrieved 4 January, 2009 from http://www.nichd.nih.gov/publications/nrp/upload/report.pdf

Fan, N., Mueller, J. H., & Marini, A. E. (1994). Solving difference problems: Wording primes coordination. *Cognition & Instruction, 12*(4), 355-369.

Fogelberg, E., Skalinder, C., Satz, P., Hiller, B., Bernstein, L., & Vitantonio, S. (2008). *Integrating literacy and math: Strategies for k-6 teachers*. New York: Guilford Press.

Foster, S. (2007). The day math and reading got hitched. *Teaching Children Mathematics, 14*(4), 196-201.

Fuson, K. C. (1992a). Research on learning and teaching addition and subtraction of whole numbers. In G. Leinhardt, R. Putnam & R. A. Hattrup (Eds.), *Analysis of arithmetic for mathematics teaching.* (pp. 53-187). Hillsdale, NJ: Lawrence Erlbaum.

Fuson, K. C. (1992b). Research on whole number addition and subtraction. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning.* (pp. 243-275). New York: Macmillan Publishing.

Fuson, K. C., Wearne, D., Hiebert, J. C., Murray, H. G., Human, P. G., Olivier, A. I., et al. (1997). Children's conceptual structures for multidigit numbers and methods of multidigit addition and subtraction. *Journal for Research in Mathematics Education, 28*(2), 130-162.

Goldin, G. A. (2000). A scientific perspective on structured, task-based interviews in mathematics education research. In A. E. Kelly & R. A. Lesh (Eds.), *Handbook of research design in mathematics and science education.* (pp. 517-545). Mahwah, NJ: Lawrence Erlbaum.

Goldin, G. A., & Shteingold, N. (2001). Systems of representations and the development of mathematical concepts. In A. A. Cuoco & F. R. Curcio (Eds.), *The roles of representation in school mathematics (NCTM yearbook).* (pp. 1-23). Reston, VA: NCTM.

Gonzales, A. (1989). The creolization of Philippine English: Evidence for English as a first language among Metro Manila children: Proceedings of the Manila studies conference. In W. V. Villacorta, I. R. Cruz & M. L. Brillantes (Eds.), *Manila: History, people and culture.* (pp. 359-373). Manila, Philippines: De La Salle University Press.

Gonzales, A., & Rafael, T. C. (1981). Transitional reading problems in English in a Philippine bilingual setting. *Reading Teacher, 35*(3), 281-286.

Helwig, R., Rozek-Tedesco, M. A., Tindal, G., Heath, B., & Almond, P. J. (1999). Reading as an access to mathematics problem solving on multiple-choice tests for sixth-grade students. *The Journal of Educational Research, 93*(2), 113-125.

Irwin, K. C., & Irwin, R. J. (2005). Assessing development in numeracy of students from different socio-economic areas: A Rasch analysis of three fundamental tasks. *Educational Studies in Mathematics, 58*(3), 289-298.

Kamii, C. (2001). Representation and abstraction in young children's numerical reasoning. In A. A. Cuoco & F. R. Curcio (Eds.), *The roles of representation in school mathematics (NCTM yearbook).* (pp. 24-34). Reston, VA: NCTM.

Kamii, C., Rummelsburg, J., & Kari, A. (2005). Teaching arithmetic to low-performing, low-SES first graders. *The Journal of Mathematical Behavior, 24*(1), 39-50.

Lesh, R. A., Post, T., & Behr, M. J. (1987). Representations and translations among representations in mathematics learning and problem solving. In C. Janvier (Ed.), *Problems of representation in the teaching and learning of mathematics.* (pp. 33-40). Hillsdale, NJ: Lawrence Erlbaum.

Maclellan, E. (2001). Representing addition and subtraction: Learning the formal conventions. *European Early Childhood Education Research Journal, 9*(1), 73-86.

Mayer, R. E. (2003). Mathematical problem solving. In J. M. Royer (Ed.), *Mathematical cognition*. Greenwich, CT: Information Age Publishing.

National Statistical Coordination Board (2007). Students' scores in achievement tests deteriorating; CARAGA and Eastern Visayas rank highest. Retrieved August 7, 2008, from http://www.nscb.gov.ph/factsheet/pdf07/FS-200705-SS2-01.asp

Nelson, J. R., & Stage, S. A. (2007). Fostering the development of vocabulary knowledge and reading comprehension though contextually-based multiple meaning vocabulary instruction. *Education and Treatment of Children, 30*(1), 1-22.

Outhred, L., & Sardelich, S. (2005). A problem is something you don't want to have. *Teaching Children Mathematics, 12*(3), 146-154.

Pearn, C., & Stephens, M. (2007). Whole number knowledge and number lines help to develop fraction concepts. In J. Watson & K. Beswick (Eds.), *Proceedings of the 30th annual conference of the Mathematics Education Research Group of Australasia.* (Vol. 2, pp. 601-610). Hobart: MERGA.

Riley, M. S., Greeno, J. G., & Heller, J. I. (1983). Development of children's problem-solving ability in arithmetic. In H. P. Ginsburg (Ed.), *The Development of Mathematical Thinking* (pp. 153-196). Orlando, FL: Academic Press.

Secada, W. G. (1991). Degree of bilingualism and arithmetic problem solving in Hispanic first graders. *The Elementary School Journal, 92*(2), 213-231.

Stothard, S. E., & Hulme, C. (1996). A comparison of reading comprehension and decoding difficulties in children. In C. Cornoldi & J. Oakhill (Eds.), *Reading comprehension difficulties: Processes and intervention.* (pp. 93-112). Mahwah, NJ: Lawrence Erlbaum.

Thomas, N. D., Mulligan, J. T., & Goldin, G. A. (2002). Children's representation and structural development of the counting sequence 1-100. *The Journal of Mathematical Behavior, 21*(1), 117-133.

Verschaffel, L. (1994). Using retelling data to study elementary school children's representations and solutions of compare problems. *Journal for Research in Mathematics Education, 25*(2), 141-165.

Verschaffel, L., Greer, B., & De Corte, E. (2007). Whole number concepts and operations. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning.* (pp. 557-628). Charlotte, NC: Information Age Publishing.

Wright, R. J., Ellemor-Collins, D., & Lewis, G. (2007). Developing pedagogical tools for intervention: Approach, methodology, and an experimental framework. In J. Watson & K. Beswick (Eds.), *Proceedings of the 30th annual conference of the Mathematics Education Research Group of Australasia.* (Vol. 2, pp. 843-852). Hobart: MERGA.

Wright, R. J., Stanger, G., Stafford, A. K., & Martland, J. (2006). *Teaching number in the classroom with 4-8 year olds*. London: Paul Chapman Publishing.

Yin, R. K. (2003). *Case study research: Design and methods* (3rd ed.). Thousand Oaks, CA: Sage Publications.

Young, C. (2002). First language first: Literacy education for the future in a multilingual Philippine society. *International Journal of Bilingual Education and Bilingualism, 5*(4), 221-232.

Zhou, Z., & Peverly, S. T. (2005). Teaching addition and subtraction to first graders: A Chinese perspective. *Psychology in the Schools, 42*(3), 259-272.

Authors:

Debbie Bautista, Macquarie University, Sydney, Australia;
e-mail: debbie.bautista@students.mq.edu.au

Joanne Mulligan, Macquarie University, Sydney, Australia

Michael Mitchelmore, Macquarie University, Sydney, Australia

Navigating around marginalizing complexities: the case of mathematics teachers in the Philippines

April 2024 · ZDM: the international journal on m... 56(3)
DOI: 10.1007/s11858-024-01571-1

Catherine P. Vistro-Yu · Debbie Verzosa

Abstract

Marginalization, widely associated with poverty, inequality, and underdevelopment restricts access to resources, limits freedom of choice, inhibits the development of individual capabilities, and makes it challenging to escape from marginalized circumstances. We examine marginality in mathematics teaching and its complexities in the tightly centralized educational system in the Philippines by identifying the different situations that contribute to how mathematics teachers can be excluded from certain educational practices that would normally fall within their mandate as professional teachers. Adopting Gatzweiler and Baumüller's (Marginality: Addressing the nexus of poverty, exclusion and ecology, Springer:27–40, 2014) framework, we interviewed 15 mathematics teachers from different geographical locations around the country. We identified and organized the drivers of marginality into biophysical and societal causal clusters that enable or constrain teachers to act as professionals. Biophysical causal clusters include mathematics curriculum and mathematics assessments among others while societal causal clusters include various directives, interpretations, and realities surrounding the biophysical causal clusters. Discussions about the complexities brought about by these clusters and how teachers act or respond as they navigate around them are highlighted. Recommendations for concrete de-marginalization actions are offered.

Development of a mobile ten frames app for Philippine K-12 schools

November 2020

Conference: 28th International Conference on Computers in Education · At: Online

Debbie Verzosa · Ma. Louise Antonette Navarro De Las Peñas · Jumela Sarmiento · Show all 5 authors · Mark L Loyola

Abstract

This paper reports on the Quick Images app, whose design framework is informed by research on ten-structured thinking and gamification principles. Inclusivity was also a major consideration, especially in the context of a developing country. Thus, the app was made freely available and required only moderate system requirements. Pilot studies revealed that the app has the potential to promote children's ability to see two-digit numbers in relation to tens and ones, which is a major goal of elementary school mathematics. Collaborations with the Philippine Department of Education to ensure the app's sustained use are also discussed.

Community Math Stations as a Learning Opportunity for Preservice Teachers

August 2020 · PRIMUS

DOI: 10.1080/10511970.2020.1809040

Debbie Verzosa

Abstract

This paper describes a project that sought to simultaneously facilitate the mathematical learning of both preservice teachers (PSTs) and schoolchildren. The project involved the organization of community Math Stations related to research-based number sense activities, and was integrated within two university-level courses. This article describes strategies for addressing the challenges associated with organizing such a project, and shows how the project contributed to the learning of PSTs with respect to content knowledge, pedagogical content knowledge, and general pedagogy. Suggestions for organizing a similar project in other settings are also discussed.

Pre-service elementary teachers' reasoning in introducing multidigit subtraction

July 2019 · International Journal of Mathematical Education In Science & Technology
51(3):1-17
DOI: 10.1080/0020739X.2019.1647573

Debbie Verzosa

Abstract

Ideally, mathematical concepts must be justified rather than simply stated. This study examined the reasoning expressed by pre-service elementary teachers when asked to describe how they would introduce the concept of multidigit subtraction to their students. The sample consisted of 230 pre-service elementary teachers from six Philippine universities located in various regions of the country. The reasoning in each response was classified to be of high- or low-quality, on the basis of whether they were anchored to intrinsic mathematical properties. Repeating phrases were also identified and were analysed qualitatively. The findings show that more than 90% of the reasoning produced by teachers relied heavily on rules and procedures, without connections to the quantities involved in the problem. The existence of responses that were repeated across a large number of participants also suggests that the pre-service teachers' reasoning's were influenced by a 'cultural script' they learned over a lifetime of participation in rule-dominated classrooms. The paper concludes by offering recommendations and future directions.

Prospects and Challenges in Implementing a New Mathematics Curriculum in the Philippines

April 2019
DOI: 10.1007/978-981-13-6312-2_11
In book: Cognitive Systems and Signal Processing

Debbie Verzosa · Catherine P. Vistro-Yu

Abstract

The Philippine Department of Education recently introduced a major revamp in the curriculum, providing for an additional two years in basic education. Three provisions of this new program directly relate to mathematics education. First was the shift of language of instruction in early primary education from English to the mother tongue. Second was the development of a new mathematics curriculum that places critical thinking and problem solving as the goal of mathematics education. Third was the extended opportunities for specialization in non-academic tracks. In this chapter, we draw upon studies in the Philippines to examine the issues and concerns that need to be addressed to derive the intended outcomes of the new curriculum. We first provide an overview of curricular changes in the Philippines. Next, we discuss the prospects and warrants of the curricular changes, given that the use of English to teach mathematics has been fraught with coping strategies, and that the relevance of school mathematics has repeatedly been questioned. Finally, we argue that achieving the intended goals is not simple, particularly in resource-poor classrooms where mathematical learning is often viewed as the ability to imitate procedures set forth by the teacher or text. In a developing country like the Philippines, there is a particular need to acknowledge the constraints within the working environment where reforms will take place. A curriculum that offers some prospects for improving mathematics education can only succeed if it follows

App-based scaffolds for writing two-column proofs

July 2018 · International Journal of Mathematical Education In Science & Technology
50(3):1-13
DOI: 10.1080/0020739X.2018.1500654

Debbie Verzosa · Ma. Louise Antonette Navarro De Las Peñas · Alva Aberin · Len Patrick Garces

Abstract

This paper presents apps designed to assist students in understanding and developing proofs in geometric theorems. These technologies focus on triangle congruence, triangle similarity and properties of parallelograms. Focus group discussions and initial testing of the apps revealed that the apps offered a more engaging medium for learning proving and were capable of facilitating proof-writing skills in geometry.

Congruence Between Context and Opportunities for Professional Development of Mathematics Teachers in the Philippines

November 2017
DOI: 10.1007/978-981-10-2598-3_7
In book: Professional Development of Mathematics Teachers

Debbie Verzosa · Maria Theresa Tulao-Fernando · Catherine P. Vistro-Yu

Abstract

Like in many other developing countries, mathematics education in the Philippines is often intertwined with macro problems that arise from the sociopolitical context of schools. We investigate the extent to which preservice and in-service education are able to prepare secondary teachers for teaching mathematics at the level of ordinary classrooms. Our analysis is based on the scholarly literature as well as on in-depth interviews with 22 classroom teachers from 12 of 17 Philippine regions who were accepted in a special credential program. We also discuss the macrostructures that exact considerable influence on classroom teaching.

Reading the World with Calculus

April 2015 · PRIMUS 25(4)
DOI: 10.1080/10511970.2014.978983

Debbie Verzosa

Abstract

It is now increasingly recognized that mathematics is not a neutral value-free subject. Rather, mathematics can challenge students' taken-for-granted realities and promote action. This article describes two issues, namely deforestation and income inequality. These were specifically chosen because they can be related to a range of calculus concepts including rates of change, optimization, Riemann sums, the Fundamental Theorem of Calculus, and partial derivatives. The aim is to give ideas about how an issue may be examined from different mathematical perspectives. The impact of discussing relevant issues in the calculus classroom is described through student reflections and course evaluation.

Using dynamic tools to develop an understanding of the fundamental ideas of calculus

February 2014 · International Journal of Mathematical Education In Science & Technology
45(2):190-199
DOI: 10.1080/0020739X.2013.790513

Debbie Verzosa · Angela Fatima Guzon ·
Ma. Louise Antonette Navarro De Las Peñas

Abstract

Although dynamic geometry software has been extensively used for teaching calculus concepts, few studies have documented how these dynamic tools may be used for teaching the rigorous foundations of the calculus. In this paper, we describe lesson sequences utilizing dynamic tools for teaching the epsilon-delta definition of the limit and the fundamental theorem of calculus. The lessons were designed on the basis of observed student difficulties and the existing scholarly literature. We show how a combination of dynamic tools and guide questions allows students to construct their understanding of these calculus ideas.

Learning to solve addition and subtraction word problems in English as an imported language

February 2013 · Educational Studies in Mathematics 82(2)
DOI: 10.1007/s10649-012-9420-z

Debbie Verzosa · Joanne Mulligan

Abstract

This paper reports an intervention phase of a design study aimed to assist second-grade Filipino children in solving addition word problems in English, a language they primarily encounter only in school. With Filipino as the medium of instruction, an out-of-school pedagogical intervention providing linguistic and representational scaffolds was implemented with 17 children. Pre-intervention, children experienced linguistic difficulties and were limited to conceptualising and solving simple additive structures. Post-intervention interviews revealed improved performance and understanding of more complex structures, but only when linguistic difficulties were minimised. The study identified socially and culturally driven barriers to learning: superficial strategies, children's engagement, and learning in an urban poor context.

CHILDREN SOLVING WORD PROBLEMS IN AN IMPORTED LANGUAGE: AN INTERVENTION STUDY

Debbie Verzosa

Abstract

This paper reports on one aspect of a two-year design study aimed to assist second-grade Filipino children solve additive word problems in English, a language they primarily encounter only in school. With Filipino as the medium of instruction, an out-of-school pedagogical intervention providing linguistic and representational scaffolds was implemented with 17 children. Pre-intervention, children experienced linguistic difficulties and were limited to conceptualising and solving simple additive structures. Post-intervention interviews revealed improved understanding of more complex structures, but only when linguistic difficulties were minimised. Filipino children from disadvantaged families are expected to learn mathematics and solve word problems in English, a language they primarily encounter only in school (Young, 2002). Thus, it is not surprising that many Filipino students who have completed two or three years of schooling are unable to solve even simple addition and subtraction word problems (Bautista, Mitchelmore, & Mulligan, 2009; Bernardo, 1999). While language problems often arise as a cause for poor performance in mathematics (Philippine Executive Report on the TIMSS, cited by Carteciano, 2005), what is not clear is whether lack of English language proficiency is the main reason for Filipino children's poor problem-solving performance. This study attempts to provide insight into these issues by addressing the following research questions: 1. Is the failure to solve problems due to linguistic difficulties and/or to an inadequate understanding of the semantic structure and associated mathematical relationships in the given problem? 2. Is it possible to improve young Filipino children's strategies for solving addition and subtraction word problems