List of Approved Research Proposals for 2025 Implementation.

#	Proponents	Proposed Title	Unit	Themes	Approved Budget
RESEA	ARCH AND DEVELOPMENT OFFI	CE			
"USM	R & D: Transforming Agricultu	ıre, Education, and Digital Systems"			
1	DHEALYN DECEE V. SABIT, REZIN G. CABANTUG, CARMEE LYN B. PAYLANGCO	Mobile App and Sensor Integration for NPK Levels Testing in Real-Time (MoNiTR)	USM-KCC		100,000.00
2	Romiel John Basan	Micro Matters: Assessing the Potential of Developing Microcredential Programs in the University of Southern Mindanao	CBDEM		165,000.00
3	Joseph Lorilla	Intelligent Feedback Management System for University of Southern Mindanao Service Personnel Using Advanced Software Development Framework and Artificial Intelligence Technologies	CEIT		200,000.00
4	John Aries Tabora	Modeling Afforestation Sites: A Decision Support Tool for Sustainable Land Management	CSM		250,000.00
5	MR. LEONARD M. PALETA MR. JUPITER G. PILONGO MR. PHILIP LESTER P. BENJAMIN	Generalized E-torsion Graph; Forcing Subsets for perfect Roman dominating sets in graphs; Convex Graph induced by a Function and a Finite Set	CSM		107,000.00
6	Jayson Baltazar Leizl Gray Oria Marry Grace Balbuena John Aldrin Sanama	Seeds of Innovation: USM's Pursuit of Breakthroughs in Cacao, Rubber, Corn, and Coffee Research	RDO/USMARDC		104,400.00
7	Leizl Gray Oria Jayson Baltazar	Digital Streamlining of USM RDE Initiatives	RDO		72,000.00

8	Kharlo Subrio Rahima Cabunto	METRICS OF SUCCESS OF R&D PROJECTS IN USM; INPUT FOR DATA- DRIVEN PERFORMANCE REVIEW	VPAA/ESO	18,000.00
9	Kharlo Subrio	TRACER STUDY FOR 2020-2023 GRADUATES OF USM	VPAA	46,127.00
10	John Aldrin Sanama	Harnessing AIIDE Integrated Instruction Among USM Faculty	RECO	65,250.00
11	Rahima Cabunto	Refining General Education Standard Through Syllabus Evaluation at USM	ESO	45,000.00
12	Gwen Iris D. Empleo	Genetic Improvement for Increased Yield, Resistance to Pest and Disease and Bean Quality in Cacao	CA	98,673.00

# Innovative Graph Theoretical Models: From E-Torsion to Roman Domination and Function-Based Convexity

LEONARD M. PALETA, PhD
Project Team Leader

PHILIP LESTER P. BENJAMIN
JUPITER G. PILONGO

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#### **ABSTRACT**

# Innovative Graph Theoretical Models: FromE-Torsion to Roman Domination and Function-Based Convexity

Project members: Leonard M. Paleta, PhD
Philip Lester P. Benjamin, PhD
Jupiter G. Pilongo, MS

#### Abstract.

This research explores three novel graph constructions that bridge distinct mathematical disciplines: the generalized e-torsion graph, convex graphs generated by a function and a finite set, and forcing perfect domination in graphs.

In 2024, the notion of (k1, k2) E-torsion graph was first introduced by Pilongo, et. al. They used the graph to represent type-(k1, k2) linear codes over the non-unital ring E. However, such graphs have few examples on small order graphs. In this paper, we will introduce (n, k) torsion graph, a generalization of (k1, k2) E-torsion graph, defined to be a graph G such that |V(G)| = n + k where n vertices have n + k-1 degrees and the k vertices have degree n. Since the order of the graph is not limited only to a power of 2 we can generate more graphs with smaller order that have the same properties as (k1, k2) E-torsion graph. We will also formulate result which immediately follows from the definition such as the behavior of central vertices and the number of edges. This study will also introduce a unary operation of a graph and binary operation of two graphs in order to construct an (n; k) torsion graph. We also provide one of the applications of the (n; k) torsion graph which is the Student-Proctor Communication Model.

The second research introduces a novel class of graphs termed "convex graphs generated by a function and a finite set," denoted as G(f,A). Unlike traditional graph convexity definitions that rely on intrinsic graph properties like paths or intervals, G(f,A) derives its structure extrinsically. Its vertex set is a finite subset of a function's domain, and an edge exists between two vertices if the underlying continuous function exhibits convexity along the segment connecting their corresponding domain points. Key properties of these graphs and some theorems were discussed.

Lastly, we introduced and explored the concept of forcing perfect domination number of graphs (fypG). Building upon the concept of a perfect dominating set—a subset of vertices where every vertex in the graph is dominated by precisely one vertex from the set—this novel graph invariant quantifies the uniqueness of such optimal dominating configurations. The fypG measures the minimum cardinality of a subset required to uniquely identify a minimum perfect dominating set (yp-set).

The study elucidates the definition of fypG through illustrative examples, demonstrating its variability. For instance, a graph with multiple minimum perfect dominating sets, like C4, exhibits a higher fypG (e.g., 2), indicating that more information is needed to distinguish among optimal solutions. Conversely, a graph possessing a unique minimum perfect dominating set yields an fypG of 0, signifying inherent and unambiguous identifiability of its optimal structure. This parameter offers a quantitative measure of the determinism and structural rigidity of graphs concerning their

perfect domination, providing insights into the inherent properties of graph structures and potentially influencing algorithmic design for optimal solution identification and network robustness assessment.

**Keywords:** generalized E-torsion graphs, forcing perfect domination, convex graphs

#### UNIVERSITY OF SOUTHERN MINDANAO



Kabacan, Philippines

	A. BASIC INFORMATION							
1.	Title	E-Torsion to Roman Do	omination and					
2.	Status	<b>x</b> □ Ongoing □ Com	pleted					
3.	Project Leader Study Leader (Indicate College/Unit )	College of Science and Math Study 1: Generalized E-Tors Study Leader: Jupiter G. Pilo College of Science and Math Study 2: Forcing subsets of I Graphs Study Leader: Leonard M. P College of Science and Math Study 3: Convex Graphs indu Set Study Leader: Philip Lester I	nematics  ion Graph ongo, PhD nematics  Perfect Roman Domination in  Paleta, PhD nematics  uced by a Function and a Finite  P. Benjamin, PhD					
	Email Address	plbenj@usm.edu.ph lmpaleta@usm.edu.ph jgpilongo@usm.edu.ph						
	Contact Number	09338245352; 09307606674						
4.	Lead Unit/College	College of Science and Math	nematics					
	Collaborating Unit/College	LEONARD M. PALETA, PhD College of Science and Mathematics  Study 1: Generalized E-Torsion Graph Study Leader: Jupiter G. Pilongo, PhD College of Science and Mathematics  Study 2: Forcing subsets of Perfect Roman Domination in Graphs Study Leader: Leonard M. Paleta, PhD College of Science and Mathematics  Study 3: Convex Graphs induced by a Function and a Finite Set Study Leader: Philip Lester P. Benjamin, PhD College of Science and Mathematics  plbenj@usm.edu.ph Impaleta@usm.edu.ph igpilongo@usm.edu.ph  t Number  09338245352; 09307606674  College of Science and Mathematics  prating Unit/College  n/a  Program    Program   Project   Study   Study   Program   Project   Study   Program   Prototype Development   Extension     Quality Learning, Skills Development, and Literacy   Social Development, and Strong Institutions   Preservation of Culture     Environmental Protection Conservation, and Risk Reduction						
1. Title  E-Torsion to Roman Dominate Function-Based Convexity  2. Status  x□ Ongoing □ Completed  LEONARD M. PALETA, PhD College of Science and Mathematic Study 1: Generalized E-Torsion Grastudy Leader: Jupiter G. Pilongo, P College of Science and Mathematic Study Leader: Jupiter G. Pilongo, P College of Science and Mathematic Study Leader: Leonard M. Paleta, F College of Science and Mathematic Study 2: Forcing subsets of Perfect Graphs Study Leader: Leonard M. Paleta, F College of Science and Mathematic Study 3: Convex Graphs induced by Set Study Leader: Philip Lester P. Benji College of Science and Mathematic Study Leader: Philip Lester P. Benji College of Science and Mathematic Philip Leader: Philip Lester P. Benji College of Science and Mathematic Study Leader: Philip Lester P. Benji College of Science and Mathematic Study Leader: Philip Lester P. Benji College of Science and Mathematic Study Leader: Philip Lester P. Benji College of Science and Mathematic Study Leader: Philip Lester P. Benji College of Science and Mathematic Study Leader: Philip Lester P. Benji College of Science and Mathematic Study Leader: Philip Lester P. Benji College of Science and Mathematic Study Leader: Philip Lester P. Benji College of Science and Mathematic Study Leader: Philip Lester P. Benji College of Science and Mathematic Study Leader: Philip Lester P. Benji College of Science and Mathematic Study Leader: Philip Lester P. Benji College of Science and Mathematic Study Leader: Philip Lester P. Benji College of Science and Mathematic Study Leader: Philip Lester P. Benji College of Science and Mathematic Philip Lester P. Benji College of Science and Mathematic Study Leader: Philip Lester P. Benji College of Science and Mathematic Study Leader: Philip Lester P. Benji College Science and Mathematic Philip Lester P. Benji College of Science and Mathematic Philip Lester P. Benji College of Science and Mathematic Philip Lester P. Benji College of Science and Mathematic Philip Lester P. Benji College of Science and Mathematic Philip	$x\square$ Project $\square$ Study							
6.	Classification	Research	☐ Development ☐ Extension					
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7.	Thematic Area	□ Social Development, and St     □ Preservation of Culture     □ Environmental Protection, G     □ Food Security and Poverty I     □ Good Health and Well-being     X□ Innovations in Science, En	trong Institutions  Conservation, and Risk Reduction  Reduction  g  Igineering, and Technology					

S SOUTHER	☐ No Poverty	☐ Reduced Inequalities				
	☐ Zero Hunger	☐ Sustainable Cities &				
The state of the s	☐ Good Health & Well-Being	Communities				
CCH, COTTAG	☐ Quality Education	☐ Responsible Consumption &				
	NARRATIVE REPORT					
	☐ Affordable and Clean Energy	☐ Life Below Water				
	☐ Decent Work and Economic	☐ Life on Land				
	Growth	☐ Peace, Justice and Strong				
	☐ Industry Innovation &	Institutions				
	Infrastructure	☐ Partnership for the Goals				
9. Project Duration	January1, 2025- December 31, 20	025				
10. Project Location	University of Southern Mindanao					
11. Total Budget Requested (Ph	Php106,978.88	Php106,978.88				

#### **B. TECHNICAL DESCRIPTION**

#### 1. Rationale / Significance

#### **Rationale**

Graph theory is one of the growing research areas in the literature of mathematics since it was first introduced by a great mathematician named Leonhard Euler regarding the problem in his published work involving the Seven Bridges of Konigsberg (Armada & Canoy, 2019). In simple terms, a graph in mathematics represents a network of points connected by lines, showing how they are related. The points are called vertices, and the lines between them are edges. Domination in graphs is a well-known and rapidly growing part of graph theory, with many practical uses (Paleta & Jamil, 2021). For example, it can help solve problems like finding the best bus routes for schools, locating army posts efficiently, designing computer networks, and planning radio station placements. Studying domination in graphs can also help us understand social networks and how relationships between people change over time in different fields. There are many different types of domination, one of which is perfect Roman domination. This concept is useful for solving problems like where to place facilities, how to design communication networks, and how to manage limited resources. On the other hand, the notion of forcing numbers originated from the study of molecular resonance structures, initially introduced by Klein and Randić, and later explored by other mathematicians (Calanza & Rara, 2022). The study of domination in graphs, including perfect Roman domination and forcing subsets, not only enriches the theoretical aspects of graph theory but also finds wide-ranging practical applications in various fields, making it a compelling area for further research and exploration.

Moreover, the concept of vertex-weighted E-torsion graphs represents a specialized area within this field, providing unique insights into graph structures with specific properties.

These structures allow for the analysis and optimization of various systems, making them invaluable in fields such as transportation planning, telecommunications, and social network analysis. By leveraging the power of vertex-weighted E-torsion graphs, researchers and practitioners can uncover hidden patterns, optimize system performance, and make informed decisions for resource allocation and network design. Furthermore, the use of vertex-weighted E-torsion graphs in these applications can lead to more efficient and effective solutions, ultimately improving productivity and enhancing the overall quality of various systems and networks. In summary, vertex-weighted E-torsion graphs have profound implications for real-world applications in network analysis, computer science, and combinatorial optimization (Priyadarsini, 2015).

On the other hand, the concept of a convex graph induced by a function and a finite set is a novel and intriguing idea that has the potential to significantly advance our understanding of convexity and its applications across various mathematical disciplines. The properties and behavior of such graphs could lead to new insights and connections, particularly in the areas of optimization, geometry, and network analysis.

One of the primary benefits of exploring convex graphs is the potential to generalize the concept of convex sets. Convex sets are a fundamental concept in many mathematical fields, including optimization and geometry. By representing convex sets using graphs, this research could provide a framework for understanding and analyzing complex data with inherent convexity structures. This could lead to new tools and techniques for solving optimization problems involving convex sets, which would be particularly valuable in fields like machine learning and data analysis.

Furthermore, the concept of convex graphs could offer a new approach to representing and analyzing problems involving convexity. This might lead to the development of more efficient algorithms for tasks like finding minimum or maximum values in convex sets. The graph structure could also be leveraged to model and analyze specific network dynamics related to convexity properties, which could be crucial in understanding complex systems and networks.

The potential applications of convex graphs are vast and diverse. In data visualization, representing convex sets through graphs could provide a more intuitive and visual way to understand and analyze complex data. This could be particularly useful in fields like finance, economics, and social network analysis, where understanding the relationships and patterns within large datasets is crucial.

In conclusion, the investigation of convex graphs induced by functions and finite sets holds significant promise for advancing our understanding of convexity, developing new algorithms, and bridging connections across different mathematical disciplines. The potential benefits of this research are numerous, and it is likely to have a lasting impact on the fields of optimization, geometry, and network analysis.

#### **Significance**

This project aims to advance the field of graph theory by introducing and exploring the concept of convex graphs. The expected outcomes of this research are likely to have significant impacts on both theoretical and applied mathematics.

#### Objectives (State the General Objectives and Specific Objectives)

General Objective: Introduce and investigate the concepts of generalized E-Torsion graphs, forcing subsets of Perfect Roman Domination in Graphs, and convex graphs generated by a function and a finite set.

#### Specific Objectives:

- 1. Introduce the concepts of generalized E-Torsion graphs, Forcing subsets of Perfect Roman Domination in Graphs, and convex graphs generated by a function and a finite set.
- 2. Discuss their basic properties.

- 3. Investigate the concepts of generalized E-Torsion graphs, forcing subsets of Perfect Roman Domination in Graphs, and convex graphs generated by a function and a finite set of some graph operations.
- 4. Provide applications of these type of graphs.

#### 2. Review of Related Literature

This section presents some of the related literature of the study.

#### 2.1. Generalized E-Torsion Graphs

#### 2.1.1 The ring E and E-Codes

The theoretical underpinnings of generalized e-torsion graphs are deeply rooted in abstract algebra, specifically ring theory and coding theory. These graphs are constructed from linear codes defined over a unique non-unital ring, denoted as E. The ring E is characterized by specific relations:  $E = \langle a, b | 2a = 2b = 0, a^2 = a, b^2 = b, ab = a, ba = b \rangle$ . Notably, E is a non-unital ring, meaning it lacks a multiplicative identity, and it is non-commutative, where the order of multiplication affects the result. It also has a characteristic of two, implying that adding any element to itself yields the additive identity. The multiplication table of E further illustrates its non-commutative and non-unital nature. E is a local ring with a unique maximal ideal  $J = \{0, c\}$ , where c = a + b, and its residue field E/J is the finite field  $F_2$  [17]. The absence of a unity element in this ring presents challenges for traditional concepts like self-duality [1].

Linear E-codes are defined as one-sided E-submodules of  $E^n$ , where n is the code length [17]. Associated with any E-code C are two crucial binary codes:

- Residue Code (res(C)): This code is formed by applying a homomorphism  $\alpha: E \to E/J = F_2$  to the elements of C, reducing them modulo the maximal ideal J [17].
- Torsion Code (tor(C)): This code consists of elements  $x \in F_2^n$  such that  $cx \in C$ , where c = a + b from ring E. It captures elements exhibiting "torsion" behavior relative to the ring structure [17].

These codes are fundamental in coding theory, particularly in the study of self-orthogonal and quasi self-dual (QSD) codes. An E-code C is self-orthogonal if the inner product of any two codewords in C is zero, meaning C is contained within its right and left duals ( $C \subseteq C^{\perp L} \cap C^{\perp R}$ ) [17]. A QSD code is a self-orthogonal E-code with a size of  $2^n$  [17]. A Type IV code is a specialized QSD code where all codewords have an even Hamming weight [1, 17]. These definitions highlight the complex algebraic environment from which e-torsion graphs emerge, pushing the boundaries of conventional coding theory.

#### 2.1.2 Definition and Construction of (k1, k2) E-Torsion Graphs

The (k1,k2) E-torsion graph, denoted as  $G_{EC}$ , provides a graph-theoretic representation derived from linear E-codes [17]. This construction bridges abstract algebraic properties with visual and structural insights.

The graph's components are defined as follows:

- Vertex Set: The vertices of  $G_{EC}$  are the binary codewords of the torsion code of C [17]. For a QSD code  $C = aB + cB^{\perp}$ , the vertices are specifically elements of the torsion code  $B^{\perp}$  [17].
- Edge Set: Edges in  $G_{EC}$  are defined based on the construction rules of E-codes, meaning the algebraic relationships within the E-code structure dictate the graph's topology [17].

This graph construction is presented as a powerful framework for visualizing and understanding complex systems related to linear codes over non-unital rings, facilitating insights into error correction and network coding [17]. The direct mapping from algebraic structures to a graph allows for a more intuitive understanding of code structure and behavior, aiding in extracting valuable information for error correction, network coding, and other relevant areas [17].

#### 2.1.3 Key Properties and Characteristics of (k1,k2) E-Torsion Graphs

Researchers have begun to systematically characterize the structural properties of (k1,k2) E-torsion graphs. For instance, when  $k_1 = 0$  and  $k_2 = 0$ , specific graph characteristics, including vertex degrees and the total number of edges, have been precisely calculated [17].

Necessary and sufficient conditions have been established for a vertex to be in the center of the graph, directly linking these conditions to the algebraic properties of the corresponding codeword [17]. To further differentiate and analyze these graphs, a **vertex-weighted** (k1,k2) E-torsion graph has been introduced. In this variant, each vertex is assigned a weight equal to the Hamming weight of its associated codeword from the torsion code. This weighting helps distinguish between isomorphic graphs generated by algebraically inequivalent E-codes, providing a finer tool for code classification [17].

A significant finding is that if the binary code B (from which the QSD code  $C = aB + cB^{\perp}$  is constructed) is self-dual, then the corresponding (k1,k2) E-torsion graph  $G_{EC}$  is a complete graph [17]. This establishes a direct link between an algebraic property (self-duality) and a fundamental graph-theoretic property (completeness) [17].

#### 2.2. Forcing Subsets of Perfect Roman Domination in Graphs

The concept of domination in graphs has numerous variations, including Roman domination and its perfect variant. This section introduces the definitions of perfect Roman domination and then discusses the concept of forcing subsets as applied to Roman dominating sets.

A dominating set S of a graph G is defined as **perfect** if each vertex of G is dominated by exactly one vertex in S [6]. The **perfect domination number**  $\gamma_p(G)$  is the minimum cardinality of a perfect dominating set of G. A perfect dominating set S with  $|S| = \gamma_p(G)$  is called a  $\gamma_p$ -set of G [6].

The study of perfect dominating sets has explored their existence and construction in various graph families, including trees, dags, and series-parallel graphs [6]. Determining if an arbitrary graph has a perfect dominating set is an NP-complete problem, even when restricted to 3-regular planar graphs [6].

Further research has investigated the perfect dominating polynomial, which is constructed by identifying families of perfect dominating sets with given cardinalities [20]. Variations such as the perfect Italian domination number have also been introduced, exploring relationships with other domination parameters

[4]. The existence of **perfect** (1,2)-dominating sets has been investigated in graphs with specific maximum degrees, noting that graphs with such sets may exhibit symmetric structures [13]. Another variant, the **perfect isolate** dominating set, combines properties of perfect and isolate dominating sets, with its minimum cardinality denoted by  $\gamma_{p0}(G)$  [3].

#### 2.2.1 Perfect Roman Domination

A perfect Roman dominating function (PRDF) on a graph G is a function  $f:V(G) \to \{0,1,2\}$  satisfying the condition that every vertex u with f(u)=0 is adjacent to exactly one vertex v for which f(v)=2 [10, 12, 16]. The weight of a perfect Roman dominating function f, denoted w(f), is the sum of the weights of the vertices,  $w(f) = \sum_{v \in V(G)} f(v)$  [10, 12, 16]. The **perfect Roman domination number** of G, denoted  $\gamma_{pR}(G)$ , is the minimum weight of a perfect Roman dominating function in G [10, 12, 16]. A PRDF f with  $w(f) = \gamma_{pR}(G)$  is called a  $\gamma_{pR}$ -function [16].

#### 2.2.2 Forcing Subsets of Roman Dominating Sets

Building on the concept of Roman domination, the notion of forcing subsets has been introduced to quantify the uniqueness of minimum Roman dominating functions. The concept of forcing domination was initially introduced by Chartrand et al. for general dominating sets [9]. This idea was later extended to Roman domination [18].

A Roman dominating function (RDF) f on a graph G = (V, E) can be represented by a set of ordered pairs  $S_f = \{(v, f(v)) : v \in V\}$  [18]. A subset T of  $S_f$  is called a **forcing subset** for  $S_f$  if  $S_f$  is the unique extension of T to a  $\gamma_R(G)$ -function (a Roman dominating function with minimum weight) [18].

The forcing Roman domination number of  $S_f$ , denoted  $f(S_f, \gamma_R)$ , is defined as the minimum cardinality of such a forcing subset for  $S_f$ :  $f(S_f, \gamma_R) = \min\{|T|: T \text{ is a forcing subset of } S_f\}$  [18].

The forcing Roman domination number of G, denoted  $f(G, \gamma_R)$ , is then defined as the minimum value among all  $f(S_f, \gamma_R)$  for every  $\gamma_R(G)$ -function f of G:  $f(G, \gamma_R) = \min\{f(S_f, \gamma_R) : f \text{ is a } \gamma_R(G)\text{-function}\}$  [18]. It is clear that  $f(G, \gamma_R) \geq 0$  [18].

This concept quantifies the degree of uniqueness of optimal Roman dominating configurations within a graph.

#### 2.3 Convex Graphs generated by a Function and a Finite Set

#### 2.3.1 Foundational Concepts in Convex Analysis

Convex graphs generated by a function and a finite set draw their fundamental principles from convex analysis, a rich area of mathematics with widespread applications [2, 7]. At its core is the concept of a convex function.

**Definition of Convex Functions**: In mathematics, a real-valued function f is defined as convex if, for any two points a and b in its domain and any  $t \in$ , the line segment connecting the points (a, f(a)) and (b, f(b)) on the function's graph lies above or on the graph of f itself. Formally, this property is expressed by the inequality:  $f(at + (1-t)b) \le tf(a) + (1-t)f(b)$  [14, 15]. Geometrically, the graph of a convex function consistently curves upwards, resembling a "cup" shape.

#### **Key Properties of Convex Functions:**

- Preservation under Operations: Convex functions exhibit desirable closure properties under common mathematical operations. For instance, the sum of two convex functions is also convex. Similarly, multiplying a convex function by any non-negative scalar results in another convex function. Linear (or more precisely, affine) functions represent a special case, as they are simultaneously both concave and convex.
- Differentiability Criterion: For functions that are twice-differentiable, convexity can be conveniently characterized by the sign of their second derivative. A twice-differentiable function is convex if and only if its second derivative is non-negative across its entire domain  $(f''(x) \geq 0)$ . This criterion provides a practical and widely used test for verifying convexity.
- Optimization Significance: Convex functions play a profoundly crucial role in optimization theory due to their highly desirable properties. A key advantage is that any local minimum of a convex function is guaranteed to be a global minimum, significantly simplifying the search for optimal solutions. Furthermore, a strictly convex function defined on an open set possesses at most one global minimum, which further streamlines optimization problems by ensuring uniqueness of the solution [2, 14, 15].

#### 2.3.2 Related Notions of Graph Convexity

The term "convexity" in graph theory is not monolithic; it encompasses various definitions, most of which are intrinsic to the graph structure itself.

#### Traditional Graph Convexity Definitions:

- Geodesic (g-) convexity: A subset S of vertices in a graph G is considered g-convex if it contains all vertices lying on any shortest path (geodesic) between any pair of vertices in S [5].
- Monophonic (m-) convexity: A set S is m-convex if it contains every vertex that lies on any induced path between vertices in S [8].
- Convex Geometry: A "convexity" on a set of vertices V (defined as a family of subsets called convex sets) forms a convex geometry if it satisfies the Krein-Milman property. This property states that every convex set is the convex hull of its extreme points [8]. For example, the monophonic alignment of a graph is a convex geometry if and only if the graph is chordal [8].

• Convex Partitions: A graph G is said to be p-convex if its vertex set can be partitioned into p convex sets [5]. Deciding whether a graph is p-convex for a fixed integer  $p \geq 2$  is an NP-complete problem, indicating its computational complexity [5].

#### 3. Methodology

This study employs a structured and rigorous approach to explore and expand the understanding of convex graphs, generalized E-Torsion graphs, and related graph-theoretical concepts such as forcing subsets of perfect Roman domination. The methodology is divided into several key steps, ensuring a comprehensive exploration and formulation of new mathematical results.

The first step involves an extensive literature review to gather relevant information and previous studies related to convex graphs, generalized E-Torsion graphs, domination theory, and graph theory in general. This review will:

- Identify gaps in current research.
- Provide context for the theoretical foundation of the study.
- Ensure that any new results are grounded in existing theory, while extending beyond current knowledge.

Key sources include academic journals, conference papers, textbooks, and relevant online databases that cover graph theory and its applications in optimization, network analysis, and geometry. Once the literature review is complete, the study will focus on the development of mathematical proofs. Both direct and indirect proof techniques will be employed to explore various properties of convex graphs, generalized E-Torsion graphs, and forcing subsets of perfect Roman domination. This step involves establishing the truth of propositions by logical reasoning and known results or utilizing contradiction, contraposition, or induction where necessary to explore less straightforward properties and relationships. This process ensures that each new result is rigorously proven and builds upon prior results in graph theory.

Based on the proofs developed, the next step is the formulation of new theorems, propositions, corollaries, and lemmas. These elements will serve as the foundation for presenting new results in the study. Each result will be carefully reviewed to ensure consistency, logical soundness, and mathematical rigor.

Once the theorems and proofs are fully developed, a clear and comprehensive framework for presenting the results will be established. This framework ensures that the results are communicated effectively and logically. The final step of the methodology involves compiling all the components into

a coherent, well-structured written report. Moreover, outputs of this research will be published to journals in mathematics indexed in Web of Science or Scopus.

#### 4. Results and Discussion

This section elaborates on the core concepts of forcing perfect domination and introduces the novel concept of the forcing perfect domination number, providing detailed examples to illustrate their definitions and implications.

#### 4.1 Forcing Perfect Domination

**Definition 1:** A dominating set S of a graph G is *perfect* if each vertex of G is dominated by exactly one vertex in S. The *perfect domination number*  $\gamma_p(G)$  is the minimum cardinality of a perfect dominating set of G. A perfect dominating set S with  $|S| = \gamma_p(G)$  is called a  $\gamma_p$ -set of G.

**Example 1:** Consider the graph *G* in Figure 1.

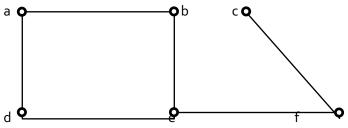


Figure 1: A graph G of order 6

Let  $S_1 = \{a, e, c\}$ ,  $S_2 = \{b, e, f\}$ ,  $S_3 = \{a, f\}$ ,  $S_4 = \{d, e, f\}$ . These sets are all dominating sets. However,  $S_1$  is the only not a perfect dominating set because d and b are adjacent to a and b, and f is adjacent to e and e. The minimum perfect dominating set is  $S_3$  and so  $\gamma_p(G) = |S_3| = 2$ .

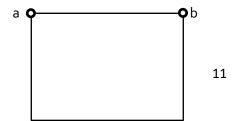
**Definition 2:** Let W be a  $\gamma_p$ -set of a graph G. A subset S of W is said to be *forcing subset* for W if W is the unique  $\gamma_p$ -set containing S. The *forcing perfect domination number* of W is given by

$$f\gamma_p(W) = \min\{|S|: S \text{ is a forcing subset for } W\}.$$

The forcing perfect domination number of *G* is given by

$$f\gamma_p(G) = \min\{f\gamma_p(W): W \text{ is a } \gamma_p - \text{set of } G\}.$$

**Example 2:** Consider the graph  $C_4$  in Figure 2.



 $_{\mathsf{C}}$  **O** Figure 2: A graph  $\mathcal{C}_{4}$ 

Let  $W_1 = \{a,c\}$ ,  $W_2 = \{a,b\}$ ,  $W_3 = \{d,c\}$ ,  $W_4 = \{b,d\}$  be the minimum perfect dominating sets of  $C_4$ . Note that the subsets  $\{a\}$ ,  $\{b\}$ ,  $\{c\}$ ,  $\{d\}$  are NOT forcing subsets since they are contained in at least two minimum perfect dominating sets of  $C_4$ . Thus, the respective sets are forcing subsets of itself, that is,  $W_1$  itself is a forcing subset of  $W_1$ ,  $W_2$  itself is a forcing subset of  $W_2$ ,  $W_3$  itself is a forcing subset of  $W_3$ , and  $W_4$  itself is a forcing subset of  $W_4$ . Hence,  $f\gamma_p(W_1) = f\gamma_p(W_2) = f\gamma_p(W_3) = f\gamma_p(W_4) = 2$ , so that  $f\gamma_p(C_4) = 2$ .

**Example 3:** Consider the graph *G* in Figure 3.

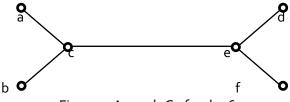


Figure 3: A graph G of order 6

Let  $W = \{c, e\}$  be the unique minimum perfect dominating set of the graph G. Note that the subsets  $\emptyset$ ,  $\{c\}$ ,  $\{e\}$ ,  $\{c, e\}$  are forcing subsets of W since they are contained in the unique minimum perfect dominating set W of the graph G. Hence,  $f\gamma_p(W) = 0$ , so that  $f\gamma_p(G) = 0$ .

#### 4.2 Convex graphs induced by a function and a finite set

In this section, we introduce the concept of convex graphs induced by a function and a finite set.

**Definition 2** Let f be a continuous function and A be a nonempty finite subset of the domain of f, we define the **convex graph induced by a function** f **and a finite set** A, G(f, A), to be the graph whose vertex set is A and for  $a, b \in A$  such that a < b,  $\overline{ab} \in E(G(f, A))$  if for all  $t \in [0, 1]$ 

$$f(at + (1-t)b) \le tf(a) + (1-t)f(b).$$

**Theorem 1** If f is convex, then G(f, A) is a complete graph for all finite subset A of the domain.

*Proof* The proof follows from the definition of G(f, A).

**Theorem 2** Let f be a smooth continuous function and A be a finite subset of the domain. Let  $\sim$  be the relation on A such that  $a \sim b$  if and only if  $\overline{ab}(G(f,A))$ . Then  $\sim$  is symmetric and transitive.

Proof Symmetric property follows immidiately from the definition of G(f, A). Suppose  $a, b, c \in A$  and  $\overline{ab}, \overline{bc} \in E(G(f, A))$ . Then, for all  $t \in [0, 1]$ ,

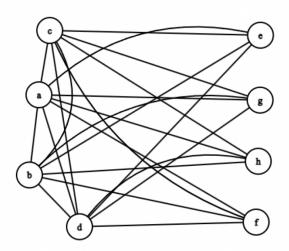
$$f(at + (1-t)b) \le tf(a) + (1-t)f(b)$$
 and  $f(bt + (1-t)c) \le tf(b) + (1-t)f(c)$ .

#### 4.3 Generalized E-Torsion Graphs

This section discusses the concept of generalized E-Torsion graphs. Some examples and theorems will be presented.

**Definition 1.** Let  $k_1, k_2 \in \mathbb{Z}^+$ . A graph G is said to be  $(k_1, k_2)$ -torsion graph if  $|V(G)| = 2^{k_1 + k_2}$  and the degree of  $2^{k_1}$  vertices is equal to  $2^{k_1 + k_2} - 1$  while the degree of the remaining  $2^{k_1 + k_2}$  vertices, if it exist, is equal to  $2^{k_1}$ .

**Example 2.** Let  $k_1 = 2$  and  $k_2 = 1$ . Then (2,1)-torsion graph has 8 vertices where 4 vertices have degree 7 and the other 4 vertices have degree 4. We have this graph G:

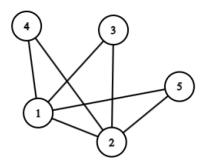


As shown in the graph, the degree of vertices a, b, c and d is 7 and the degree of vertices e, f, g and h is 4. Therefore, G is a (2,1)-torsion graph.

Note that  $(k_1, k_2)$  E-torsion graph contains  $2^{k_1+k_2}$  vertices. This means we have few examples for small orders of such graph. To widen the coverage of such graph with almost the same properties, we introduce the (n, k) torsion graph.

**Definition 3.** Let G be a graph and |V(G)| = n + k, where  $n \in \mathbb{N}$  and k is a nonnegative integer. Then G is said to be a (n,k) torsion graph if there are n vertices whose degree is n + k - 1 and there are k vertices whose degree is n.

**Example 4.** Consider the graph G:



Note that n = 5 and by looking at the graph, there are 2 vertices with degree 5-1=4, or they are connected to all vertices by an edge. This means k = 2. The rest 3 vertices have degree equal to 2 which is equal to k. Hence, G is a generalized E-torsion graph.

Corollary 5. A generalized E-torsion graph is a connected graph.

*Proof.* From the definition, there are vertices that are connected to all other vertices by an edge. Hence, it is connected.  $\Box$ 

**Theorem 6.** If G is a generalized E-torsion graph of order n such that there are k central vertices, then  $|E(G)| = \frac{2nk-k^2-k}{2}$ .

*Proof.* Note that there are k vertices of n-1 degree and there are n-k vertices of k degrees. Hence, the number of edges is

$$|E(G)| = \frac{k(n-1) + (n-k)k}{2}$$
  
=  $\frac{nk - k + nk - k^2}{2}$   
=  $\frac{2nk - k^2 - k}{2}$ .

**Lemma 7.** Let G be a generalized E-torsion graph. Then r(G) = 1.

*Proof.* Note that the k vertices of G is connected to all other vertices by an edge. Hence, the eccentricity of those vertices is 1. Since G is a connected graph, then r(G) = 1.

**Proposition 8.** Let G is a generalized E-torsion graph of order n and  $x \in V(G)$ . Then deg(x) = n - 1 iff x is a central vertex.

*Proof.*  $x \in V(G)$  such that deg(x) = n - 1 if and only if the eccentricity of x is 1. That happends if and only if,  $x \in C(G)$  by Lemma 7. That is if and only if x is a central vertex.

**Corollary 9.** A complete graph is a generalized E-torsion graph.

*Proof.* The complete graph is a special case of generalized E-torsion graph when n = k.

**Proposition 10.** If G is a generalized E-torsion graph, then G is regular iff G is a complete graph.

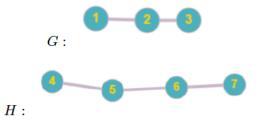
*Proof.* G is regular iff n = k iff all vertices have degree equal to n - 1 iff G is a complete graph.

# 5 Build-up construction of generalized E-torsion graph

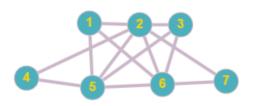
To simplify the construction of generalized E-torsion graph, we define the following operation.

**Definition 11.** Let G and H be graphs such that C(G) and C(H) is their respective center. Then the **central join** of G and H, denoted by  $G +_C H$ , is the graph such that  $V(G +_C H) = V(G) \cup V(H)$  and  $E(G +_C H) = E(G) \cup E(H) \cup E(G + C(H)) \cup E(C(G) + H)$ .

#### Example 12.



Then  $G +_C H$ :



We can see that  $C(G) = \{2\}$  and  $C(H) = \{5,6\}$ . Using the definition of central join of G and H, we connect the vertex 2 to every vertex in H by an edge and the same thing happens for vertex 5 and 6 to the graph G.

Corollary 13. The central join of graphs is commutative.

*Proof.* Follows from the definition.

**Theorem 14.** Let G and H be generalized E-torsion graphs. Then  $G +_C H$  is also a generalized E-torsion graph.

Proof. Let  $|V(G)| = n_1$  and  $|V(H)| = n_2$  such that  $|C(G)| = k_1$  and  $|C(H)| = k_2$ . By the definition of central join, if  $x \in C(G)$ , then the degree of x in  $G +_C H$  is  $n_1 + n_2 - 1$ . Same with  $y \in C(H)$ . This means that there are  $k_1 + k_2$  vertices with degree  $n_1 + n_2 - 1$ . Suppose there exist  $x \in V(G) - C(G)$ , then deg(x) in G is  $k_1$ , thus, in  $G +_C H$ ,  $deg(x) = k_1 + k_2$  from the definition. The same argument for if  $y \in V(H) - C(H)$ . Take  $n = n_1 + n_2$  and  $k = k_1 + k_2$ . This means that  $G +_C H$  is also a generalized E-torsion graph.

Corollary 15. Let G be a generalized E-torsion graph. Then  $G + K_n$  is also a generalized E-torsion graph where  $K_n$  is a complete graph of order n.

*Proof.* Note that  $C(K_n) = V(K_n)$ . Also,  $E(C(G) \cup K_n) \subseteq E(G + K_n)$ . This mean that

$$E(G +_C K_n) = E(G) \cup E(K_n) \cup E(G + C(K_n)) \cup E(C(G) \cup K_n)$$

$$= E(G) \cup E(K_n) \cup E(G + K_n) \cup E(C(G) \cup K_n)$$

$$= E(G) \cup E(K_n) \cup E(G + K_n),$$

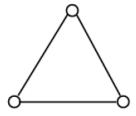
$$= E(G + K_n).$$

We can also construct a generalized E-torsion graph from a graph by using the concept of subdivision of graphs. This method will use the following unary operation of a graph.

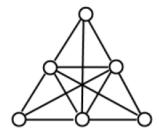
**Definition 16.** Let G be a graph. Then the subdivision semi self-join of a graph G is the graph  $(G)_{ss}$  such that  $V((G)_{ss}) = \bigcup_{e \in E(G)} V(SG(e, 1))$  and  $E((G)_{ss}) = \{xy : x \neq y, x \in V((G)_{ss}) - V(G), y \in V((G)_{ss})\}.$ 

Remark 17.  $V((G)_{ss}) - V(G)$  is the set of new vertices obtained from the subdivision which will be connected to all vertices of  $V((G)_{ss})$  while each original vertex is not connected by an edge to other original vertices.

Example 18. Let G be this graph



Then  $(G)_{ss}$  is



Corollary 19. If G is a graph of order n with m edges, then  $(G)_{ss}$  is of order n + m with  $mn + \frac{m(m-1)}{2}$  edges.

*Proof.* From the definition, the number of additional vertices is the number of edges. Thus,  $|V((G)_{ss})| = n + m$ . Now, note that the obtained the vertices from the subdivision is connected by an edge to n original vertices which means we already have mn vertices. Also, each new vertices will be connected by an edge to m-1 other new vertices. From that, we have  $\frac{m(m-1)}{2}$  additional edges.  $\square$ 

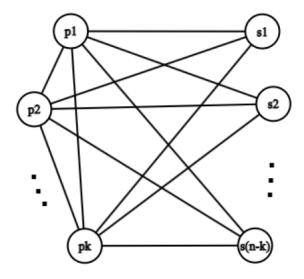
**Theorem 20.** Let G be a graph with at least one edge. Then  $(G)_{ss}$  is a generalized E-torsion graph.

### 6 Graph induced by sets of the form $\{1, 2, \dots, n\}$

Let  $A_n = \{1, 2, \dots, n\}$  and  $m \leq n$ . Then the graph G with  $V(G) = \{A_1, \dots, A_n\}$  and  $(A_i, A_j) \in E(G)$  if and only if  $A_i \cap A_j \subseteq A_m$ , for  $i \neq j$ , is a generalized E-torsion graph.

#### 7 Student-Proctor Communication Model

One of the applications of generalized E-torsion graph is creating a Student-Proctor Communication Model. In an examination, with possibly multiple proctors, students are not allowed to talk to their fellow students, but only to proctors, while proctors can communicate with all of other proctors and all students. If there are k proctors and n-k students, each proctor is connected to n-1 communication tools. On the other hand, each student is connected to only k communication tools. This problem can be modelled using the following graph:



p's corresponds to the proctors and s's corresponds to the students. The communication model obtained corresponds to a generalized E-torsion graph.

#### 5. Conclusion and Recommendation

#### 6. Accomplishment (6 Ps)

6Ps	Description (example)
Publication	1 publication drafted
Patent	1 patent applied
Products	2 products (product 1, product 2)
People Services	Number of people benefited
Place and Partnership	MOA drafted
Policies	1 policy drafted

Indicate the accomplishment of each study of the project or each component of the study.

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#### 8. Problems Met and Recommended Action

-unable to process personal services (honoraria) on schedule due to delay in SO as attachment

Recommended action: Submit request earlier for SO processing

#### 9. **Attachment**s:

Attachment A – Supplementary Table/Figure, Photo documentation

Attachment B- Budget Utilization

Component	Allocation	Utilized	% Utilized
Office Supplies	6,311.16	6,311,16	100%
Other supplies	7,151.27	7,151.27	100%
Technical and scientific equipment	40,916.45	40,916.45	100%
Representation	5,000	1,500	30%
Communication	17,600	8,800	50%
Personal Services (Honoraria)	30,000	0	o%
Total	106,978.88	64,678.88	60.46%

Attachment C - Workplan



#### UNIVERSITY OF SOUTHERN MINDANAO

Kabacan, Philippines

#### **WORK PLAN SCHEDULE**

	Innovative Graph Theoretical Models: From
TITLE:	E-Torsion to Roman Domination and
	Function-Based Convexity
COLLECTION DEPARTMENT (LINUT)	College of Science and Mathematics
COLLEGE/DEPARTMENT/UNIT:	Department of Mathematics and Statistics
	Study 1: Generalized E-Torsion Graph
	Study Leader: Jupiter G. Pilongo, PhD
PROPONENT(S):	Study 2: Forcing subsets of Perfect Roman Domination in Graphs Study Leader: Leonard M. Paleta, PhD
	Study 3: Convex Graphs induced by a Function and a Finite Set Study Leader: Philip Lester P. Benjamin, PhD

Total Duration (in months)	Duration (in months) 12		January 2025		Planned En	d Decembe	r 2025			
				Schedule of Activities						
Objectives	Expected Outputs	Activiti	Activities		Year 1					
					2 <sup>nd</sup> Quarter	3 <sup>rd</sup> Quarter	4 <sup>th</sup> Quarter			
Introduce the concepts of generalized E-Torsion graphs, Forcing subsets of Perfect Roman Domination in Graphs, and convex graphs generated a function and a finite set.	Torsion graphs, Forcing subs	books		Search for relevant articles through published	Define the concepts of generalized E- Torsion graphs, Forcing subsets of Perfect					

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Kabacan, Philippines

#### **WORK PLAN SCHEDULE**

	generated by a function and a finite set.	Define the concepts of generalized E-Torsion graphs, Forcing subsets of Perfect Roman Domination in Graphs, and convex graphs.  Provide examples of generalized E-Torsion graphs, Forcing subsets of Perfect Roman Domination in Graphs, and convex graphs	journals, books Literature review	Roman Domination in Graphs, and convex graphs.  Provide examples of generalized E- Torsion graphs, Forcing subsets of Perfect Roman Domination in Graphs, and convex graphs		
Discuss basic properties	Basic properties are established.	Establish basic properties of generalized E-Torsion graphs, Forcing subsets of Perfect Roman Domination in Graphs, and convex graphs.		Establish basic properties of generalized E-Torsion graphs, Forcing subsets of Perfect Roman Domination in Graphs, and convex graphs.		
Investigate the concepts of generalized E-Torsion graphs, forcing subsets of Perfect Roman Domination in Graphs, and convex graphs generated by a	Provide some important theorems.	State and prove theorems related to generalized E-Torsion graphs, Forcing subsets of Perfect Roman Domination in Graphs, and convex graphs.			State and prove theorems related to generalized E- Torsion graphs,	

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Kabacan, Philippines

#### **WORK PLAN SCHEDULE**

function and a finite set of some graph operations.				Forcing subsets of Perfect Roman Domination in Graphs, and convex graphs.	
Provide applications of these type of graphs.	Applications of the graphs introduced are provided.	Provide applications of generalized E-Torsion graphs, Forcing subsets of Perfect Roman Domination in Graphs, and convex graphs.			Provide applications of generalized E-Torsion graphs, Forcing subsets of Perfect Roman Domination in Graphs, and convex graphs.

#### For Faculty and Staff Researchers

I hereby declare and confirm with my signature that the REPORT is exclusively the result of my own autonomous work based on my research and literature published, which is referenced immediately after the information is presented and listed in the reference section. I also declare that no part of the work submitted has been made in an inappropriate way, whether by plagiarizing, infringing on any third person's copyright, or falsifying data. Finally, I declare that no part of the REPORT submitted has been used for any other paper in another higher education or research institution.

Printed Name and Signature	
Date	



### UNIVERSITY OF SOUTHERN MINDANAO Kabacan, Philippines



#### **BUDGET SUMMARY**

TITLE: Innovative Graph Theoretical Models: FromE-Torsion to Roman Domination and Function-Based Convexity

PROPONENTS: Leonard Paleta, Philip Lester Benjamin, Jupiter Pilongo

FUND CLUSTER: Fund 01 BUDGET ALLOCATION: 106, 978.88

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Prepared By:

LEONARD M. PALETA, PhD

Project Leader

Cartified By

Director, RDO

Reviewed b

SHERBEN MAE P. VILLARUZ

Recommending.

EIMER M. ESTILLOSO

VP for Administration & Finance

Debie Marie Versoza

VP for Research and Extension

Approved By:

JONALD L. PIMENTEL, PhD

President

USM-RES-F11-Rev.1.2020.02.18

#### UNIVERSITY OF SOUTHERN MINDANAO

<u>Kabacan, Cotabato</u> Project Procurement Management Plan

			PROJECT P	ROCUREMENT	MANAG	EMENT	PLAN (	PPMP	1							
END-USER/UNIT:		СЅМ						LEON	ARD M.	PALETA,	PhD					
Charge	d to Fund:		Fund 0	1	377135											
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PART I.	Procurement of Common Supplies (CSE) through PS-DBM (Please refer PPMP-CSE Part I attached)		8,298.88	NP-53.5 Agency-to- Agency								i i				
II.C	Printers Consumables		6,080.00			Mariting.	Section.				E CONTRACT	THE REAL	MARCH			
	Epson L120/L220/L210/L220/L121/L360/L310 Ink, Black 664	10	3,800.00		10											
	Epson L120/L220/L210/L220/L121/L360/L310 Ink, Cyan 664	2	760.00		2											
	Epson L120/L220/L210/L220/L121/L360/L310 Ink, Magenta 664	2	760.00		2											
	Epson L120/L220/L210/L220/L121/L360/L310 Ink, Yellow 664	2	760.00		2											
11.Q	Procurement of Semi-expendable I.C.T. Equipment (less than P50,000.000 per unit)		40,000.00													
20.00m	Printer 3in1 with ADF	2	40,000.00	Competitive Bidding	2	1022-2	2, 2 2	Land State	Dawn.	The Samuel	I Market	Carlo mai	San Contract	Acres 100	Harris de la constante de la c	1
II.AC	Procurement of Communication Supplies and Accessories		17,600.00	Competitive Bidding					(SAN)		1 2000	<b>Light</b>				
II.AF	Cell Cards (smart)	32	17,600.00	Competitive Bidding	8	A SHOW WAT	51-		В		8	3		3 Especia	8	6
	Payment of other professional services		30,000.00	Park - Sept	A CONTRACT						1 3500	(47 mg)				
	Other Professional Services		30,000.00		3000	3000	3000	300	0 3000	3000	3000	300	0	300	30	00
II.AO	Other Maintenance and Operating Expenses		5,000.00					White-			1 Marian y	1000				
	Payment of Representation		5,000.00		S PERSON		T. Comment				1		a receive			
TOTAL BU	DGET:	=	106,978.88	-												
	TIMATED BUDGET:		106,978.88													

Certified by:

MLEONARD M.PALETA, PhD Department/College/Project Head

## UNIVERSITY OF SOUTHERN MINDANAO PROJECT PROCUREMENT MANAGEMENT PLAN 2025

HOTE: PLEASE HIDE COLUMNS WITH NO "TOTAL AMOUNT FOR THE YEAR" ENTRIES BEFORE PRINTING

Department/College/Project: Department Head/College Dean/Project Leader: Contact Person (if different from Head):	LEONARD M.F	CSM  LEONARD M PALETA, PhD  PHILIP LESTER P. BENJAMIN, PhD			Funding Agency (External):         Fund 01           Contact Number:         09338245352           Contact Number:         09338245352									
Item & Specifications	. Unit of Measure	. Unit of Measure			Monthly Quantity Requirement							Total Quantity for the year	Price Catalogue (as of 30 June 2024 based on	
		Jan Feb Mar Q1	, Q1 TNUOMA	April May June Q2	Q2 AMOUNT	Q2 MOUNT July Aug Sept		Q3 AMOUNT	Oct Nov Dec O		4 Q4 AMOUNT		PS-DBM APP- CSE for 2025)	
PART I. AVAILABLE AT PS-DBM (MAIN WAREHOUSE AND DEPOTS) MOOE				A CONTRACTOR OF THE PARTY OF TH	All and									
Other Supplies and Materials Expenses	THE REAL PROPERTY OF THE PERSON OF THE PERSO		SAME DE LA COMPANIE D	CAN BE STOLD TO BE STOLD THE STOLD TO	SECRETARIA DE LA COMPANSION DE LA COMPAN	A STATE OF THE STA	ESTABLISHED IN	DE GUELLE SAN					distribution of the second	1235 TEA
ALCOHOL OR ACETONE BASED ANTISEPTICS	The second secon	·····································	KONTHAND PROMISE	· 数据中间内数据 人名西莫特 中国语言的	Charles and the second	<b>可能的基本的</b> 的现在分词	NAME OF PERSONS	CALL SON AND AND AND AND AND AND AND AND AND AN	Sandahirin "Wilsolan"	Panylight UNITED	TO SHEET SHEET AND	THE PERSON NAMED IN		of the Constitution
2 12191601-AL-E03 ALCOHOL, Ethyl, 1 Gallon	gallon	3 0 0 3	1,071.27	0 0 0 0	0.00	0 0 0	0	0.00	0 0	0 0	0.00	3	357.09	1,071.2
Office Supplies	THE RESIDENCE OF THE PARTY OF T	经过多的证明的证明的证明的	A STATE OF THE STA	THE PROPERTY OF STREET OF STREET	A CONTRACTOR	<b>罗罗以</b> 安克斯·阿拉	SECURIOR SE	光点的 计程序字 医红	<b>设置的原始联系</b>	門用書物	CO SCHOOL STANDS			Contract of the
ARTS AND CRAFTS EQUIPMENT AND ACCESSORIES AND SUPPLIES		Manager of the Control of the Contro	STATE OF THE STATE	And the state of t	Anadaside		G-1/00 500 70	(4) 图图 \$100 年间	AND THE REAL PROPERTY.	55 to 60 K	AND DESCRIPTION OF THE PARTY OF	A STREET, ST. OF ST.	SECTION SECTION	40 00000
10 60121524-SP-006 SIGN PEN Fine Tip Blue	piece	50 0 0 50	2,236.00	0 0 0 0	0.00	0 0	0	0.00	0 0	0 0	0.00	50	44.72	2,236.
INFORMATION AND COMMUNICATION TECHNOLOGY (ICT) EQUIPMENT AND DEVICES AND ACCESSOR	RIES	Surger to The State of State o	ELLYMPIA STORAGE	<b>网络</b> 美国人民国共和国人民国共和国人民国	and the end of petal	REPRODUCTION CO.	The state of the same	STATE OF THE PARTY	iorition states	The Control of the		WEATHER DAY	CALCULATION OF THE PARTY OF THE	and the same
54   43202010-FD   FLASH DRIVE, 64 GB capacity	unit	5 0 0 5	916.45	0 0 0 0	0.00	0 0	0	0.00	0 0	0 0	0.00	5	183.29	916.
OFFICE EQUIPMENT AND ACCESSORIES AND SUPPLIES	(1) 10 · 10 · 10 · 10 · 10 · 10 · 10 · 10	THE STREET STREET, STR	第19年3月2日本東京日本の大阪	The second of the State of the	A south or the last	STATE OF THE PARTY	12000000000	A STATE OF THE PARTY OF		MANUS AND	SOLIDER LINE	AND DESCRIPTION OF THE PERSON		Manager St.
81 44121801-CT-R02 CORRECTION TAPE, 8 meters	piece	10 0 0 10	168.80	0 0 0 0	0.00		0	0.00	0 0	0 0	0.00	10	16.88	168.
85 44103202-DS-M01 DATER STAMP	piece	1 0 0 1	543.69	0 0 0 0	0.00		0	0.00	0 0	0 0	0.00	1	543.69	543.
110 44121708-MW-B02 MARKER Whiteboard, Blue	piece	50 0 0 50	603.00	0 0 0 0	0.00	0 0	0	0.00	0 0	0 0	0.00	50 50	12.06	543. 603. 603.
111 44121708-MW-803 MARKER, Whiteboard, Red	piece	50 0 0 50	603.00	0 0 0 0	0.00		0	0.00	0 0	0 0	0.00	3	55.89	167.
116 44121706-PE-L01 PENCIL, lead/graphile, with eraser, one (1) dozen per box	box	3 0 0 3	167.67	0 0 0 0	0.00		0	0.00	0 0	0 0	0.00	3	247.00	741
122 44121615-ST-S01   STAPLER, standard type	piece	3 0 0 3	741.00	0 0 0 0	0.00	0 0	U	0.00	0 0	0 1 0 1	0.00	-	247.00	/41
PAPER MATERIALS AND PRODUCTS	A STATE OF THE STA	AND CONTRACTOR OF THE PARTY OF	a polyment and a solution		0.00	T 0 T 0 T	** 27.44 minus	0.00	10101	0 1 0 1	0.00	10	124.80	1,248
	pack	10 0 0 10	1,248.00	0 0 0 0	0.00   0	0 0	0	0.00	10101	0 1 0 1	0.00	10	1 124.00	1,240
142 14111704-TT-P02 TOILET TISSUE PAPER, 2 pty, 12 rolls in a pack														

PHILIPLESTED D. REN IAMIN PHO

Date Prepared:

Certified by:

LEONARD M.PALETA, PhD

Department/College/Project Head

JONALD DEIMENTEL

President

SPECIAL BUDGET Fund 05/06 CY 2025

Agency	:	UNIVERSITY OF SOUTHERN MINDANAO	
College/Institute		Fund 05	
Source of Fund :		Tuition & Other School Fees	
(Pls. Check if 164)		Laboratory Fees	
		Other Fees	
Breakdown		e Brought Forward	
	CT 20.	o - 1st Quarter - 2nd Quarter	
		3rd Quarter	
	Total /	- 4th Quarter Amount Proposed	106,978.88
		EXPENDITURE PROGRAM	
MAINTENANCE & OT	HER OP	ERATING EXPENSES (MOOE)	Amount
	Trave	ling Expenses	
	Schol	ng Expenses arship	
	Suppli	es & Materials	54,378.88
		Office Supplies Expenses Accountable Forms Expenses	6,311.16
		Food Supplies Expenses Medical, Dental and Laboratory Supplies Expenses	
		Fuel, Oil and Lubricants Expenses	
		Agricultural and Marine Supplies Expenses Textbooks and Instructional Materials Expenses	
		Semi-Expendable Expenses- Office Equipment	•
		Semi-Expendable Expenses- ICT Equipment Semi-Expendable Expenses- Medical Equipment	40,916.45
		Semi-Expendable Expenses- Printing Equipment	
		Semi-Expendable Expenses- Sports Equipment Semi-Expendable Expenses- Technical & Scientific Equipment	
		Semi-Expendable Expenses- Other Equipment	
		Semi-Expendable Furniture & Fixtures Semi-Expendable Books	
		Other Supplies and Materials Expenses	7,151.27
	Utility	Expenses Water Expenses	
		Electricity Expenses	
	Comn	nunication Expenses Postage & Deliveries	17,600.00
		Telephone Expenses	17,600.00
	Cable	Internet Subscription Expenses , Satellite, Telegraph and Radio Expenses	
	Other	Professional Services	30,000.00
		rial Services ity Services	
	Other	General Services	The second secon
		rs and Maintenance - Infrastructure Assets rs and Maintenance - Buildings and Other Structures	
	Repai	rs and Maintenance - Machinery and Equipment	
	Repai	rs and Maintenance - Transportation Equipment rs and Maintenance - Furniture and Fixtures	
	Renai	rs and Maintenance - Other Property, Plant and Equipment	
	RM-	rs and Maintenance - Semi-expendable Machinery and Equ Semi-expendable Furniture and Fixtures	upment
	RM-S	Semi-expendable Other Property, Plant and Equipment	
	Taxes	cial Assistance/Subsidy Duties and Licenses	
		y Bond Premiums	
		nce Expenses Maintenance and Operating Expenses	5,000.00
	TOTA	LMOOE	106,978.88
CAPITAL OUTLAY			
		Land Improvements Infrastructure	
	Buildin	gs	
		Buildings Structures	
	Machin	nery	
		Equipment juipment	
	Medica	al equipment	
		g Equipment Equipment	
	Techn	ical & Scientific Equipment	
		Machinery & Equipment Vehicles	
		re & Fixtures	
	Other	r Books Property, Plant & Equipment	
	Patent	s/Copyrights uter Software	
		CAPITAL OUTLAY	
			106,978.88
	GKAN	DIOTAL	
pared by:			Submitted by:
- Mos	)	nu nu n	WLEONARD M.PALETA, PhD
LIP LESTER P.	BENJAN	IN, PNU	Department/College/Project Head